Today

Radiometry

Announcement

Quiz #2 is available in Blackboard.

Due date: 11:59pm EST, Thursday, Feb. 6th

Open book and open notes

Announcement

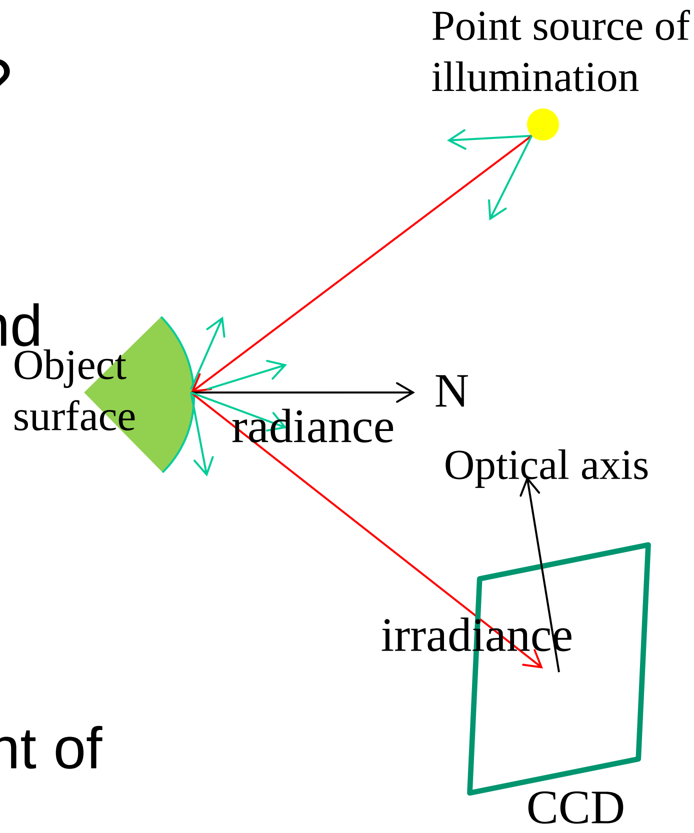
Homework #2 has been posted in Blackboard.

Due on 1:15pm EST, Thursday, Feb 13th.

Recall: Basic Radiometry

Questions:

- how “bright” will surfaces be?
- what is “brightness”?
  - – measuring light
  - – interactions between light and surfaces



The brightness is affected by

- Illumination
- Surface radiance **($wsr^{-1}m^{-2}$)**: amount of radiation in a specific direction
- Image irradiance **(wm^{-2})**: power of light per unit area a CCD array element receives

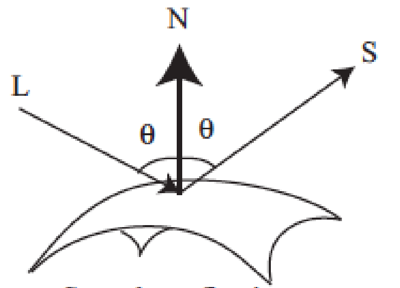
Recall: Important Reflection Modes

Specular reflection (mirror like)

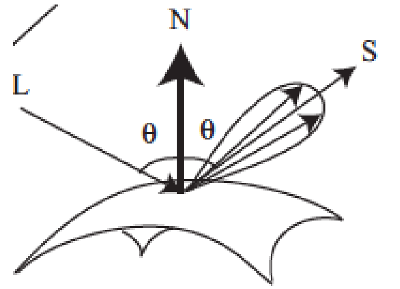
- Pure mirror:
  - incoming, outgoing directions and normal are coplanar
  - incoming, outgoing angles to normal are equal
- Most specular surfaces:
  - some light leaves the surface along directions near to the specular direction as well

Diffuse reflection

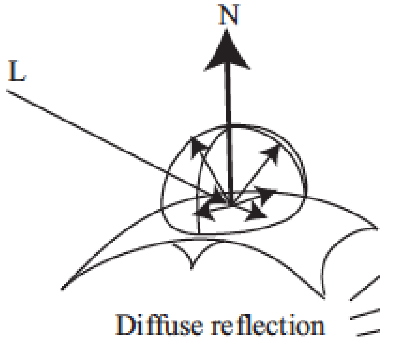
- Light leaves in equal amounts in each direction
  - so surface looks equally bright from each viewing direction
  - Application: diffusion umbrella



Specular reflection



Specularity



Diffuse reflection

Recall: Diffuse Reflection

Light leaves the surface evenly in all directions

- E.g., cotton cloth, carpets, matte paper, matte paints, most "rough" surfaces

Described by one parameter: Albedo (reflection coefficient)

- percentage of light arriving that leaves
- range 0-1
  - practical range is 0.05-0.9
  - Albedo of earth is 0.3-0.35

Light leaving is (Albedo)x(Light arriving)

- Does not depend on the observation direction
- Ambiguity: A surface could be dark because
  - It reflects a small percentage of the light arriving
  - There isn't very much light arriving

How much light arrives: Lambertian Reflection

Assume source is far away

- So light travels in parallel rays
- (Light arriving) proportional to (number of rays striking surface)

