First formulation of SLAM

Alberto Quattrini Li

2015-09-22

University of South Carolina

What is SLAM?

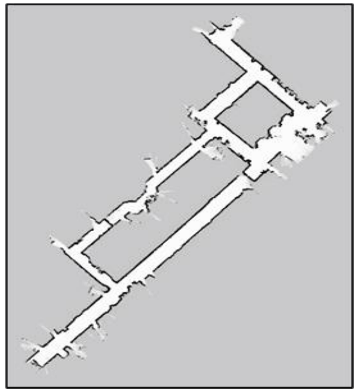
- **Localization**: estimating the robot’s location
- **Mapping**: building a map
- **SLAM**: computing the robot’s pose and the environment map simultaneously

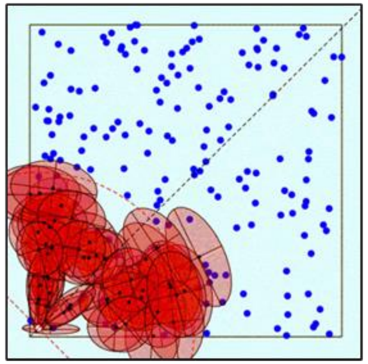
Definition of the SLAM problem

- Input:
  - Robot's controls
    - $u_{1:T} = \{u_1, u_2, \ldots, u_T\}$
  - Observations
    - $z_{1:T} = \{z_1, z_2, \ldots, z_T\}$
- Output
  - Poses of the robots
    - $x_{0:T} = \{x_0, x_1, x_2, \ldots, x_T\}$
  - Map of the environment
    - m

Map representations

- Grid-based
  - Occupancy grid with typically fixed resolution
- Landmark-based
  - The map consists of a set of isolated landmarks
  - A landmark is described, e.g., by a pose location wrt a frame





Landmark-based SLAM

- The robot learns the locations of the landmarks while localizing itself
- State variables
  - Robot pose
  - Coordinates of each of the landmarks
- The problem involves different aspects
  - Landmark extraction
  - Data association
  - State estimation
  - State update
  - Landmark update

First formulation of SLAM

- Smith et al. [1990] present
  - *Stochastic map*: representation for spatial relationships between objects
  - A set of procedures for
    - Reading information from it
    - Building/updating it

Spatial Relationship

- A *spatial relationship* is represented by the vector of its *spatial variables*: e.g., the position and orientation of one, in the frame of reference of the other in 2D.

$$\mathbf{x_1} = \mathbf{x} = \begin{bmatrix} x_1 \\ y_1 \\ \phi_1 \end{bmatrix}$$

Uncertain spatial relationship in 2D

- An *uncertain spatial relationship* is represented by a *probability distribution* over its spatial variables, e.g., with a mean and a covariance matrix.

$$\hat{\mathbf{x}} \triangleq E(\mathbf{x}) \qquad \mathbf{C}(\mathbf{x}) \triangleq E((\mathbf{x}-\hat{\mathbf{x}})(\mathbf{x}-\hat{\mathbf{x}})^{\mathsf{T}})$$

$$\hat{\mathbf{x}} = \hat{\mathbf{x}}_1 = \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{\phi} \end{bmatrix}, \quad \mathbf{C}(\mathbf{x}) = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{x\phi} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{y\phi} \\ \sigma_{x\phi} & \sigma_{y\phi} & \sigma_\phi^2 \end{bmatrix}$$

Stochastic map

- A stochastic map models n uncertain spatial relationships with the *system state vector* (all spatial variables wrt world reference frame) and with the associated *system covariance matrix*

$$\mathbf{x} = \begin{bmatrix} \mathbf{x_1} \\ \mathbf{x_2} \\ \vdots \\ \mathbf{x_n} \end{bmatrix} \qquad \hat{\mathbf{x}} = \begin{bmatrix} \hat{\mathbf{x}}_\mathbf{1} \\ \hat{\mathbf{x}}_\mathbf{2} \\ \vdots \\ \hat{\mathbf{x}}_\mathbf{n} \end{bmatrix} \qquad \mathbf{C}(\mathbf{x}) = \begin{bmatrix} \mathbf{C}(\mathbf{x_1}) & \mathbf{C}(\mathbf{x_1}, \mathbf{x_2}) & \cdots & \mathbf{C}(\mathbf{x_1}, \mathbf{x_n}) \\ \mathbf{C}(\mathbf{x_2}, \mathbf{x_1}) & \mathbf{C}(\mathbf{x_2}) & \cdots & \mathbf{C}(\mathbf{x_2}, \mathbf{x_n}) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}(\mathbf{x_n}, \mathbf{x_1}) & \mathbf{C}(\mathbf{x_n}, \mathbf{x_2}) & \cdots & \mathbf{C}(\mathbf{x_n}) \end{bmatrix}$$

where $\mathbf{C}(\mathbf{x_i}, \mathbf{x_j}) \triangleq E((\mathbf{x_i} - \hat{\mathbf{x}}_\mathbf{i})(\mathbf{x_j} - \hat{\mathbf{x}}_\mathbf{j})^\intercal)$

$$\mathbf{C}(\mathbf{x_j}, \mathbf{x_i}) = \mathbf{C}(\mathbf{x_i}, \mathbf{x_j})^\intercal$$

Estimating the first two moments of unknown multivariate probability distributions

- Consider the non-linear mapping $\mathbf{y} = \mathbf{f}(\mathbf{x})$
- approximate using Taylor Series

$$\mathbf{y} = \mathbf{f}(\hat{\mathbf{x}}) + \mathbf{F}_{\mathbf{x}}(\mathbf{x} - \hat{\mathbf{x}}) + \cdots,$$

where

$$\mathbf{F}_{\mathbf{x}} \triangleq \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}}(\hat{\mathbf{x}}) \triangleq \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_r}{\partial x_1} & \frac{\partial f_r}{\partial x_2} & \cdots & \frac{\partial f_r}{\partial x_n} \end{bmatrix}_{\mathbf{x}=\hat{\mathbf{x}}}.$$

- The first-order estimate of the mean: $\hat{\mathbf{y}} \approx \mathbf{f}(\hat{\mathbf{x}}).$

And the first-order estimate of the covariances:

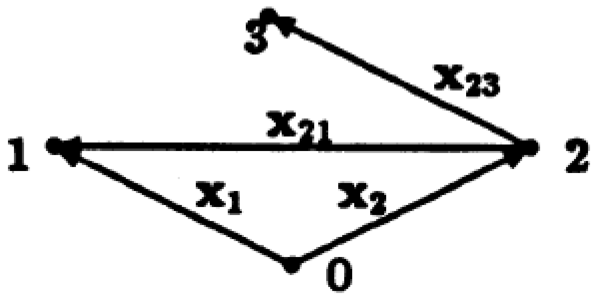
$$\begin{aligned} \mathbf{C}(\mathbf{y}) &\approx \mathbf{F}_{\mathbf{x}}\mathbf{C}(\mathbf{x})\mathbf{F}_{\mathbf{x}}^T, \\ \mathbf{C}(\mathbf{y},\mathbf{z}) &\approx \mathbf{F}_{\mathbf{x}}\mathbf{C}(\mathbf{x},\mathbf{z}), \\ \mathbf{C}(\mathbf{z},\mathbf{y}) &\approx \mathbf{C}(\mathbf{z},\mathbf{x})\mathbf{F}_{\mathbf{x}}^T. \end{aligned}$$

How to read from the map

- In a real system, it is useful to get the information from the stochastic map wrt a different frame than the world frame
  - e.g., motion of the robot or its observations wrt robot’s frame
- Estimate the resultant relationship between initial and final frames
  - Compounding operation
  - Reversal operation

Compounding operation

- For example, given $_{02}$ and x_{23} how do we compute the resultant relationship x_{03}?



Compounding operation

- Given two spatial relationships $\mathbf{x}_{ij}$ and $\mathbf{x}_{jk}$, calculate the resultant relationship $\mathbf{x}_{ik}$

$$
\begin{aligned}
\mathbf{x}_{ik} &\triangleq \mathbf{x}_{ij} \oplus \mathbf{x}_{jk} \\
&= \begin{bmatrix} x_{jk}\cos\phi_{ij} - y_{jk}\sin\phi_{ij} + x_{ij} \\ x_{jk}\sin\phi_{ij} + y_{jk}\cos\phi_{ij} + y_{ij} \\ \phi_{ij} + \phi_{jk} \end{bmatrix}
\end{aligned}
$$

Compounding operation

- The first-order estimate of the mean of the compounding operation is

$$\hat{\mathbf{x}}_{ik} \approx \hat{\mathbf{x}}_{ij} \oplus \hat{\mathbf{x}}_{jk}$$

- And the first-order estimate of the covariance is

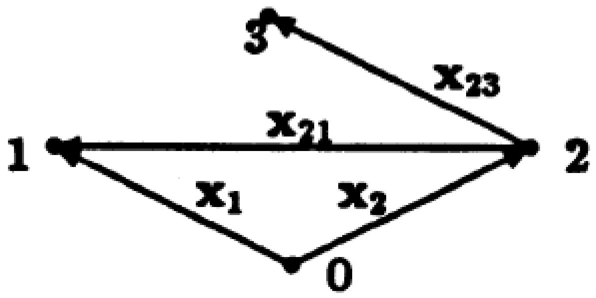
$$\mathbf{C}(\mathbf{x}_{ik}) \approx \mathbf{J}_{\oplus} \begin{bmatrix} \mathbf{C}(\mathbf{x}_{ij}) & \mathbf{C}(\mathbf{x}_{ij}, \mathbf{x}_{jk}) \\ \mathbf{C}(\mathbf{x}_{jk}, \mathbf{x}_{ij}) & \mathbf{C}(\mathbf{x}_{jk}) \end{bmatrix} \mathbf{J}_{\oplus}^{T}$$

where

$$\mathbf{J}_{\oplus} \triangleq \frac{\partial(\mathbf{x}_{ij} \oplus \mathbf{x}_{jk})}{\partial(\mathbf{x}_{ij}, \mathbf{x}_{jk})} = \frac{\partial \mathbf{x}_{ik}}{\partial(\mathbf{x}_{ij}, \mathbf{x}_{jk})} = \begin{bmatrix} 1 & 0 & -(y_{ik} - y_{ij}) & \cos\phi_{ij} & -\sin\phi_{ij} & 0 \\ 0 & 1 & (x_{ik} - x_{ij}) & \sin\phi_{ij} & \cos\phi_{ij} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Reversal operation

- For example, how to compute x_{21}? We need first x_{20}



Reversal operation

- Given $\mathbf{x}_{ij}$, calculate $\mathbf{x}_{ji}$

$$\mathbf{x}_{ji} \triangleq \ominus \mathbf{x}_{ij} \triangleq \begin{bmatrix} -x_{ij}\cos\phi_{ij} - y_{ij}\sin\phi_{ij} \\ x_{ij}\sin\phi_{ij} - y_{ij}\cos\phi_{ij} \\ -\phi_{ij} \end{bmatrix}$$

- The estimate of the mean

$$\hat{\mathbf{x}}_{ji} = \ominus \hat{\mathbf{x}}_{ij}$$

- The estimate of the covariances

$C(x_{ji}) \approx \mathbf{J}_{\ominus}\, C(x_{ij})\, \mathbf{J}^T{}_{\ominus}$

$$\mathbf{J}_{\ominus} \triangleq \frac{\partial \mathbf{x}_{ji}}{\partial \mathbf{x}_{ij}} = \begin{bmatrix} -\cos\phi_{ij} & -\sin\phi_{ij} & y_{ji} \\ \sin\phi_{ij} & -\cos\phi_{ij} & -x_{ji} \\ 0 & 0 & -1 \end{bmatrix}$$

Composite Operations

- Compounding and reversal operations can be combined to compute any sequence of relationships
- Recursive head-to-tail $\mathbf{x}_{il} = \mathbf{x}_{ij} \oplus \mathbf{x}_{jl} = \mathbf{x}_{ij} \oplus (\mathbf{x}_{jk} \oplus \mathbf{x}_{kl}) =$

