Announcement

Homework 4 has been posted online and in Blackboard.

Due: 1:15pm EST, Thursday, April 3rd

Announcement

We will have Quiz #5 in class, Tuesday, April 1st

Open book and open notes

We will have 5 more quizzes. No late quizzes will be accepted without preapproval.

Reminder: Final Project Presentation Schedule

Presentation days:

April 22 and 24

A team project needs a combined presentation.

Each individual presentation has 10 + 1 minutes Q&A
Each team presentation has 13 + 2 minutes Q&A

Send me an email (tongy@cse.sc.edu) that includes:

- Your name and your preferred presentation date
- Earlier email has higher priority in choosing the date

Today

Early vision on multiple images

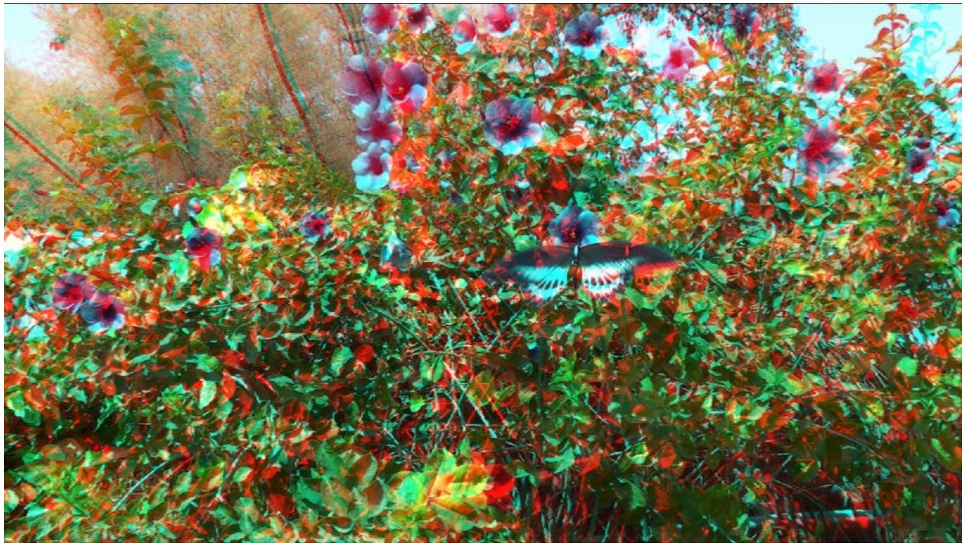
- Epipolar geometry

Stereopsis

Stereo camera
California Museum of Photography

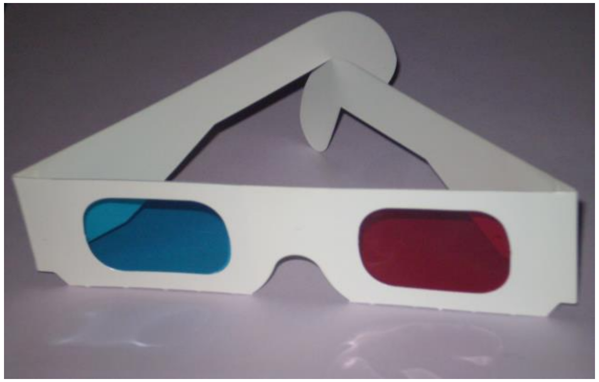
“Mark Twain at Pool Table", no date,
UCR Museum of Photography

More Stereo Images



http://realvision.ae/blog/2009/09/stereoscopic-3d-in-classrooms/

https://www.dpreview.com/



By Snaily - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=1612671

Recover 3D from Images

Goal: recover the 3D property of an object including

- 3D coordinates
- Depth
- 3D shape

Two typical ways:

- **Reconstruct from image cues**
  - Single image
  - Multiple images

Human vision
Kinect

- **Obtain depth information directly using special devices**
  - Laser scanner
  - Radar
  - Ultrasound

Shape from X -- Image Cues

Shading

http://theindependentartschool.com/art-blog/2011/02/07/drawing-and-shading-a-sphere/

Texture

Visual cliff
http://brisray.com/optill/ovision1.htm

Focus/defocus

Motion

http://www.cs.ucla.edu/~soatto/research/projects.html

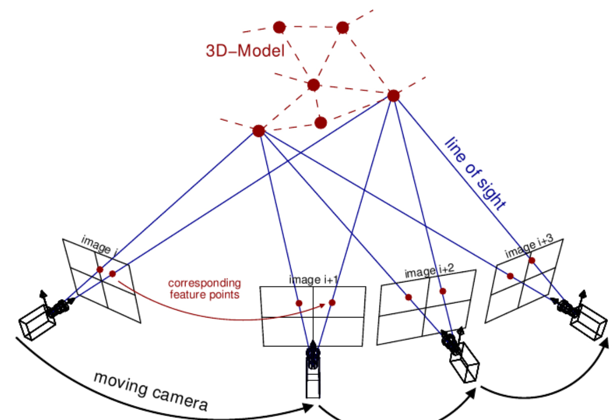
Many others

- Geometry
- Shadow
- Specularity
- etc.

Shape from Multiple Images: Stereo

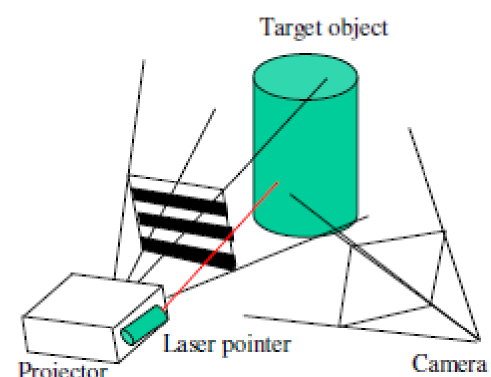
Reconstruct a 3D scene/object from images observed from multiple view points