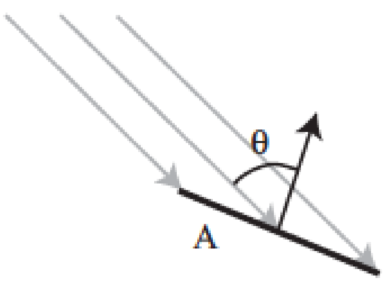
Lambertian reflection: Ideal diffuse reflection

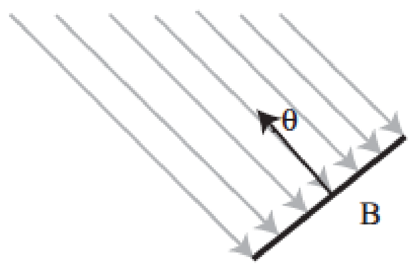
- Surface radiance follows the Lambert’s cosine law

$$I_R = \rho \mathbf{L} \cdot \mathbf{N}$$

I_R: surface radiance; ρ: surface albedo;

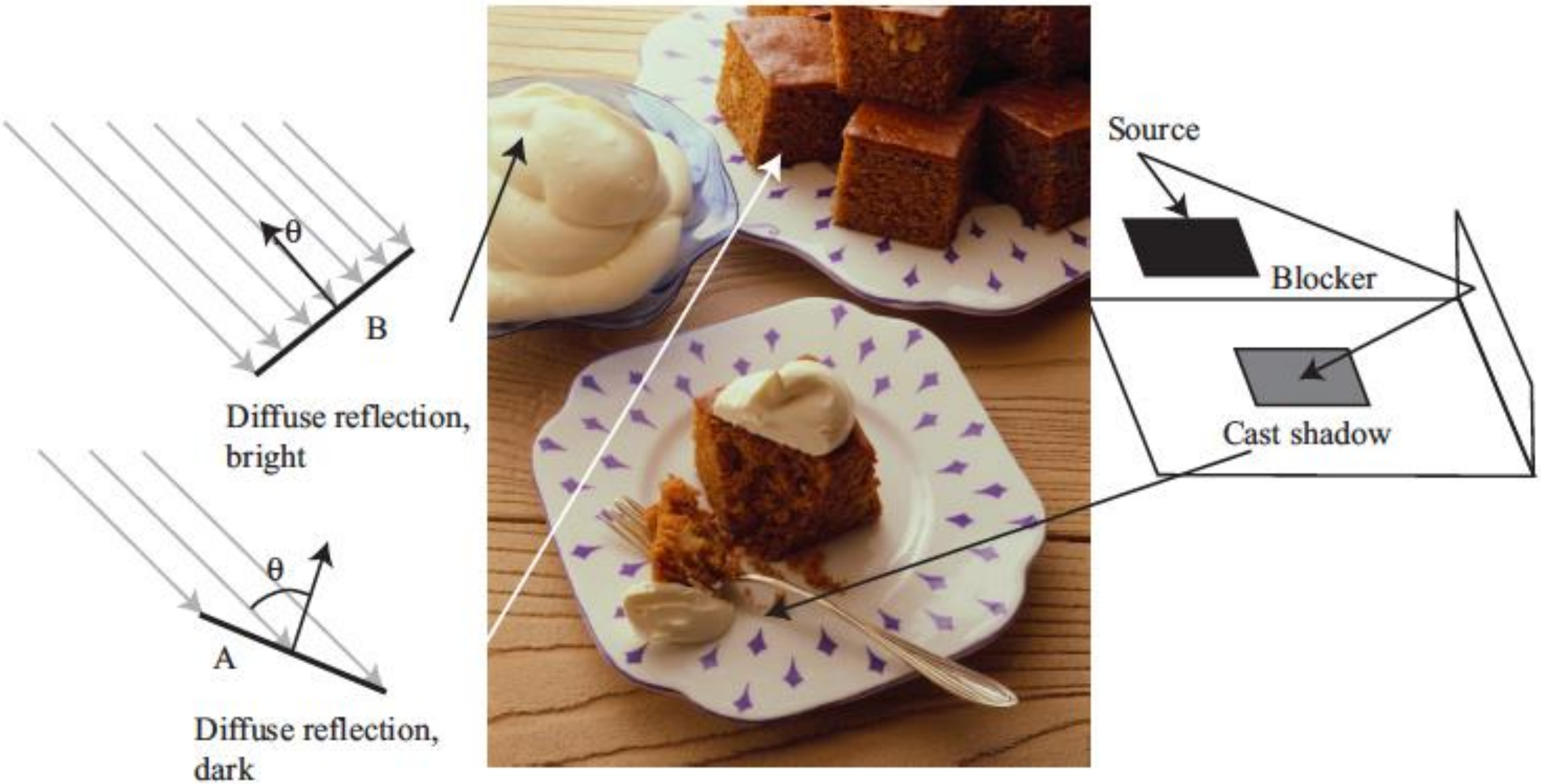
$\mathbf{L}$: intensity and direction of incoming light; $\mathbf{N}$: surface normal





Which surface will be brighter with the same number of rays arriving?