  $= \mathbf{x}_{ik} \oplus \mathbf{x}_{kl} = (\mathbf{x}_{ij} \oplus \mathbf{x}_{jk}) \oplus \mathbf{x}_{kl}$
  - Compounding operation is associative, but not commutative
- Combine compounding and reversal operations (head-to-head)

$$\mathbf{x}_{ij} \ominus \mathbf{x}_{kj} = \mathbf{x}_{ij} \oplus (\ominus \mathbf{x}_{kj})$$

- Tail-to-tail combinations come from observing two things from the same point: $\mathbf{x}_{jk} = (\ominus \mathbf{x}_{ij}) \oplus \mathbf{x}_{ik}$

Composite Operations

- To estimate the mean of a complex relationship, just solve the estimate equations recursively
- e.g., tail-to-tail

$$\hat{\mathbf{x}}_{jk} = \hat{\mathbf{x}}_{ji} \oplus \hat{\mathbf{x}}_{ik} = \ominus \hat{\mathbf{x}}_{ij} \oplus \hat{\mathbf{x}}_{ik}$$

$$\mathbf{C}(\mathbf{x_{jk}}) \approx \mathbf{J}_{\oplus} \begin{bmatrix} \mathbf{C}(\mathbf{x_{ji}}) & \mathbf{C}(\mathbf{x_{ji}}, \mathbf{x_{ik}}) \\ \mathbf{C}(\mathbf{x_{ik}}, \mathbf{x_{ji}}) & \mathbf{C}(\mathbf{x_{ik}}) \end{bmatrix} \mathbf{J}_{\oplus}^{\mathsf{T}} \approx \mathbf{J}_{\oplus} \begin{bmatrix} \mathbf{J}_{\ominus}\mathbf{C}(\mathbf{x_{ij}})\mathbf{J}_{\ominus}^{\mathsf{T}} & \mathbf{J}_{\ominus}\mathbf{C}(\mathbf{x_{ij}}, \mathbf{x_{ik}}) \\ \mathbf{C}(\mathbf{x_{ik}}, \mathbf{x_{ij}})\mathbf{J}_{\ominus}^{\mathsf{T}} & \mathbf{C}(\mathbf{x_{ik}}) \end{bmatrix} \mathbf{J}_{\oplus}^{\mathsf{T}}$$

General spatial relationship

- For any spatial relationship among world locations

$$\mathbf{y} = \mathbf{g}(\mathbf{x})$$

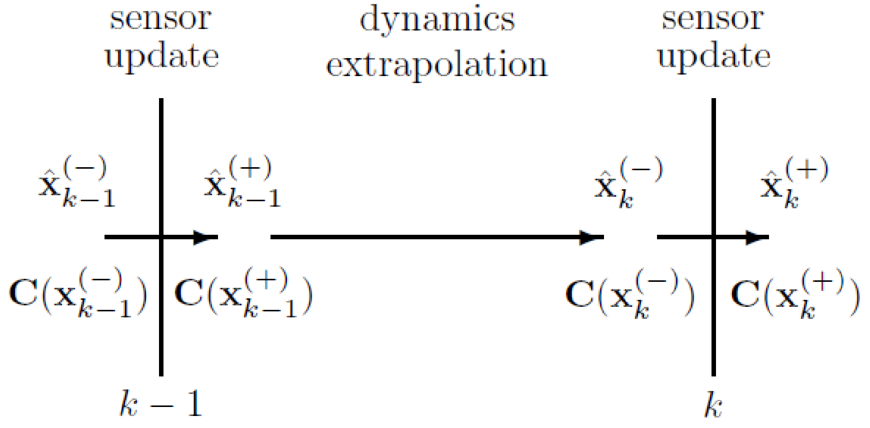
- The estimated mean and covariance of the relationship

$$\hat{\mathbf{y}} \approx \mathbf{g}(\hat{\mathbf{x}})$$

$$\mathbf{C}(\mathbf{y}) \approx \mathbf{G_x}\mathbf{C}(\mathbf{x})\mathbf{G_x}^\intercal$$

Build/update the map

- The map changes when
  - An object (e.g., the robot) moves
  - New spatial information is obtained
- Assumption
  - New spatial information is obtained at discrete moments k and is instantaneous
  - As an object moves, no measurements of external objects are made



Moving object

- The system dynamics model is given by

$$\mathbf{x_k^{(-)}} = \mathbf{f}(\mathbf{x_{k-1}^{(+)}}, \mathbf{y_{k-1}})$$

where

$$\mathbf{y_{k-1}} = \mathbf{u_{k-1}} + \mathbf{w}$$

$$\mathbf{\hat{y}_{k-1}} = \mathbf{u_{k-1}}$$

$$\mathbf{C}(\mathbf{y_{k-1}}) = \mathbf{C}(\mathbf{w})$$

Moving object

- Given the estimates of the state vector and variance matrix at state *k-1*

$$\hat{\mathbf{x}}_{\mathbf{k}}^{(-)} \approx \mathbf{f}(\hat{\mathbf{x}}_{\mathbf{k-1}}^{(+)}, \hat{\mathbf{y}}_{\mathbf{k-1}})$$

$$\mathbf{C}(\mathbf{x}_{\mathbf{k}}^{(-)}) \approx \mathbf{F}_{(\mathbf{x},\mathbf{y})} \begin{bmatrix} \mathbf{C}(\mathbf{x}_{\mathbf{k-1}}^{(+)}) & \mathbf{C}(\mathbf{x}_{\mathbf{k-1}}^{(+)}, \mathbf{y}_{\mathbf{k-1}}) \\ \mathbf{C}(\mathbf{y}_{\mathbf{k-1}}, \mathbf{x}_{\mathbf{k-1}}^{(+)}) & \mathbf{C}(\mathbf{y}_{\mathbf{k-1}}) \end{bmatrix} \mathbf{F}_{(x,y)}^{\intercal}$$

$$\mathbf{F}_{(\mathbf{x},\mathbf{y})} = \begin{bmatrix} \mathbf{F}_{\mathbf{x}} & \mathbf{F}_{\mathbf{y}} \end{bmatrix} \triangleq \frac{\partial(\mathbf{f}(\mathbf{x},\mathbf{y}))}{\partial(\mathbf{x},\mathbf{y})}(\hat{\mathbf{x}}_{\mathbf{k-1}}^{(+)}, \hat{\mathbf{y}}_{\mathbf{k-1}})$$