- **Passive stereo**
  - Multiple (at least two) cameras



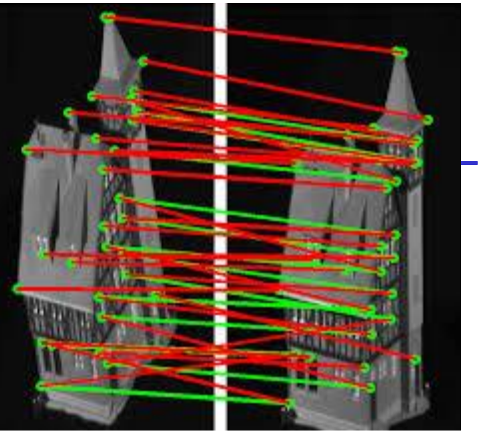
http://www.tnt.uni-hannover.de/project/motionestimation/

- **Active stereo**
  - Structured light
  - Usually consist of a camera and a projector
    - Kinect



Kawasaki et al (ACCV2006)

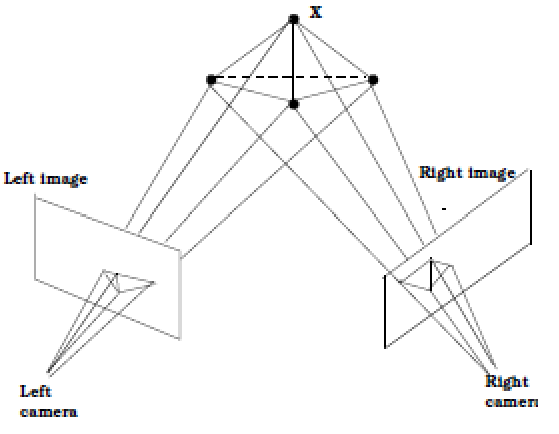
Passive Stereo



pages.cs.wisc.edu

General strategy:

- **Camera calibration**
- **Correspondence**
  - Detect a set of tokens (points, lines, and conic curves)
  - Build correspondence for the tokens in multiple (at least two) images
- **3D reconstruction from matched 2D points**
  - Image rectification
  - 3D triangulation from rectified images

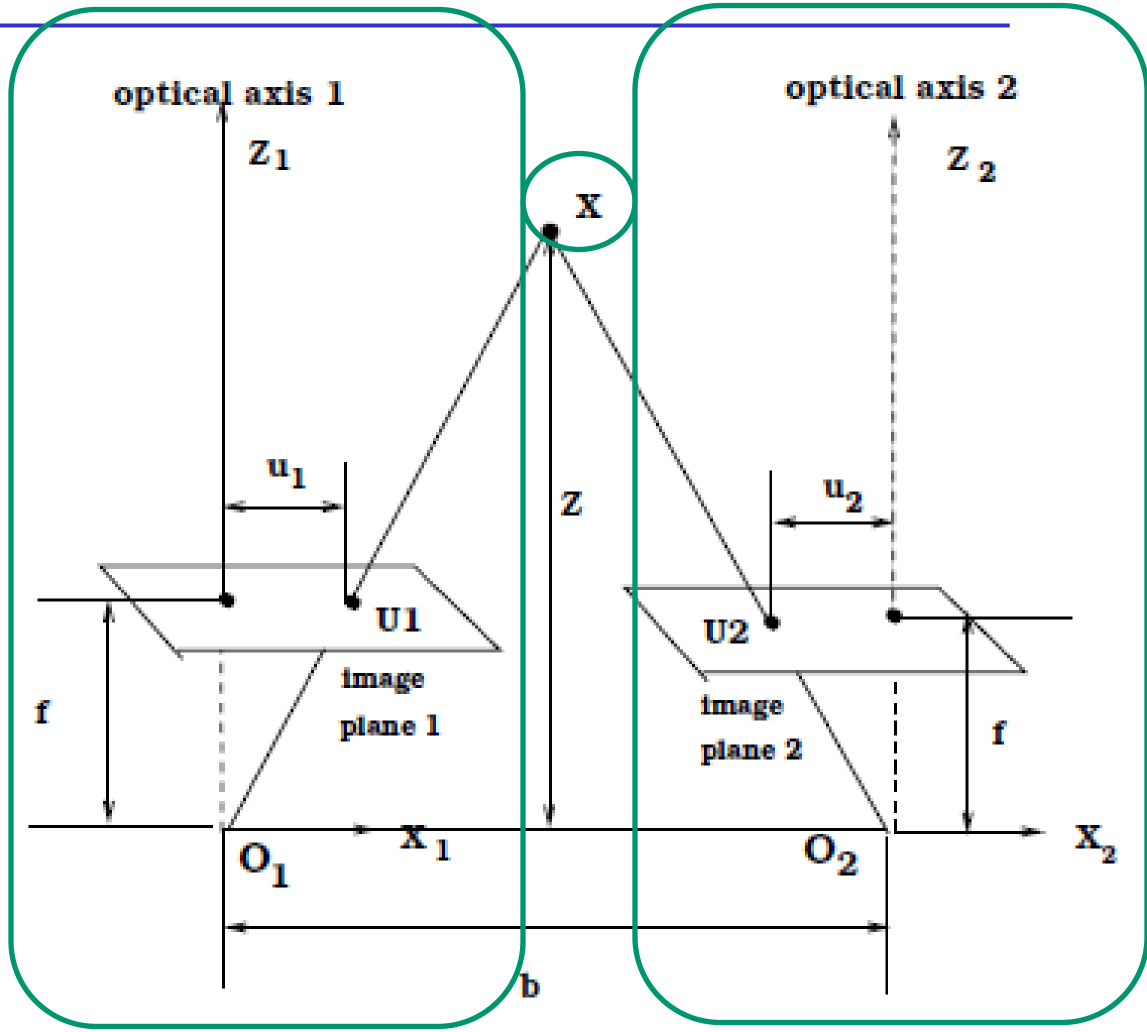


courtesy of Dr. Qiang Ji

A Simple Case: 3D Triangulation – Rectified Geometry

Rectified geometry:

- two image planes are coplanar and parallel to the line O_1O_2



courtesy of Dr. Qiang Ji

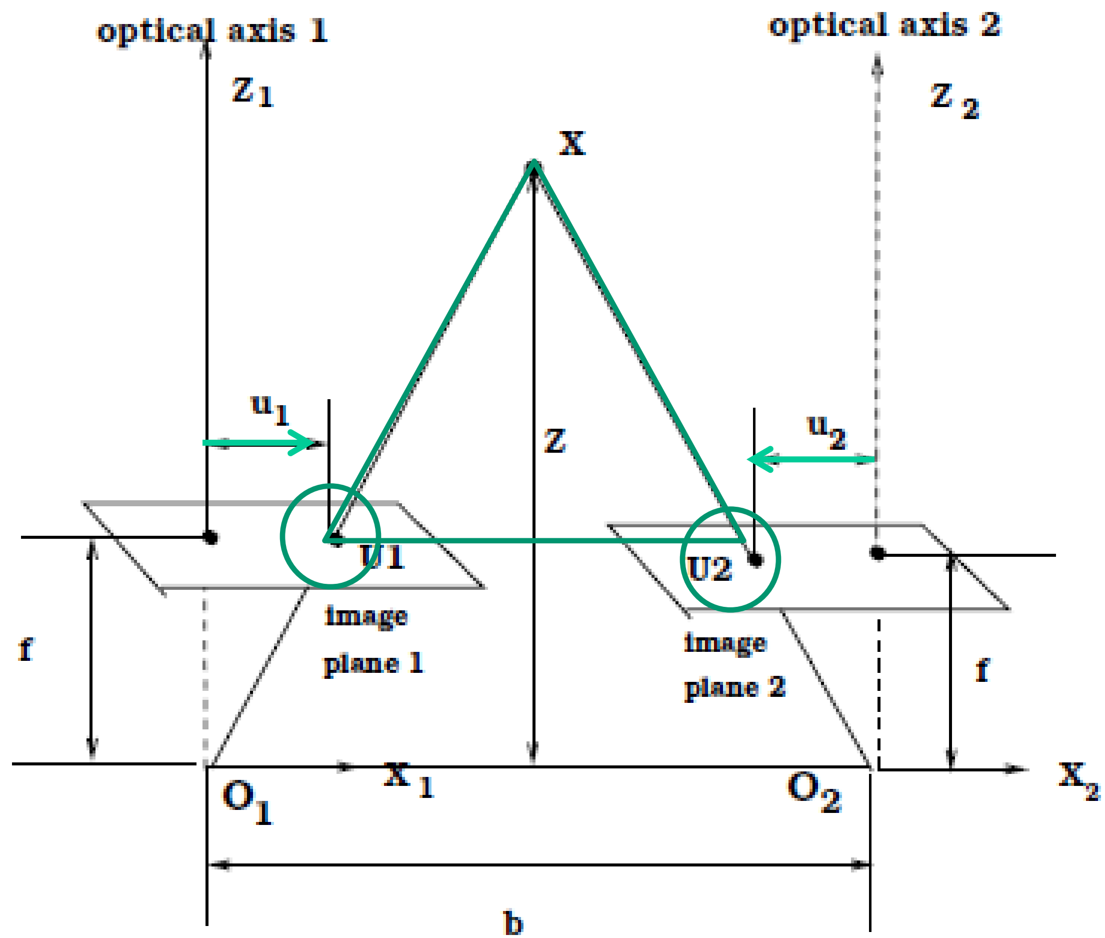
A Simple Case: 3D Triangulation – Rectified Geometry

Rectified geometry:

Hence, triangle XU_1U_2 is similar to triangle XO_1O_2

$$\frac{Z-f}{Z} = \frac{b-u_1+u_2}{b}$$

$$Z = \frac{fb}{u_1-u_2}$$



courtesy of Dr. Qiang Ji

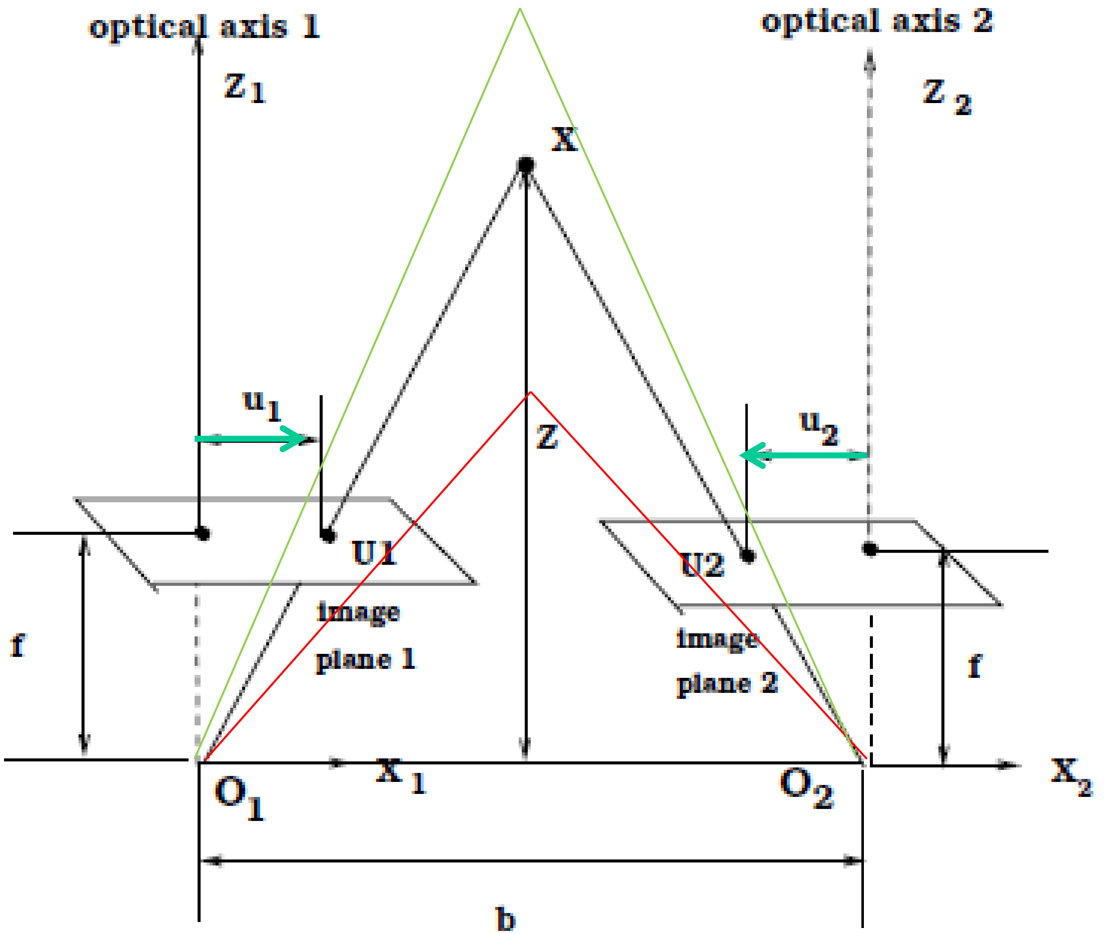
3D Triangulation – Rectified Geometry

$$Z = \frac{fb}{u_1 - u_2}$$

b: base distance
$u_1 - u_2$: disparity

$$u_1 - u_2 = \frac{fb}{Z}$$

Disparity is inversely proportional to the depth.



