Improved Techniques for Grid Mapping with Rao-Blackwellized PFs

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Introduction

Murphy et al. introduced Rao-Blackwellized PFs for SLAM

The main problem of RBPFs

- # of particles to build an accurate map
- Particle Depletion

To fix those issues

To fix those issues

To increase the performance of RBPF:

- Proposal distribution considers accuracy of the sensors
 - Less estimation error leads to less particles
- An adaptive resampling to prevent particle depletion
 - Do the resampling whenever is needed

But what is RBPFs?

Rao-Blackwellized Particle Filters

To estimate $p(x_{1:t}, m \mid z_{1:t}, u_{1:t-1})$ in which

- *m* is map
- $X_{1:t} = X_1 + X_2 + \dots + X_t$ is robot's trajectory
- $Z_{1:t} = Z_1 + Z_2 + \dots + Z_t$ is the observation
- $u_{1:t-1} = u_1 + u_2 + \dots + u_{t-1}$ is the odometry measurement

Rao-Blackwellized Particle Filters(Cntd)

By using factorization:

$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t-1}) = p(m \mid x_{1:t}, z_{1:t}).p(x_{1:t} \mid z_{1:t}, u_{1:t-1})$$

- The first part, $p(m \mid x_{1:t}, z_{1:t})$ is nothing but mapping with known poses
- The posterior $p(x_{1:t} \mid z_{1:t}, u_{1:t-1})$ is estimated by applying PF.

What kind of PF is that?

Sampling Importance Resampling(SIR)

Each particle has a potential trajectory of the robot.

As well as, an environment map of its own.

A RBSIR algorithm incrementally uses odom & sensor measurements for mapping.

How?

1- Sampling

Obtaining the next generation $\{x_t^{(i)}\}$ from $\{x_{t-1}^{(i)}\}$ by sampling form

proposal distribution π

 π is usually a probabilistic odometry motion model

2- Importance Weighting

Importance Sampling Principle:

$$W_t^{(i)} = p(x_{1:t}^{(i)} \mid z_{1:t}, u_{1:t-1}) / \pi(x_{1:t}^{(i)} \mid z_{1:t}, u_{1:t-1})$$

Proposal distribution π is in general not equal to target distribution

We can do it in a recursive way(by some assumption for efficiency)

$$W_{t}^{(i)} = W_{t-1}^{(i)} p(z_{t} \mid m_{t-1}^{(i)}, x_{t}^{(i)}).p(x_{t}^{(i)} \mid x_{t-1}^{(i)}, u_{t-1}) / \pi(x_{t} \mid x_{1:t-1}^{(i)}, z_{1:t}, u_{1:t-1})$$

3- Resampling

Proportional to importance weight

With replacement

4- Map Estimation

The map estimate for each particle

$$p(m^{(i)} | x_{1:t}^{(i)}, z_{1:t})$$

is computed based on its trajectory $x_{1:t}^{(i)}$ and the history of observations $z_{1:t}$

Improved Proposal Distribution

Local approximation of the posterior $p(x_t | m_{t-1}^{(i)}, x_{t-1}^{(i)}, z_t, u_{t-1})$ around the maximum likelihood function.

- 1. Using a scan-matcher to determine the meaningful area
- 2. K Sample in the meaningful area
- 3. Evaluated based on target distribution
- 4. $\mu_t^{(i)} \& \sum_t^{(i)}$ are determined for K sample points

$$egin{array}{lll} \mu_t^{(i)} &=& rac{1}{\eta^{(i)}} \cdot \sum_{j=1}^K x_j \cdot p(z_t \mid m_{t-1}^{(i)}, x_j) \ && \cdot p(x_j \mid x_{t-1}^{(i)}, u_{t-1}) \ \Sigma_t^{(i)} &=& rac{1}{\eta^{(i)}} \cdot \sum_{j=1}^K p(z_t \mid m_{t-1}^{(i)}, x_j) \ && \cdot p(x_j \mid x_{t-1}^{(i)}, u_{t-1}) \ && \cdot (x_j - \mu_t^{(i)}) (x_j - \mu_t^{(i)})^T \end{array}$$