Moving object

- The robot makes an uncertain relative motion

$$\mathbf{x}'_{\mathbf{R}} = \mathbf{x}_{\mathbf{R}} \oplus \mathbf{y}_{\mathbf{R}}$$

- Thus, only a small portion of the map should be updated

$$\hat{\mathbf{x}}'_{\mathbf{R}} \approx \hat{\mathbf{x}}_{\mathbf{R}} \oplus \hat{\mathbf{y}}_{\mathbf{R}}$$

$$\hat{\mathbf{x}}^{(+)}_{\mathbf{k-1}} = \left[\begin{array}{c} \\ \hline \hat{\mathbf{x}}_{\mathbf{R}} \\ \hline \\ \end{array}\right] \qquad \hat{\mathbf{x}}^{(-)}_{\mathbf{k}} = \left[\begin{array}{c} \\ \hline \hat{\mathbf{x}}'_{\mathbf{R}} \\ \hline \\ \end{array}\right]$$

$$\mathbf{C}(\mathbf{x}^{(-)}_{\mathbf{k}}) = \left[\begin{array}{c|c|c} & \mathbf{C}(\mathbf{x}, \mathbf{x}'_{\mathbf{R}}) & \\ \hline \mathbf{C}(\mathbf{x}'_{\mathbf{R}}, \mathbf{x}) & \mathbf{C}(\mathbf{x}'_{\mathbf{R}}) & \\ \hline & & \\ \end{array}\right]$$

$$\mathbf{C}(\mathbf{x}'_R, \mathbf{x}_i) \approx \mathbf{J}_{1\oplus}\mathbf{C}(\mathbf{x}_R, \mathbf{x}_i)$$

New spatial information (1)

- New object is added to the map

$$\hat{\mathbf{x}}^{(+)} = \left[\begin{array}{c} \hat{\mathbf{x}}^{(-)} \\ \\ \hline \hat{\mathbf{x}}_{\mathbf{n+1}} \end{array}\right] \qquad \mathbf{C}(\mathbf{x}_{\mathbf{k}}^{(-)}) = \left[\begin{array}{c|c} \mathbf{C}(\mathbf{x}^{(-)}) & \mathbf{C}(\mathbf{x}, \mathbf{x}_{\mathbf{n+1}}) \\ \hline \mathbf{C}(\mathbf{x}_{\mathbf{n+1}}, \mathbf{x}) & \mathbf{C}(\mathbf{x}_{\mathbf{n+1}}) \end{array}\right]$$

If independent of the estimates of other object locations

$$\mathbf{x}_{\mathbf{n+1}} = \mathbf{x}_{\mathrm{new}}$$
$$\hat{\mathbf{x}}_{\mathbf{n+1}} = \hat{\mathbf{x}}_{\mathrm{new}}$$
$$\mathbf{C}(\mathbf{x}_{\mathbf{n+1}}) = \mathbf{C}(\mathbf{x}_{\mathrm{new}})$$
$$\mathbf{C}(\mathbf{x}_{\mathbf{n+1}}, \mathbf{x}_{\mathbf{i}}) = \mathbf{C}(\mathbf{x}_{\mathrm{new}}, \mathbf{x}_{\mathbf{i}}) = \mathbf{0}$$

Otherwise

$$\mathbf{x}_{\mathbf{n+1}} = \mathbf{g}(\mathbf{x}, \mathbf{z})$$
$$\hat{\mathbf{x}}_{\mathbf{n+1}} = \mathbf{g}(\hat{\mathbf{x}}, \hat{\mathbf{z}})$$
$$\mathbf{C}(\mathbf{x}_{\mathbf{n+1}}) = \mathbf{G}_{\mathbf{x}}\mathbf{C}(\mathbf{x})\mathbf{G}_{\mathbf{x}}^{\intercal} + \mathbf{G}_{\mathbf{z}}\mathbf{C}(\mathbf{z})\mathbf{G}_{\mathbf{z}}$$
$$\mathbf{C}(\mathbf{x}_{\mathbf{n+1}}, \mathbf{x}_{\mathbf{i}}) = \mathbf{C}(\mathbf{x}_{\mathbf{n+1}}, \mathbf{x}_{\mathbf{i}})$$
$$\mathbf{C}(\mathbf{x}_{\mathbf{n+1}}, \mathbf{x}) = \mathbf{G}_{\mathbf{x}}\mathbf{C}(\mathbf{x})$$

New spatial information (2)

- An already-existing object is sensed, thus some constraints are added to the existing relationships
- The measurement

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{v}.$$

- The expected value of the sensor value and its covariance

$$\hat{\mathbf{z}} \approx \mathbf{h}(\hat{\mathbf{x}}).$$

$$\mathbf{C}(\mathbf{z}) \approx \mathbf{H_x C(x) H_x^{\mathit{T}}} + \mathbf{C}(\mathbf{v}),$$

where

$$\mathbf{H_x} \triangleq \frac{\partial \mathbf{h}_k(\mathbf{x})}{\partial \mathbf{x}} \left(\hat{\mathbf{x}}_k^{(-)} \right)$$

New spatial information (2)

- For example, if the sensor measures the relative location of the observed object

$$\mathbf{z} = \mathbf{x}_{21} = \ominus\mathbf{x}_2 \oplus \mathbf{x}_1.$$

$$\hat{\mathbf{z}} = \hat{\mathbf{x}}_{21} = \ominus\hat{\mathbf{x}}_2 \oplus \hat{\mathbf{x}}_1.$$

$$\mathbf{C}(\mathbf{z}) = {}_{\ominus}\mathbf{J}_{\oplus} \begin{bmatrix} \mathbf{C}(\mathbf{x}_2) & \mathbf{C}(\mathbf{x}_2, \mathbf{x}_1) \\ \mathbf{C}(\mathbf{x}_1, \mathbf{x}_2) & \mathbf{C}(\mathbf{x}_1) \end{bmatrix} {}_{\ominus}\mathbf{J}_{\oplus}^T + \mathbf{C}(\mathbf{v})$$