Example of Disparity Map

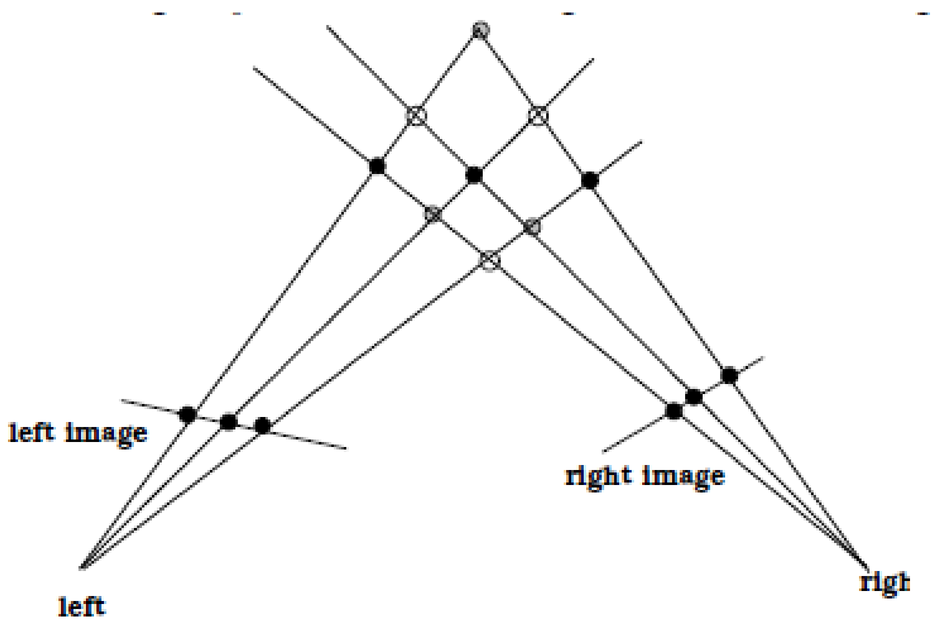
Groundtruth disparity map

Head and lamp from University of Tsukuba Stereo Dataset

http://cvlab-home.blogspot.com/2012/05/h2fecha-2581457116665894170-displaynone.html

Establish Correspondence

Problem: there are multiple images taken for the same object. We need to determine which point in one image is corresponding to the point in the others.

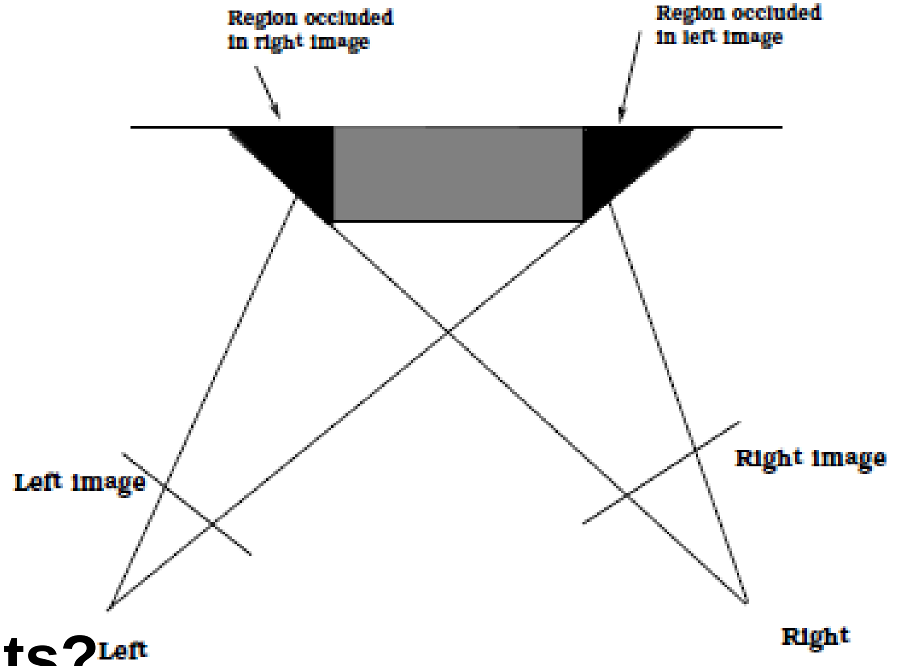


courtesy of Dr. Qiang Ji

Establish Correspondence

Challenges:

- Information associated with each image is not sufficient
- Image points in one image may not have corresponding point in another image due to
  - Field of view doesn't overlap
  - Occlusion
  - Miss detection



How to find corresponding points?

courtesy of Dr. Qiang Ji

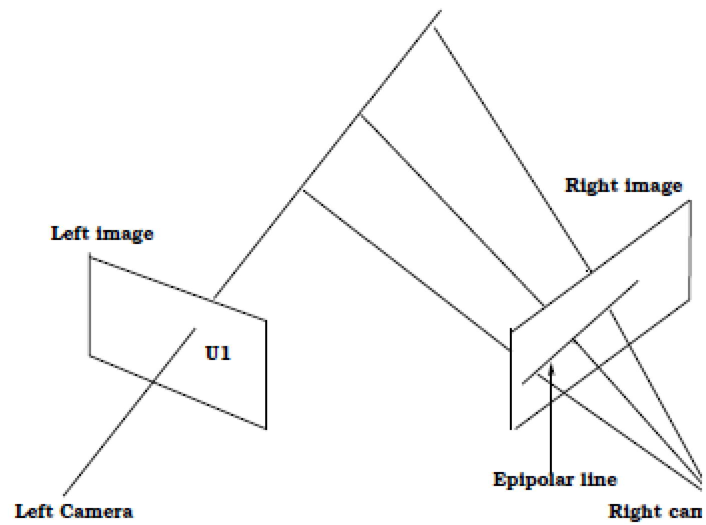
Establish Pointwise Correspondence using Epipolar Constraint

Epipolar constraint: For a 2D image point U_l in the left image, its corresponding point in the right image must lie on the conjugate epipolar line.