How much light arrives: Lambertian Reflection

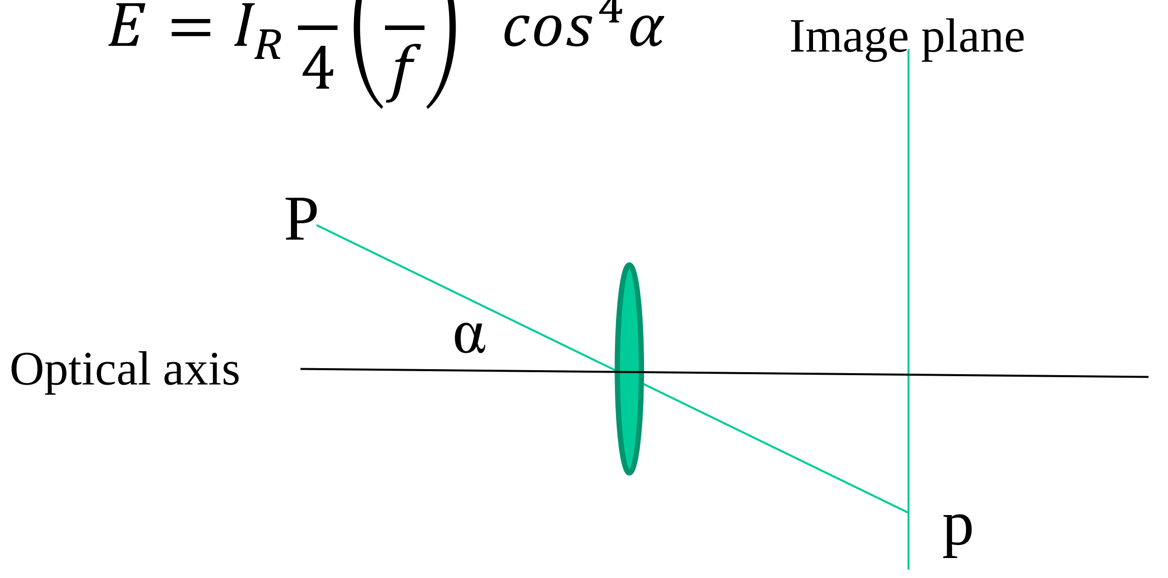


Fundamental Equation of Radiometric Image Formation

What is the relationship between the surface radiance and image irradiance?

For a thin lens with diameter d and focal length f, we have

$$E = I_R \frac{\pi}{4}\left(\frac{d}{f}\right)^2 cos^4\alpha$$



For small angular aperture (FOV), the image irradiance is proportional to the surface radiance.

Fundamental Equation of Radiometric Image Formation

illumination ⇒ **intensity**

$$I = \beta\rho\frac{\pi}{4}\left(\frac{d}{f}\right)^2 cos^4\alpha\mathbf{L}\cdot\mathbf{N}$$

↓

Sensor determined

Notes: Radiometry

Image brightness is affected by

- amount of light arriving at surface
- surface type (diffuse, specular) and amount reflected at surface
- camera sensitivity

➡ Low albedo surface in bright light vs high albedo surface in low light: each might reflect about the same amount

Most surfaces can be modeled as diffuse + specular

- generally, find and remove specularities
- treat the rest as diffuse

Illumination and Shading

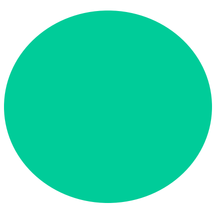
The point is in the shadow if L=0

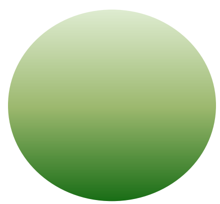
Shadow

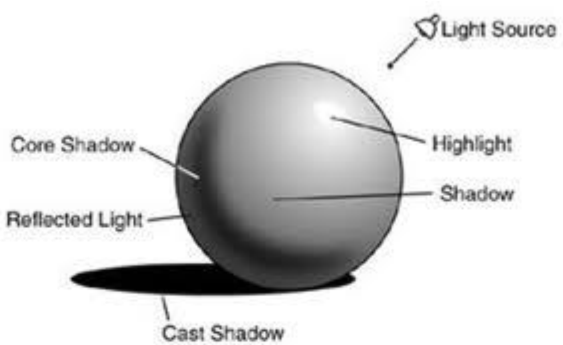
👍: tells us the geometry of the object and the relative position to the light source

👎: cause trouble for recognition

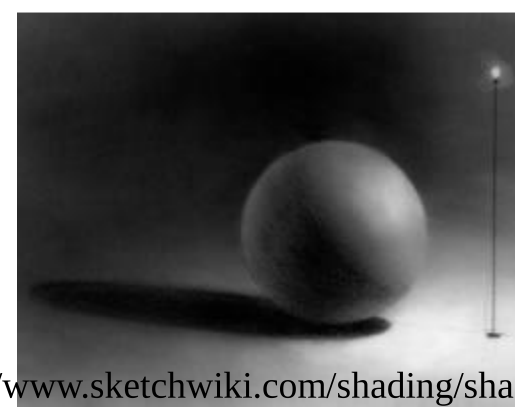
Is it a ball or a circle?