- Given the sensor model, the Kalman filter equations can be used for updating the state estimate

$$\hat{\mathbf{x}}_k^{(+)} = \hat{\mathbf{x}}_k^{(-)} + \mathbf{K}_k \left[\mathbf{z}_k - \mathbf{h}_k(\hat{\mathbf{x}}_k^{(-)})\right],$$

$$\mathbf{C}(\mathbf{x}_k^{(+)}) = \mathbf{C}(\mathbf{x}_k^{(-)}) - \mathbf{K}_k \mathbf{H}_{\mathbf{x}} \mathbf{C}(\mathbf{x}_k^{(-)}),$$

$$\mathbf{K}_k = \mathbf{C}(\mathbf{x}_k^{(-)}) \mathbf{H}_{\mathbf{x}}^T \left[\mathbf{H}_{\mathbf{x}} \mathbf{C}(\mathbf{x}_k^{(-)}) \mathbf{H}_{\mathbf{x}}^T + \mathbf{C}(\mathbf{v})_k\right]^{-1}.$$

Example

a) The robot starts from $[0,0,0]$ coinciding the world reference frame origin
b) The robot senses object #1.
c) The robot moves.
d) The robot senses a different object #2.
e) Now the robot senses object #1 again.

Step a)

- The stochastic map is initialized

$$\hat{\mathbf{x}} = [\hat{\mathbf{x}}_R] = [0]$$

$$\mathbf{C}(\mathbf{x}) = [\mathbf{C}(\mathbf{x}_R)] = [0]$$

Step b)

- Object #1 is sensed and added to the stochastic map

$$\hat{\mathbf{x}} = \begin{bmatrix} \hat{\mathbf{x}}_R \\ \hat{\mathbf{x}}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \hat{\mathbf{z}}_1 \end{bmatrix}$$

$$\mathbf{C}(\mathbf{x}) = \begin{bmatrix} \mathbf{C}(\mathbf{x}_R) & \mathbf{C}(\mathbf{x}_R, \mathbf{x}_1) \\ \mathbf{C}(\mathbf{x}_1, \mathbf{x}_R) & \mathbf{C}(\mathbf{x}_1) \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}(\mathbf{z}_1) \end{bmatrix}$$

Step c)

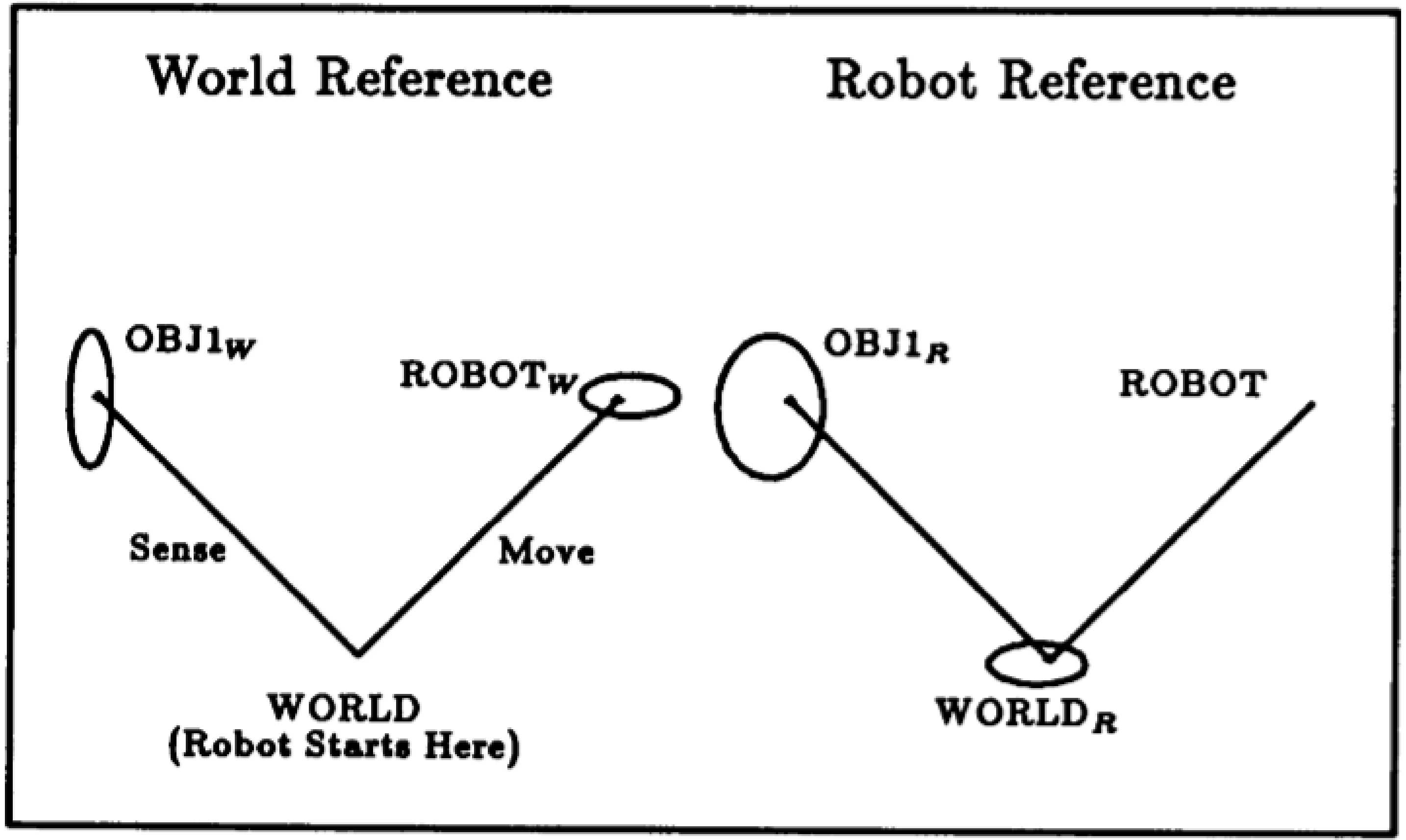
- The robot moves and so the entry related to the robot is updated

$$\hat{\mathbf{x}} = \begin{bmatrix} \hat{\mathbf{x}}_R \\ \hat{\mathbf{x}}_1 \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{y}}_R \\ \hat{\mathbf{z}}_1 \end{bmatrix}$$

$$\mathbf{C}(\mathbf{x}) = \begin{bmatrix} \mathbf{C}(\mathbf{x}_R) & \mathbf{C}(\mathbf{x}_R, \mathbf{x}_1) \\ \mathbf{C}(\mathbf{x}_1, \mathbf{x}_R) & \mathbf{C}(\mathbf{x}_1) \end{bmatrix} = \begin{bmatrix} \mathbf{C}(\mathbf{y}_R) & \mathbf{0} \\ \mathbf{0} & \mathbf{C}(\mathbf{z}_1) \end{bmatrix}$$