Epiploar line is the projection of the 3D line, which passes through U_l and the left camera center, on the right image.

Advantages:

- Simplify the searching for the corresponding points from 2D to 1D
- improve the efficiency significantly

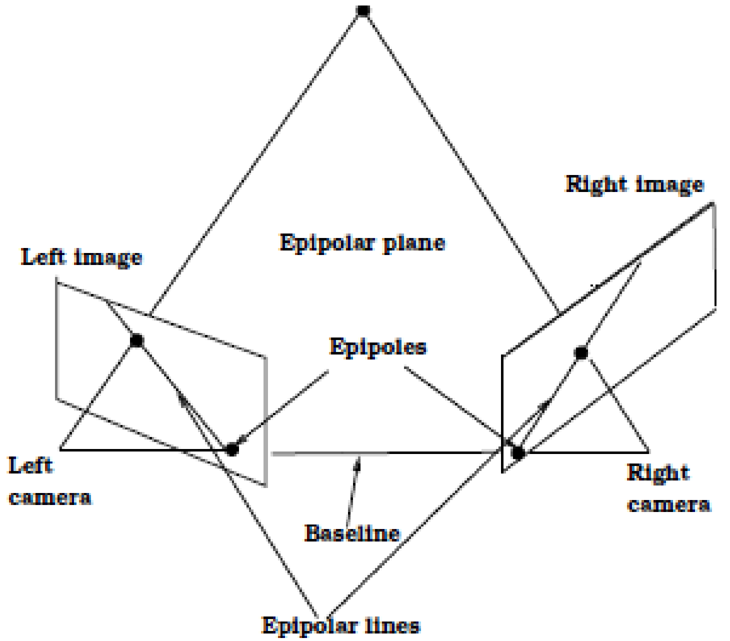


courtesy of Dr. Qiang Ji

Epipolar Geometry

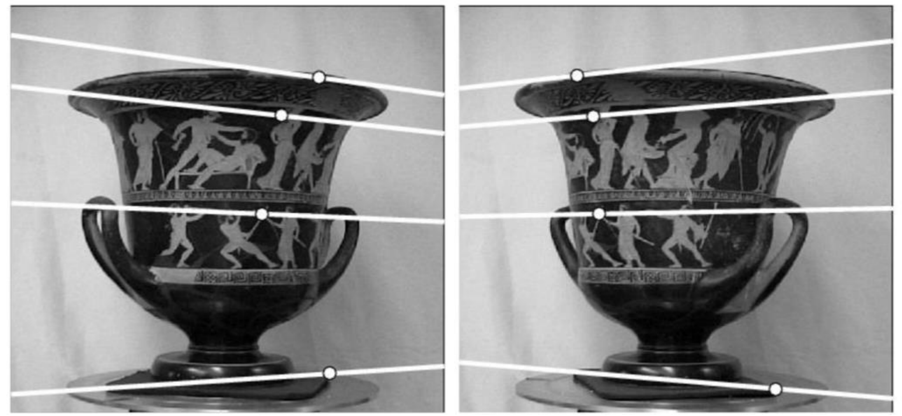
All epipolar lines go through an epipole on one image.

The epipole on the left (right) image is the projection of the optical center of the right (left) image on the left (right) image plane.