http://www.paulcarneyarts.com/shading.html



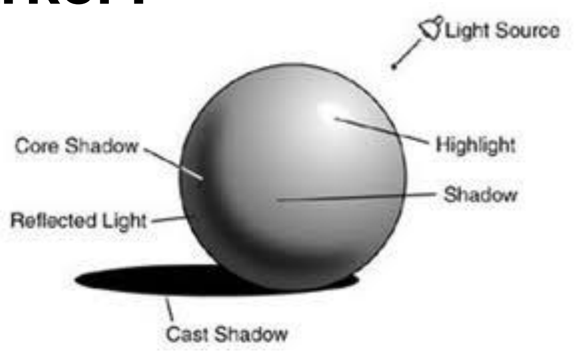
http://www.sketchwiki.com/shading/shading-sphere.php

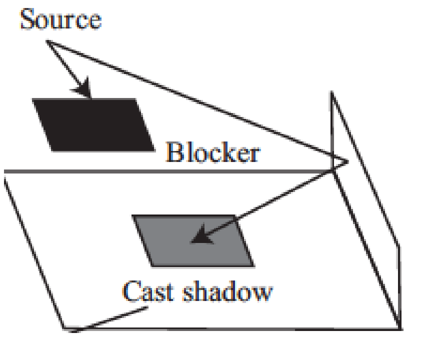
Shadows

Most shadows aren't dark

- because shadow points get light from other surfaces, not just light source
- Cast shadow – created by the blocker
- Form shadow – on the side of object opposite to light
- Core shadow – dark edge between the illuminated part and form shadow

Why cast shadow is darker?





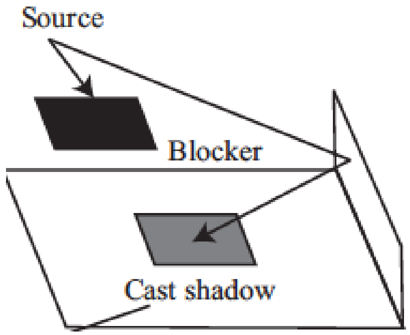
http://www.paulcarneyarts.com/shading.html

Shadows

Most shadows aren't dark

- because shadow points get light from other surfaces, not just light source

Why cast shadow is more darker?



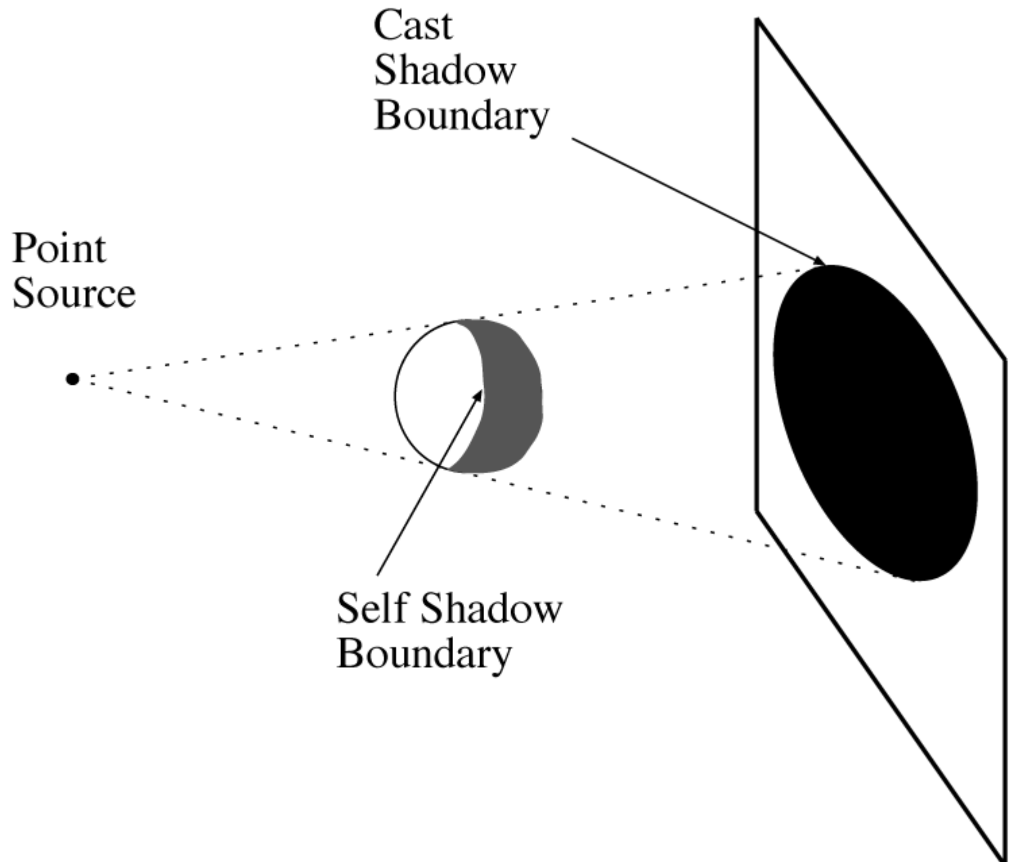
Area sources

- Large, bright areas
- Yield smooth, blurry shadows
  - Points that can see the whole source are brighter
  - Points that can see only part of the source are darker (penumbra)
  - Points that can see no part of the source are darkest (umbra)