Object #1 wrt robot frame

$$OBJ1_R = (\ominus ROBOT_W) \oplus OBJ1_W$$
$$= WORLD_R \oplus OBJ1_W$$



Step d)

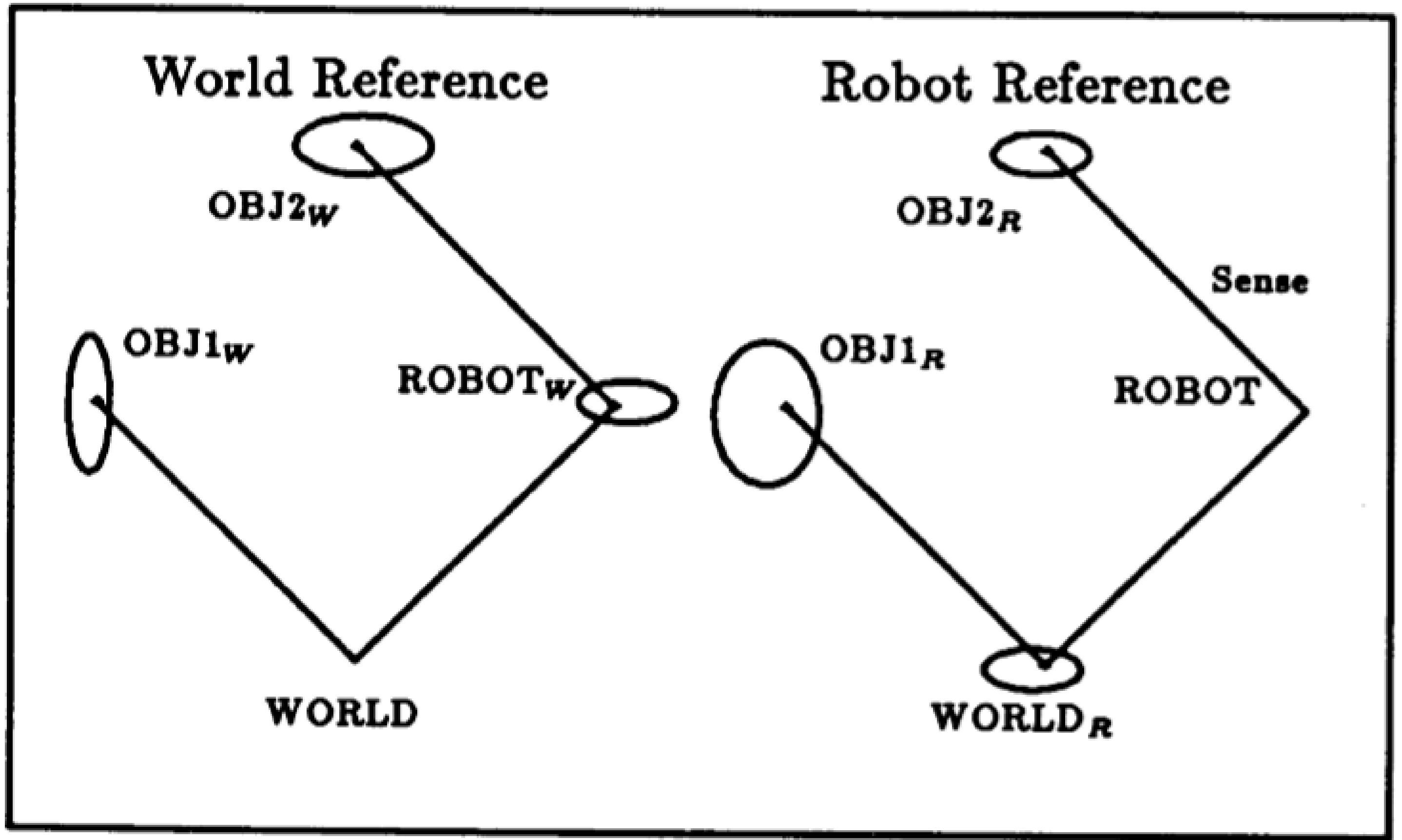
- A new object is sensed, from the robot reference frame.
- The stochastic map is updated accordingly

$$\hat{\mathbf{x}} = \begin{bmatrix} \hat{\mathbf{x}}_R \\ \hat{\mathbf{x}}_1 \\ \hat{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{y}}_R \\ \hat{\mathbf{z}}_1 \\ \hat{\mathbf{y}}_R \oplus \hat{\mathbf{z}}_2 \end{bmatrix}$$

$$\mathbf{C}(\mathbf{x}) = \begin{bmatrix} \mathbf{C}(\mathbf{x}_R) & \mathbf{C}(\mathbf{x}_R, \mathbf{x}_1) & \mathbf{C}(\mathbf{x}_R, \mathbf{x}_2) \\ \mathbf{C}(\mathbf{x}_1, \mathbf{x}_R) & \mathbf{C}(\mathbf{x}_1) & \mathbf{C}(\mathbf{x}_1, \mathbf{x}_2) \\ \mathbf{C}(\mathbf{x}_2, \mathbf{x}_R) & \mathbf{C}(\mathbf{x}_2, \mathbf{x}_1) & \mathbf{C}(\mathbf{x}_2) \end{bmatrix} = \begin{bmatrix} \mathbf{C}(\mathbf{y}_R) & \mathbf{0} & \mathbf{C}(\mathbf{y}_R)\mathbf{J}_{1\oplus}^T \\ \mathbf{0} & \mathbf{C}(\mathbf{z}_1) & \mathbf{0} \\ \mathbf{J}_{1\oplus}\mathbf{C}(\mathbf{y}_R) & \mathbf{0} & \mathbf{C}(\mathbf{x}_2) \end{bmatrix}$$