courtesy of Dr. Qiang Ji

Examples



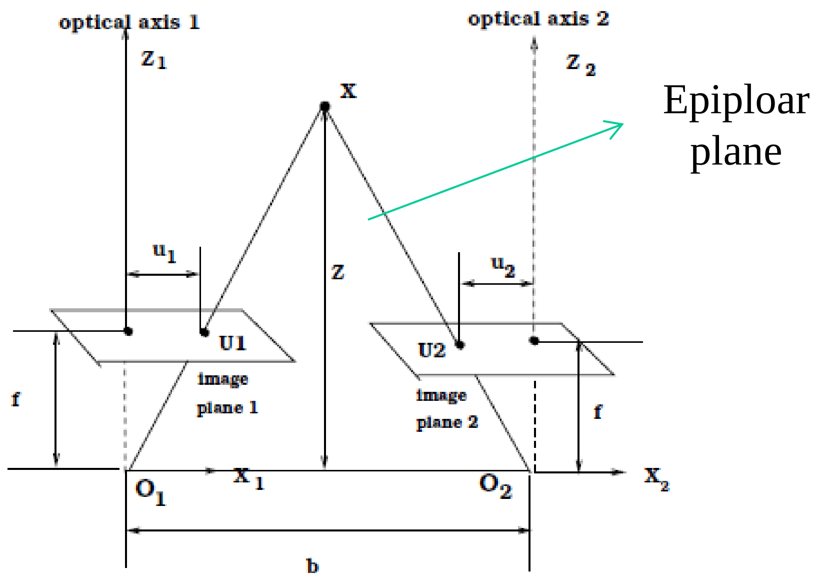
Epipolar Geometry

In rectified images,

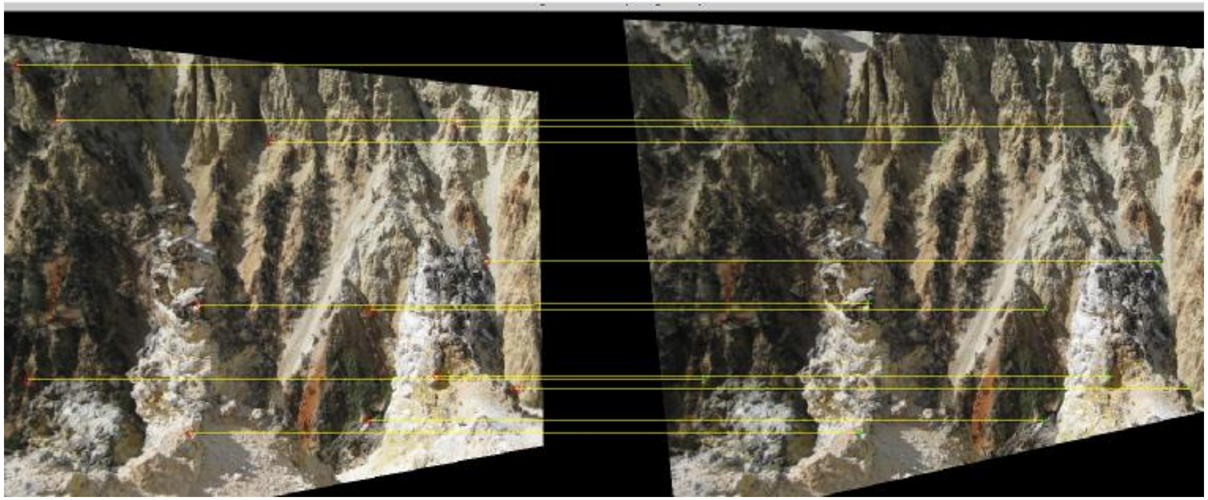
- the epipolar lines are parallel; ⟹ The epipole is at infinity
- conjugate epipolar lines are collinear and parallel to the base line.

Why?

• The conjugate epipolar lines are the intersections of the epipolar plane with the two image planes
• The two image planes are coplanar and parallel to the base line



Examples



Epipolar Geometry

P: a 3D point

$\boldsymbol{P_l}$**:** the coordinate of P on the left camera coordinate system

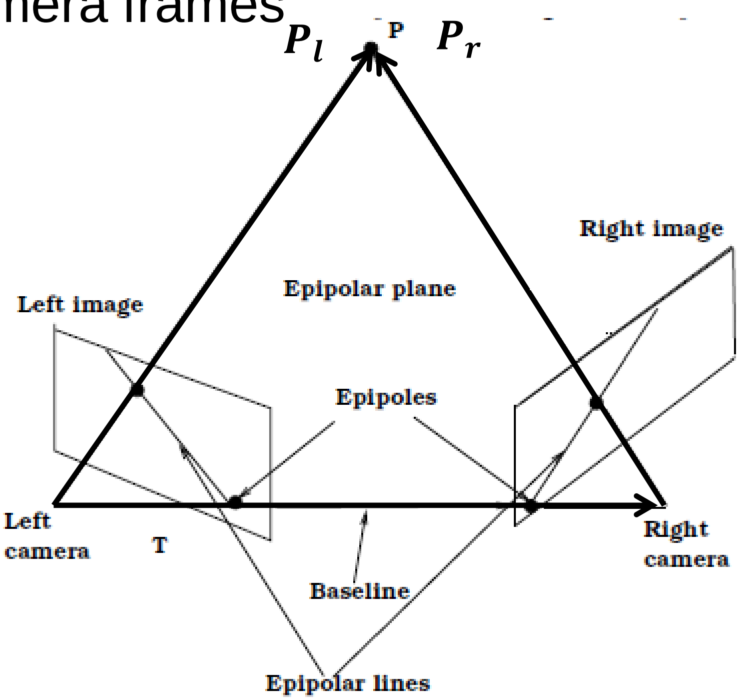
$\boldsymbol{P_r}$**:** the coordinate of P on the right camera coordinate system

T: the relative translation between two camera frames

$$\boldsymbol{O_r}^{\boldsymbol{l}} = \boldsymbol{O_r}^{\boldsymbol{r}} + \boldsymbol{T}$$

The right camera center measured in the left frame

The right camera center measured in the right frame



Epipolar Geometry

P: a 3D point

$\boldsymbol{P_l}$: the coordinate of P on the left camera coordinate system

$\boldsymbol{P_r}$: the coordinate of P on the right camera coordinate system

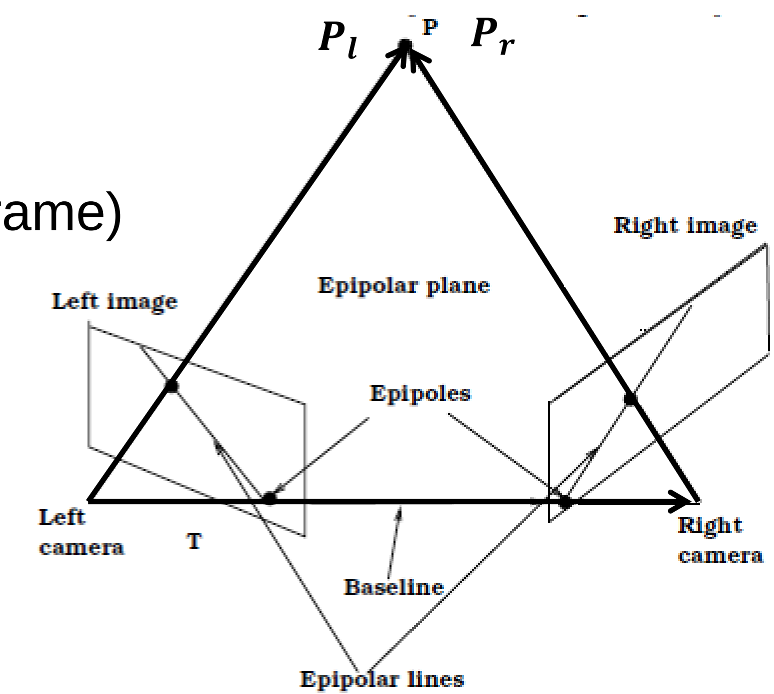
R: the relative rotation between two

camera frames (from right frame to left frame)

$$\boldsymbol{P_l} = \mathbf{R}\boldsymbol{P_r} + \boldsymbol{T}$$

$$\Updownarrow$$

$$\boldsymbol{P_r} = \mathbf{R}^t(\boldsymbol{P_l} - \boldsymbol{T})$$



Epipolar Geometry

$\boldsymbol{P_l}$ **,** $\boldsymbol{P_l - T}$ **and T** are coplanar → in the same epipolar plane

$$(\boldsymbol{T} \times \boldsymbol{P_l})^t \boxed{(\boldsymbol{P_l} - \boldsymbol{T})} = \mathbf{0}$$