Shadows Cast by a Point Source

A point that can't see the source is in shadow

For point sources, the geometry is simple



Shadows Cast by an Area Source

Area sources

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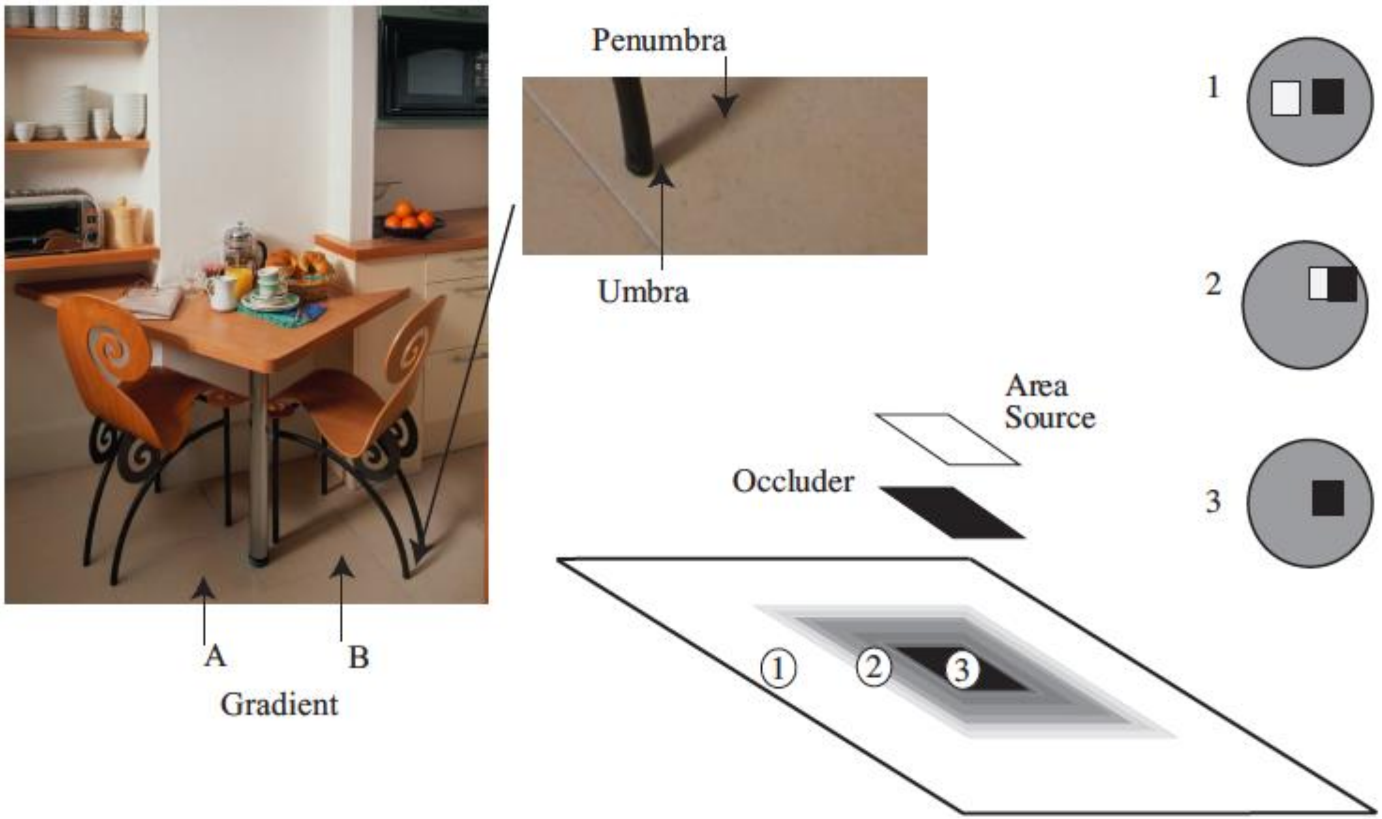
Shadows Cast by an Area Source



Penumbra: Points that can see only part of the source are darker
Umbra: Points that can see no part of the source are darkest

Information from Shading

Recover some information about the world from shading

- Photometric stereo (shape from shading)
  - recover shape and albedo of surfaces from multiple shaded images
- Recovering surface albedo
  - from image data
- Radiometric calibration
  - how much light is required to produce a particular number in the image

Applications in graphics: rendering of a scene with different illumination and different surface materials

Photometric Stereo

Assume:

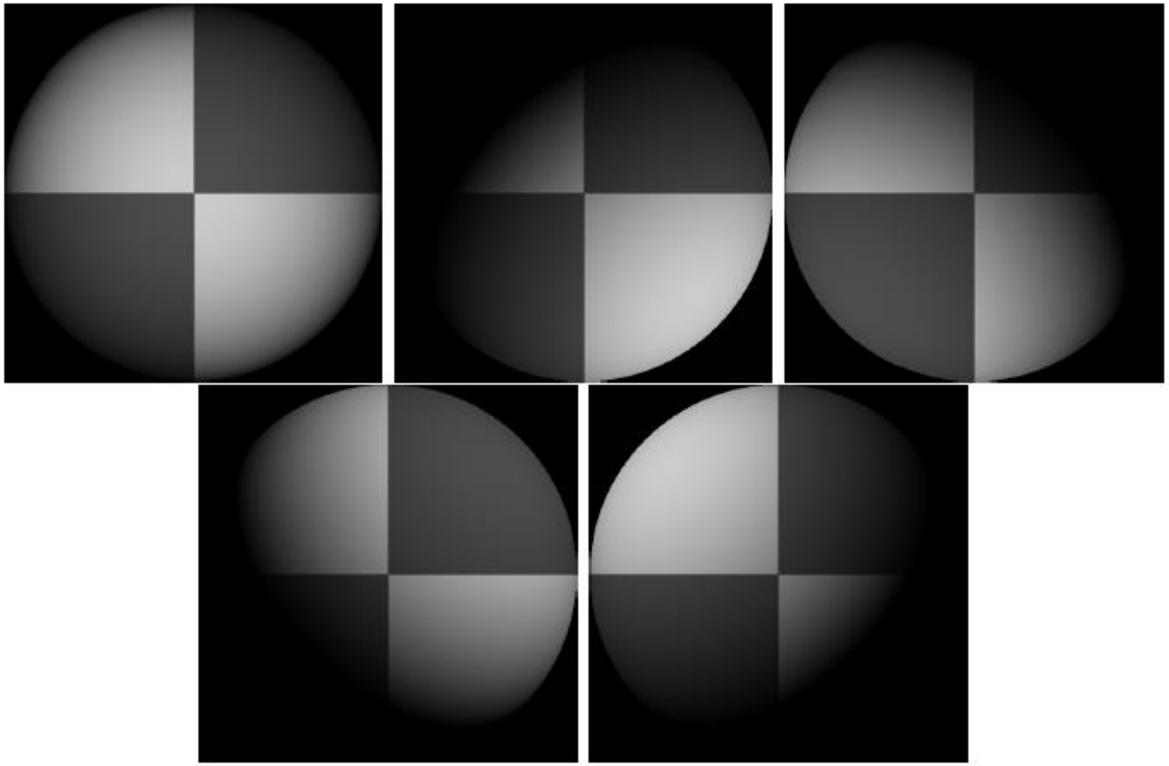
- A set of point sources that are infinitely distant
- A set of pictures of an object, obtained in exactly the same camera/object configuration but using different illumination sources
- Pictures taken based on an orthographic camera model
- A Lambertian object (or the specular component has been identified and removed)

What do we want

- Reconstruct a patch of surface in 3D space

Example

Five synthetic images of a sphere with different sources

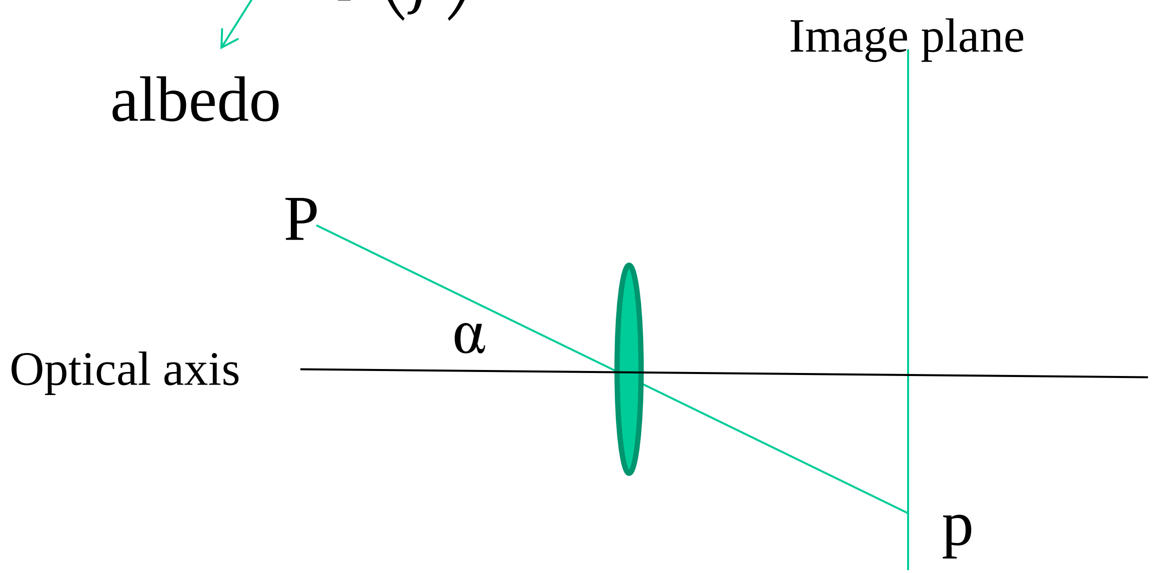


Fundamental Equation of Radiometric Image Formation

For a thin lens with diameter d and focal length f, we have

$$I = \beta\rho\frac{\pi}{4}\left(\frac{d}{f}\right)^2 cos^4\alpha\mathbf{L}\cdot\mathbf{N}$$

albedo



For small angular aperture, the image irradiance is proportional to the surface radiance.