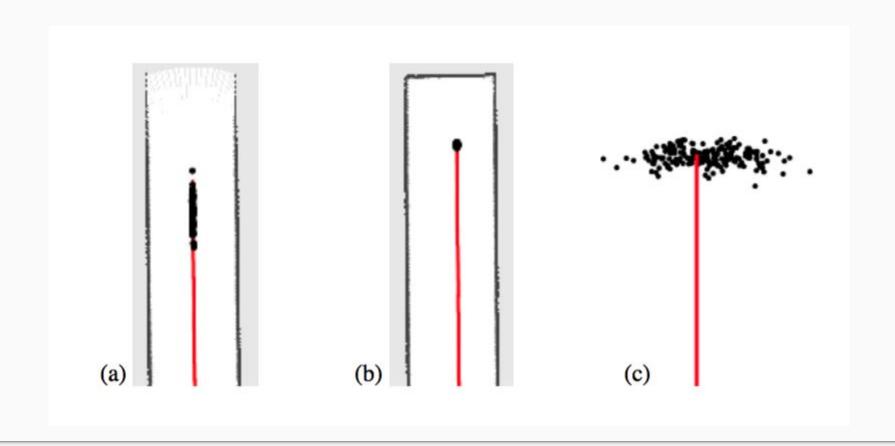
Improved Proposal

Using this proposal distribution weights can be computed as:

$$w_{t}^{(i)} = w_{t-1}^{(i)} \cdot p(z_{t} \mid m_{t-1}^{(i)}, x_{t-1}^{(i)}, u_{t-1})$$

$$= w_{t-1}^{(i)} \cdot \int p(z_{t} \mid m_{t-1}^{(i)}, x') \cdot p(x' \mid x_{t-1}^{(i)}, u_{t-1}) dx$$

$$\simeq w_{t-1}^{(i)} \cdot \sum_{j=1}^{K} p(z_{t} \mid m_{t-1}^{(i)}, x_{j}) \cdot p(x_{j} \mid x_{t-1}^{(i)}, u_{t-1})$$



Adaptive Resampling

Effective Sample Size

$$N_{\text{eff}} = \frac{1}{\sum_{i=1}^{N} \left(\tilde{w}^{(i)}\right)^2},$$

 $N_{\rm eff}$ can be regarded as a measure of the dispersion of importance weights.

Each time N_{eff} drops below the threshold N/2 resampling is needed.

for all $x_i \in \{x_1, \ldots, x_K\}$ do Require: $\mu_t^{(i)} = \mu_t^{(i)} + x_i \cdot p(z_t \mid m_{t-1}^{(i)}, x_i) \cdot p(x_t \mid x_{t-1}^{(i)}, u_{t-1})$ S_{t-1} , the sample set of the previous time step z_t , the most recent laser scan $\eta^{(i)} = \eta^{(i)} + p(z_t \mid m_{t-1}^{(i)}, x_i) \cdot p(x_t \mid x_{t-1}^{(i)}, u_{t-1})$ u_{t-1} , the most recent odometry measurement end for $\mu_t^{(i)} = \mu_t^{(i)}/\eta^{(i)}$ **Ensure:** \mathcal{S}_t , the new sample set for all $x_i \in \{x_1, ..., x_K\}$ do $\Sigma_t^{(i)} = \Sigma_t^{(i)} + (x_j - \mu^{(i)})(x_j - \mu^{(i)})^T \cdot p(z_t \mid m_{t-1}^{(i)}, x_j) \cdot p(x_j \mid x_{t-1}^{(i)}, u_{t-1})$ $\mathcal{S}_t = \{\}$ for all $s_t^{(i)} \in \mathcal{S}_{t-1}$ do $\langle x_{+}^{(i)}, w_{+}^{(i)}, m_{+}^{(i)} \rangle = s_{+}^{(i)}$ end for $\Sigma_{t}^{(i)} = \Sigma_{t}^{(i)}/\eta^{(i)}$ // sample new pose // scan-matching $x_{t}^{\prime(i)} = x_{t-1}^{(i)} \oplus u_{t-1}$ $x_t^{(i)} \sim \mathcal{N}(\mu_t^{(i)}, \Sigma_t^{(i)})$ $\hat{x}_{t}^{(i)} = \operatorname{argmax}_{x} p(x \mid m_{t-1}^{(i)}, z_{t}, x_{t}^{(i)})$ // update importance weights $w_t^{(i)} = w_{t-1}^{(i)} \cdot \eta^{(i)}$ if $\hat{x}_{t}^{(i)} =$ failure then end if $x_{t}^{(i)} \sim p(x_{t} \mid x_{t-1}^{(i)}, u_{t-1})$ // update map $w_t^{(i)} = w_{t-1}^{(i)} \cdot p(z_t \mid m_{t-1}^{(i)}, x_t^{(i)})$ $m_t^{(i)} = \text{integrateScan}(m_{t-1}^{(i)}, x_t^{(i)}, z_t)$ else // update sample set // sample around the mode $S_t = S_t \cup \{\langle x_t^{(i)}, w_t^{(i)}, m_t^{(i)} \rangle \}$ for $k = 1, \ldots, K$ do end for $|x_k \sim \{x_i \mid |x_i - \hat{x}^{(i)}| < \Delta\}$ end for $N_{ ext{eff}} = rac{1}{\sum_{i=1}^{N} \left(ilde{w}^{(i)}
ight)^2}$ // compute Gaussian proposal if $N_{\rm eff} < T$ then $\mu_t^{(i)} = (0,0,0)^T$ $\mathcal{S}_t = \text{resample}(\mathcal{S}_t)$ end if