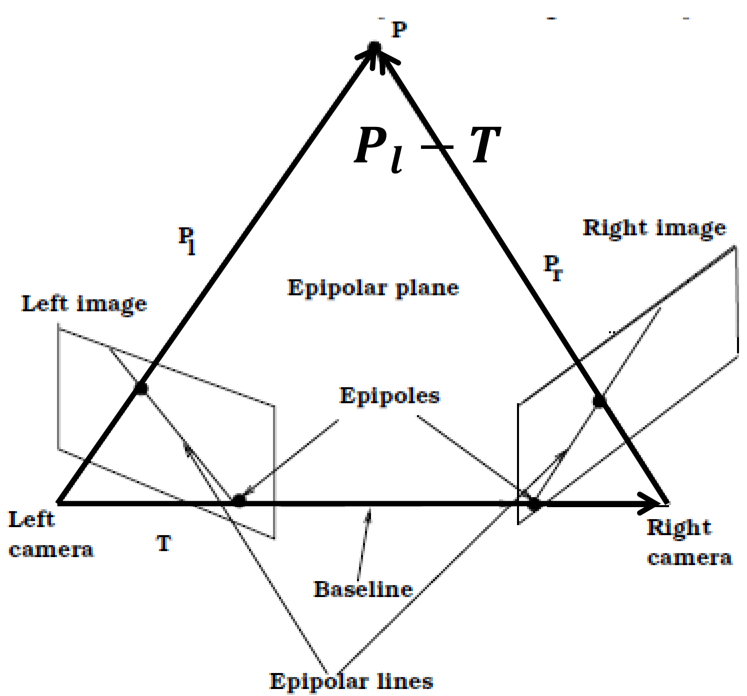
Normal of the plane

$$(\boldsymbol{T} \times \boldsymbol{P_l})^t \mathbf{R} \boldsymbol{P_r} = \mathbf{0}$$

Let

$$\mathbf{S} = \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{bmatrix}$$

$$\boldsymbol{T} \times \boldsymbol{P_l} = \boldsymbol{S} \boldsymbol{P_l}$$



Epipolar Geometry

$$(\boldsymbol{S}\boldsymbol{P}_l)^t\mathbf{R}\boldsymbol{P}_r = \mathbf{0} \leftrightarrow \boldsymbol{P}_l^t\,\mathbf{S}^t\mathbf{R}\boldsymbol{P}_r = 0$$

Let $\mathbf{E} = \mathbf{S}^t\mathbf{R}$

$$\boldsymbol{P}_l^t\,\mathbf{E}\boldsymbol{P}_r = \mathbf{0}$$

E is called the **essential matrix**

Since $\text{rank}(\mathbf{S}) = 2$**,** $\text{rank}(\mathbf{R}) = 3 \Rightarrow \text{rank}(\mathbf{E}) = 2$

The essential matrix

- establishes a link between the epipolar constraint and the relative position of the two camera systems
- has 5 degree of freedom

Fundamental Matrix

$\boldsymbol{U_l}$: the homogeneous coordinate of the projection of point P on the left image plane

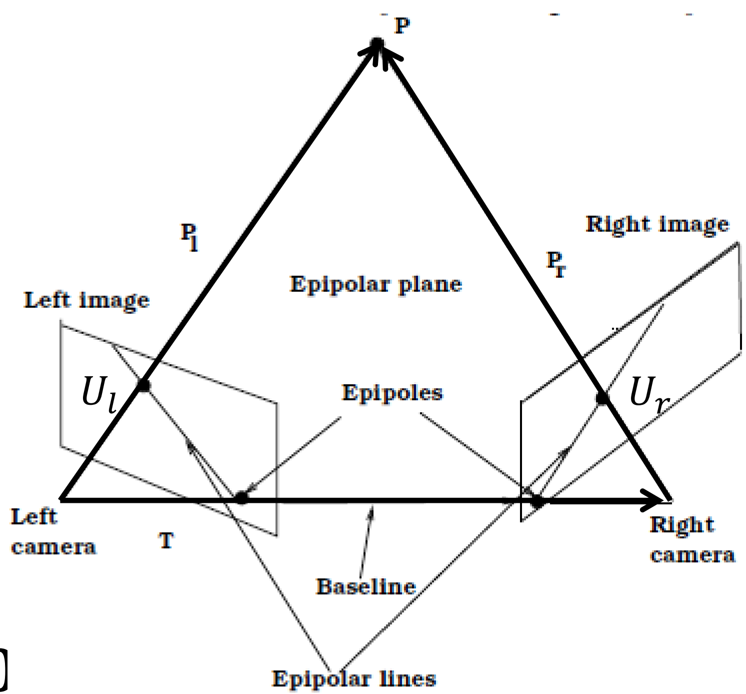
$\boldsymbol{U_r}$: the homogeneous coordinate of the projection of point P on the right image plane

$$\lambda_l \boldsymbol{U_l} = \mathbf{W}_l \boldsymbol{P_l} \qquad \lambda_r \boldsymbol{U_r} = \mathbf{W}_r \boldsymbol{P_r}$$

Intrinsic parameters for left and right cameras

Substitute to $P_l^t \mathbf{E} P_r = 0$

$$U_l^t \mathbf{W}_l^{-t} \mathbf{E} \mathbf{W}_r^{-1} U_r = 0$$



Fundamental Matrix

Fundamental matrix

$$\mathbf{F} = \mathbf{W_l}^{-\mathbf{t}}\mathbf{E}\mathbf{W_r}^{-\mathbf{1}}$$

$$\boldsymbol{U}_{\boldsymbol{l}}^{\boldsymbol{t}}\boxed{\mathbf{F}}\boldsymbol{U}_{\boldsymbol{r}} = \mathbf{0}$$

A pair of corresponding image points are related by **F**!