Imaging Model

Assume a small aperture lens, fix the camera and the surface in position

Image value at (x, y) is

$$I(x,y) = \beta\rho(x,y)\frac{\pi}{4}\left(\frac{d}{f}\right)^2 cos^4\alpha\mathbf{L}(x,y)\cdot\mathbf{N}(x,y)$$

Group the constant parameters together, we have

$$I(x,y) = k\rho(x,y)\mathbf{L}(x,y)\cdot\mathbf{N}(x,y) = \mathbf{g}(x,y)\cdot\mathbf{v}(x,y)$$

where $\mathrm{g}(x,y) = \rho(x,y)\mathrm{N}(x,y)$ and $\mathrm{v}(x,y) = k\mathrm{L}(x,y)$

Can be measured

Notice g(x, y) tells us about the surface, v(x,y) about the source and camera

Normal and Albedo from Many Pictures

We take *n* photographs, using *n* different sources, and measure

$$\mathbf{V} = \begin{pmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \\ \cdots \\ \mathbf{V}_n^T \end{pmatrix}_{n\times 3} \qquad \mathbf{i}(x,y) = \begin{pmatrix} I_1(x,y) \\ I_2(x,y) \\ \cdots \\ I_n(x,y) \end{pmatrix}_{n\times 1}$$

Mask the shadow point

We have $\mathbf{i}(x,y) = \mathbf{V}\mathbf{g}(x,y)$ → $M\mathbf{i}(x,y) = M\mathbf{V}\mathbf{g}(x,y)$

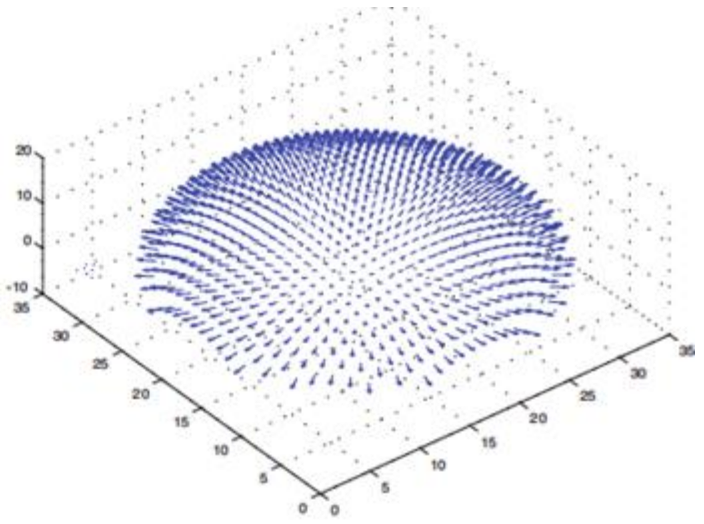
Least-squared estimation can be used to solve for $\mathbf{g}(x,y)$

Albedo $\rho(x,y) = \|\mathbf{g}(x,y)\|$ → Should be in [0, 1]

Normal $\mathbf{N}(x,y) = \dfrac{\mathbf{g}(x,y)}{\|\mathbf{g}(x,y)\|}$

Shape from Normals

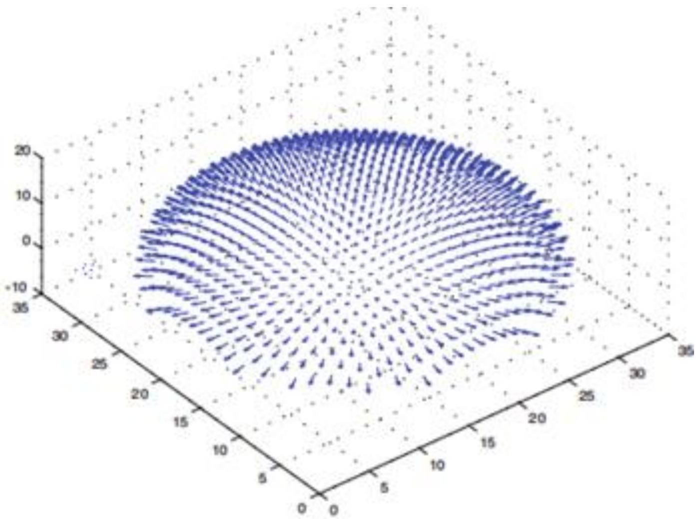
Now we have



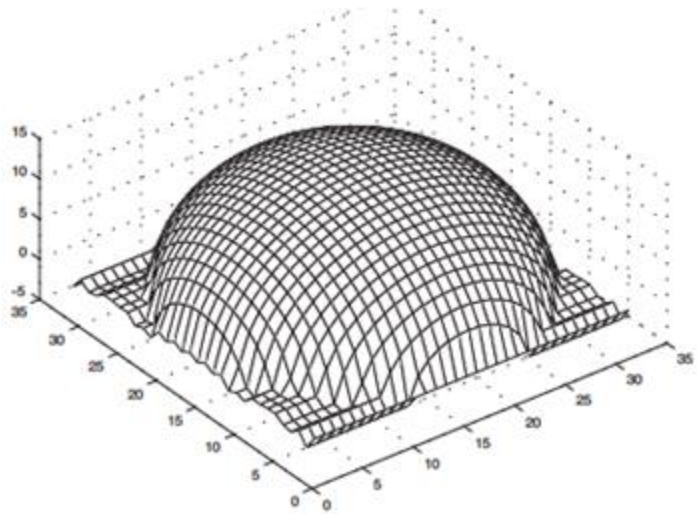
$\mathbf{N}(x, y)$

Shape from Normals

Now we have



$\mathbf{N}(x, y)$



Shape from Normals

How to calculate the surface normal at (x,y)?