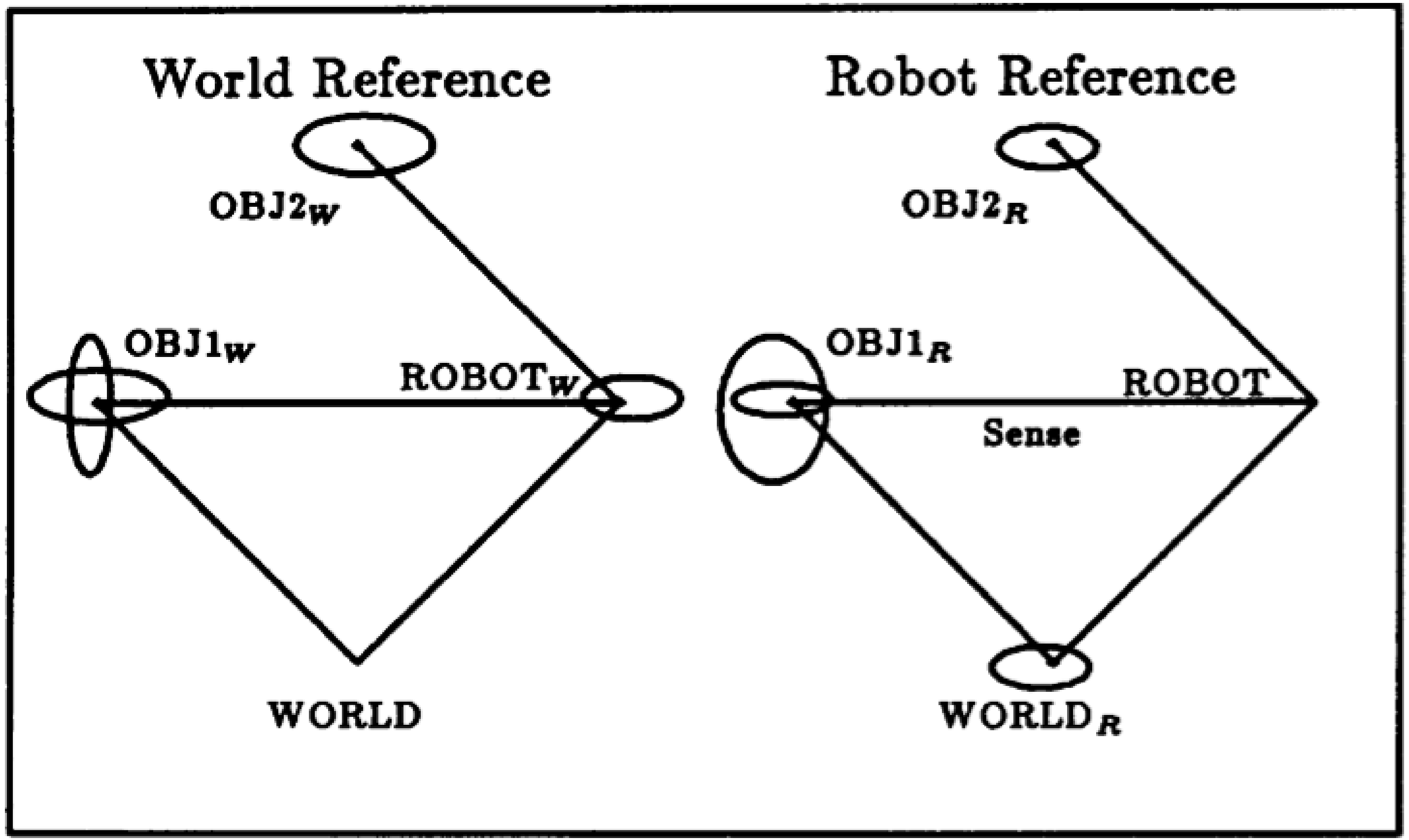
Sensing Object#2

- $OBJ2_W = ROBOT_W \oplus OBJ2_R$



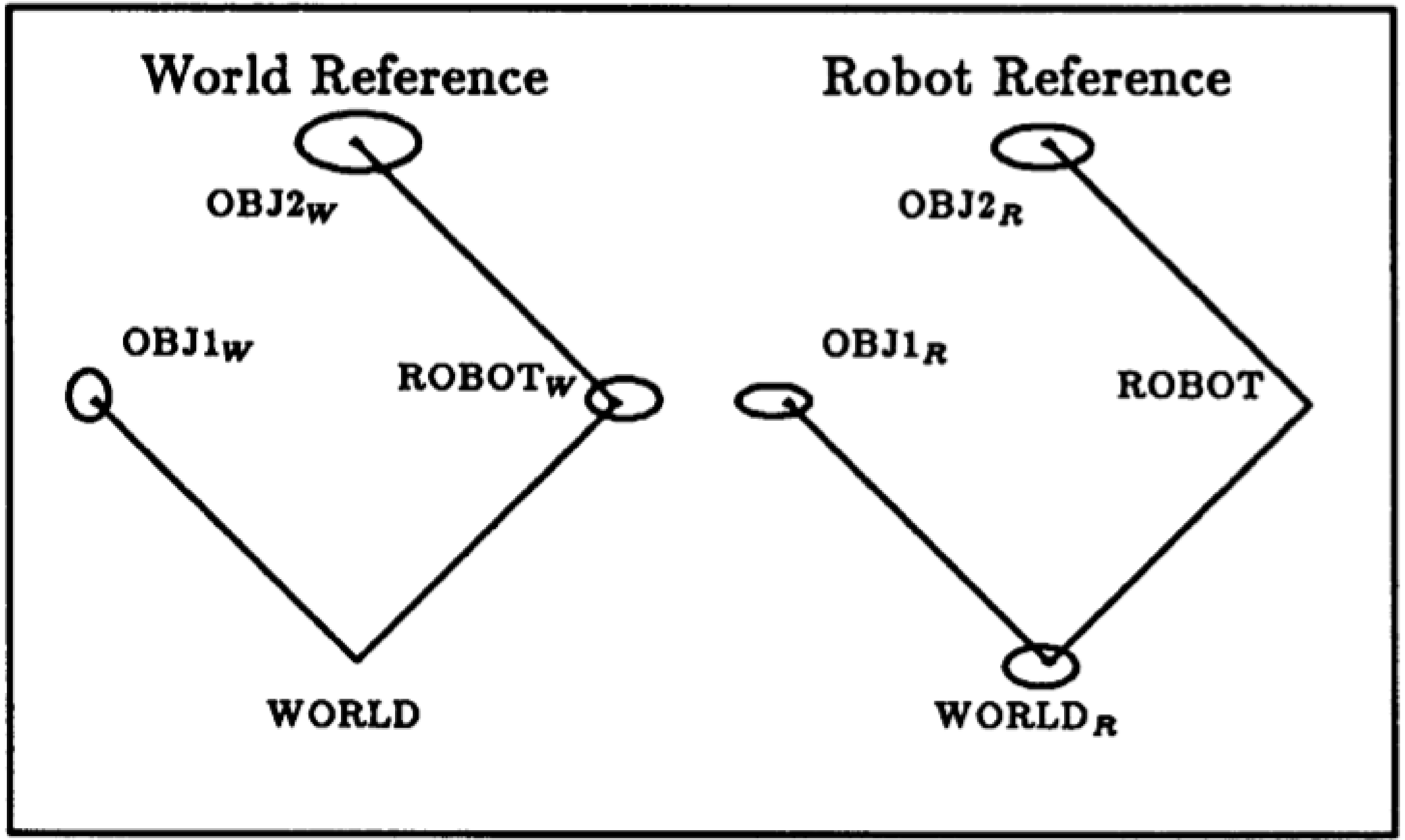
Step e)