Complexity

| Operation | Complexity |
|--|------------|
| Computation of the proposal distribution | O(N) |
| Update of the grid map | O(N) |
| Computation of the weights | O(N) |
| Test if resampling is required | O(N) |
| Resampling | O(NM) |

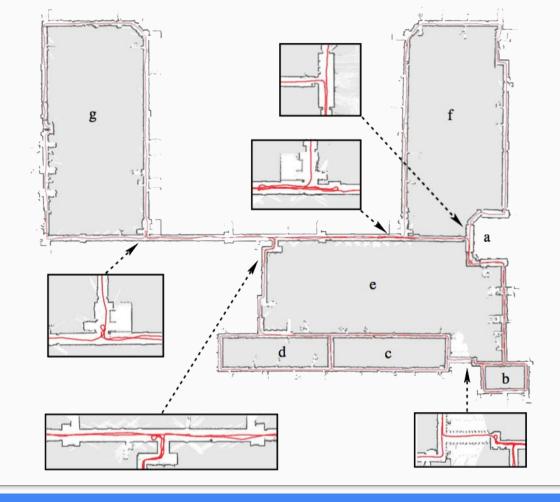


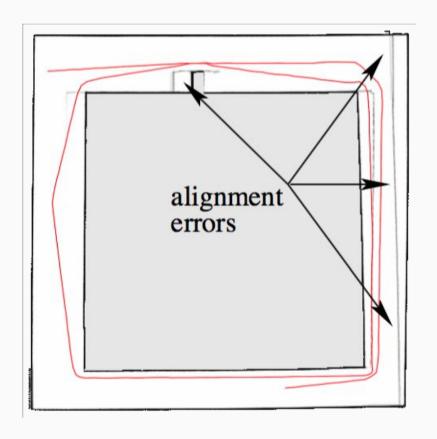


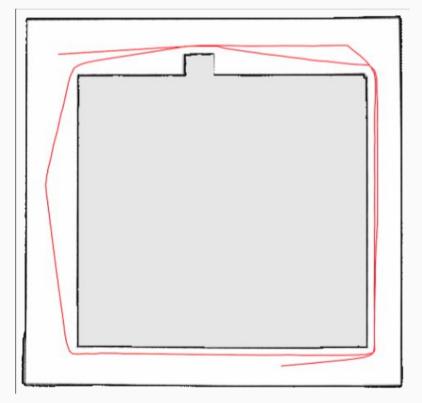












References

G. Grisetti, C. Stachniss, and W. Burgard. Improved Techniques for Grid Mapping with Rao-Blackwellized Particle Filters, Robotics, IEEE Transactions on, 2007.

Thanks for listening!