- **A 3D surface can be represented as a function of X and Y:** $Z = f(X, Y)$ **or** $F(X, Y, Z) = Z - f(X, Y) = 0$

- **Then the surface normal at** (X, Y, Z) **is** $\nabla F(X, Y, Z)$ **for** $F(X, Y, Z) = 0$

$$\mathbf{N}(X, Y) = \frac{1}{\sqrt{1 + \left(\frac{\partial f}{\partial X}\right)^2 + \left(\frac{\partial f}{\partial Y}\right)^2}} \left[-\frac{\partial f}{\partial X}, -\frac{\partial f}{\partial Y}, 1\right]^T$$

Shape from Normals

Assume we use the orthographic model

$$x = X \text{ and } y = Y$$

$$\mathbf{N}(x, y) = \frac{1}{\sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}} \left[-\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1\right]^T$$

Shape from Normals

Let the measured unit normal at (x, y) is

$$\mathbf{N}(x,y) = \begin{bmatrix} a(x,y) \\ b(x,y) \\ c(x,y) \end{bmatrix} = \frac{1}{\sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}} \begin{bmatrix} -\frac{\partial f}{\partial x} \\ -\frac{\partial f}{\partial y} \\ 1 \end{bmatrix}$$

then we know

$$\frac{\partial f}{\partial x} = -\frac{a(x,y)}{c(x,y)} \quad \text{and} \quad \frac{\partial f}{\partial y} = -\frac{b(x,y)}{c(x,y)}$$

Reconstruct Depth Map

Using differential equation to approximate

$$f(x+\Delta x, y) = f(x,y) + \frac{\partial f}{\partial x}\Delta x$$
$$f(x, y+\Delta y) = f(x,y) + \frac{\partial f}{\partial y}\Delta y$$

Let the top-left corner of the depth map to be zero and then increase *x* and *y* with the step length of 1 pixel each time to construct the whole depth map

Example

$\rho(x,y) = |\mathbf{g}(x,y)|$ $\mathbf{N}(x,y) = \mathbf{g}(x,y)/|\mathbf{g}(x,y)|$ Reconstructed surface

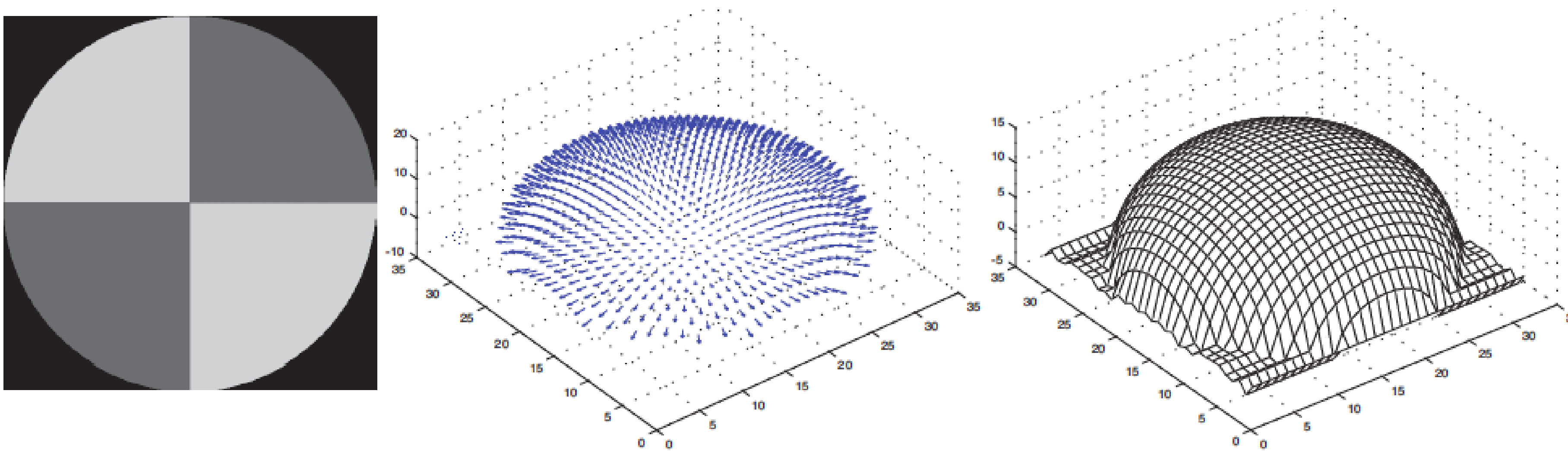


FIGURE 2.12: The image on the **left** shows the magnitude of the vector field $\boldsymbol{g}(x, y)$ recovered from the input data of Figure 2.11 represented as an image—this is the reflectance of the surface. The **center** figure shows the normal field, and the **right** figure shows the height field.