- $OBJ1_W = ROBOT_W \oplus OBJ1_R$



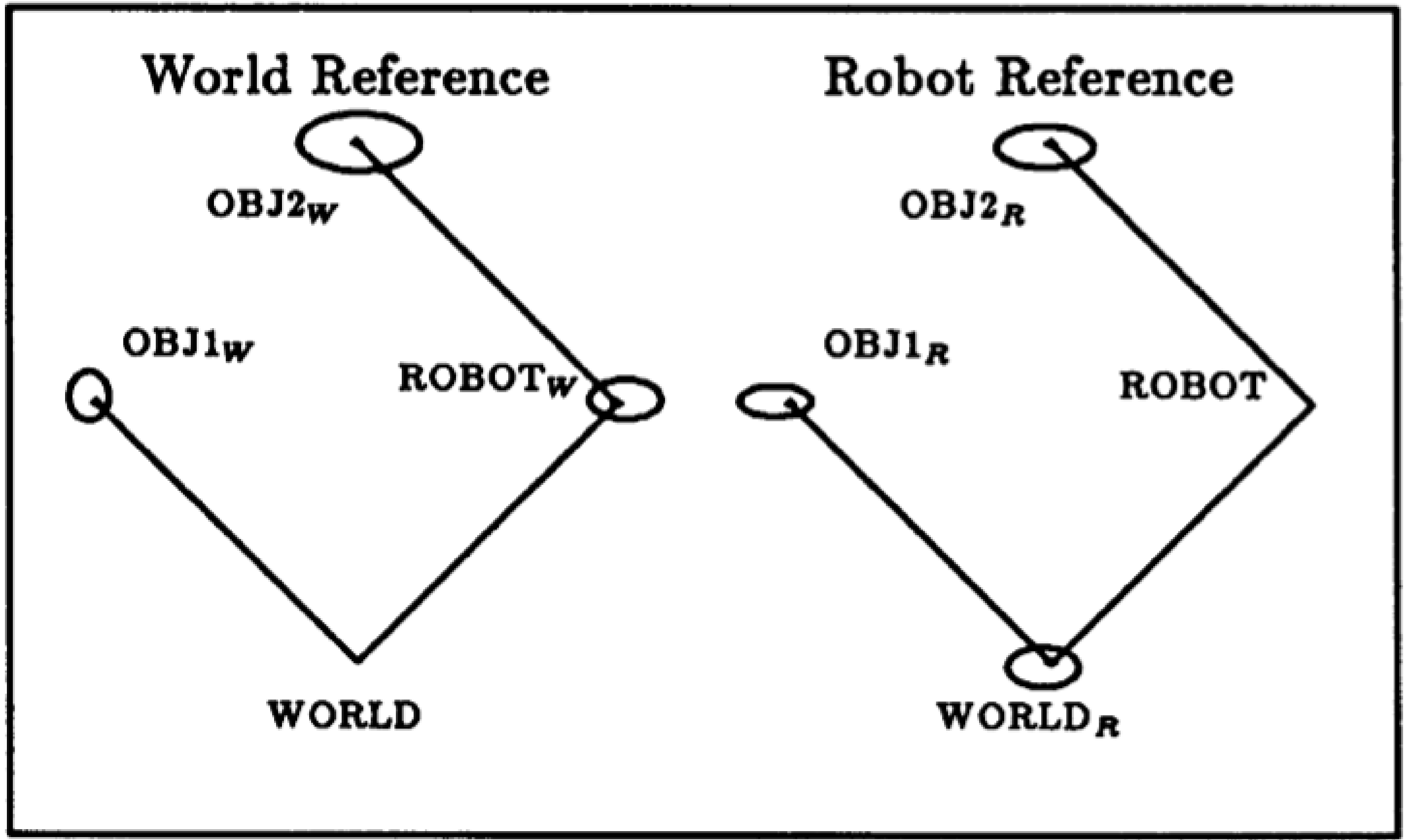
Combining observations (update)

- $OJB1_W = OJB1_W(new) \otimes OBJ1_W(old)$
- $OJB1_R = OJB1_R(new) \otimes OBJ1_R(old)$



Combining observations (update)

- $ROBOT_W(new) = OJB1_W \oplus (\ominus OBJ1_R)$
- $ROBOT_W = ROBOT_W(new) \otimes ROBOT_W(old)$



Discussion

- Data association?
- Partial observation?
- Non-unimodal Gaussians?
- Complexity?
- What happens if data association is wrong?
- Dynamic landmarks?
- Can be applied to certain decision-making problems
- …