A fundamental matrix encodes

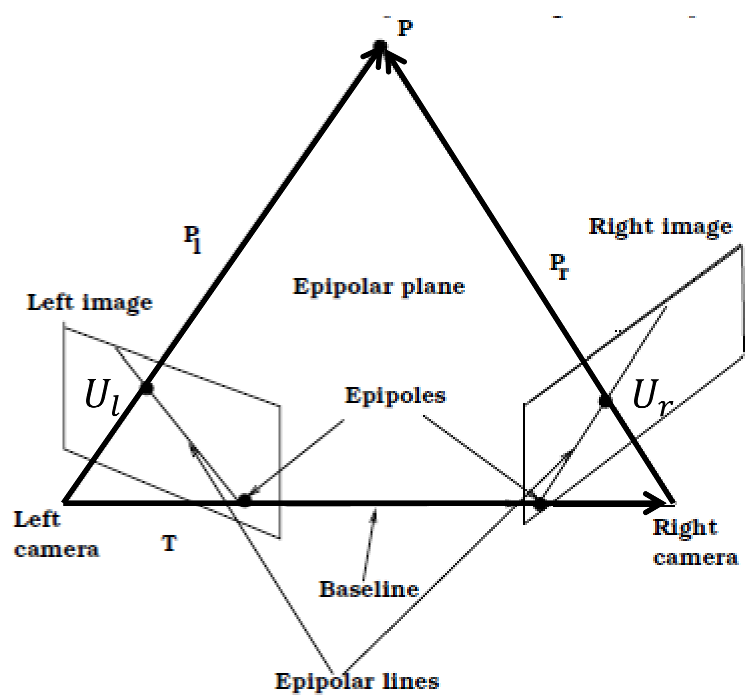
- the relative orientation and translation

between two camera systems

- the intrinsic parameters of the cameras

What is the rank of **F**?

rank(**F**)=2 because of **E**



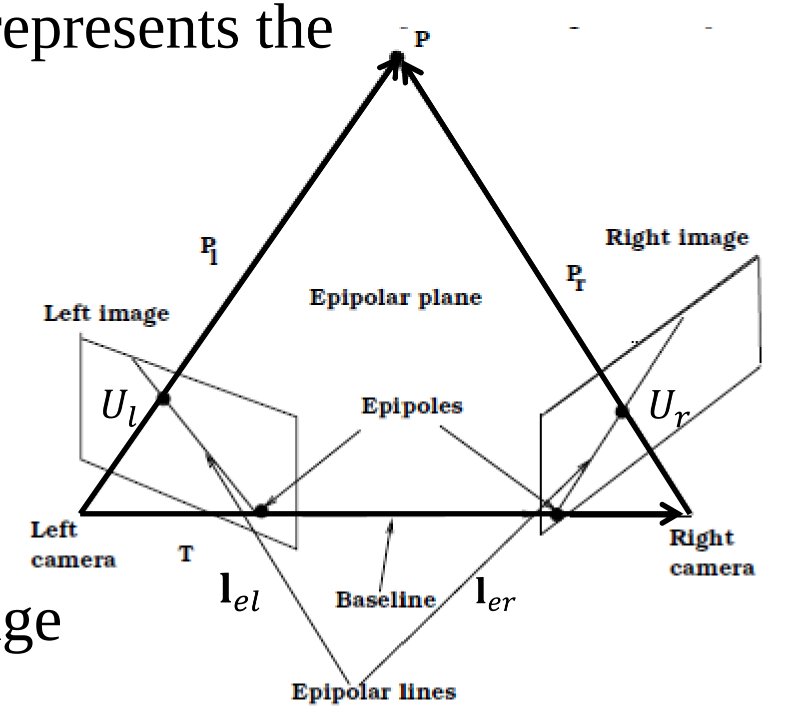
Information from Fundamental Matrix

$$U_l^t \mathbf{F} U_r = 0$$

Epipolar constraint: $U_l^t(\mathbf{F}U_r) = 0$ and $(\mathbf{F}^t U_l)^t\ U_r = 0$

Recall line function: $\mathbf{l} \cdot \mathbf{p} = \mathbf{0}$, where $\mathbf{l}$ represents the line parameters

$\mathbf{l}_{el} = \mathbf{F}U_r$



U_l is on a line determined by $\mathbf{l}_{el}$

- Epipolar line on the left image
- Corresponding to U_r on the right image

Information from Fundamental Matrix

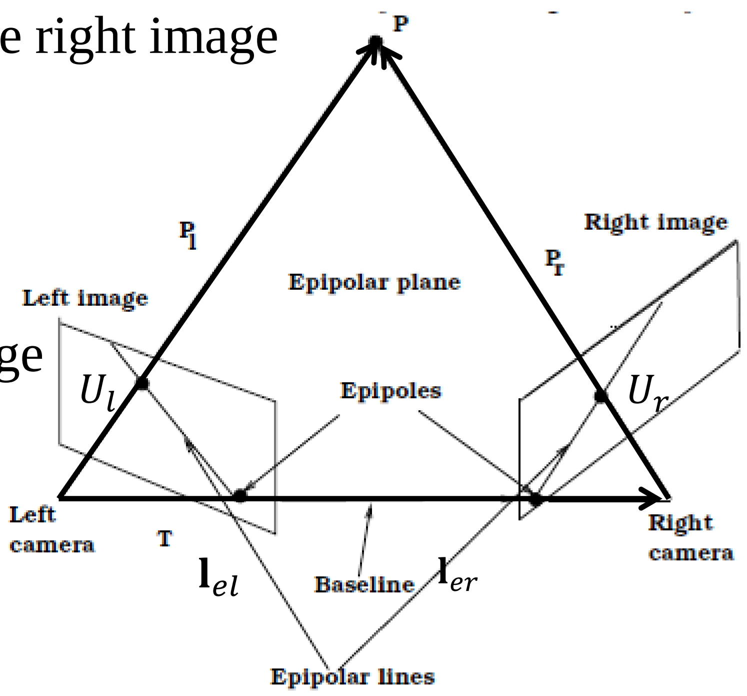
$$U_l^t \mathbf{F} U_r = 0$$

- Epipolar constraint: $U_l^t(\mathbf{F}U_r) = \mathbf{0}$ and $(\mathbf{F}^t U_l)^t \; U_r = \mathbf{0}$
- $\mathbf{l}_{el} = \mathbf{F}U_r$ Epipolar line on the left image corresponding to U_r on the right image

Similarly

$\mathbf{l}_{er} = \mathbf{F}^t U_l$ Epipolar line on the right image corresponding to U_l on the left image

Given F and a point on one image, we can estimate the corresponding epipolar line



Information from Fundamental Matrix

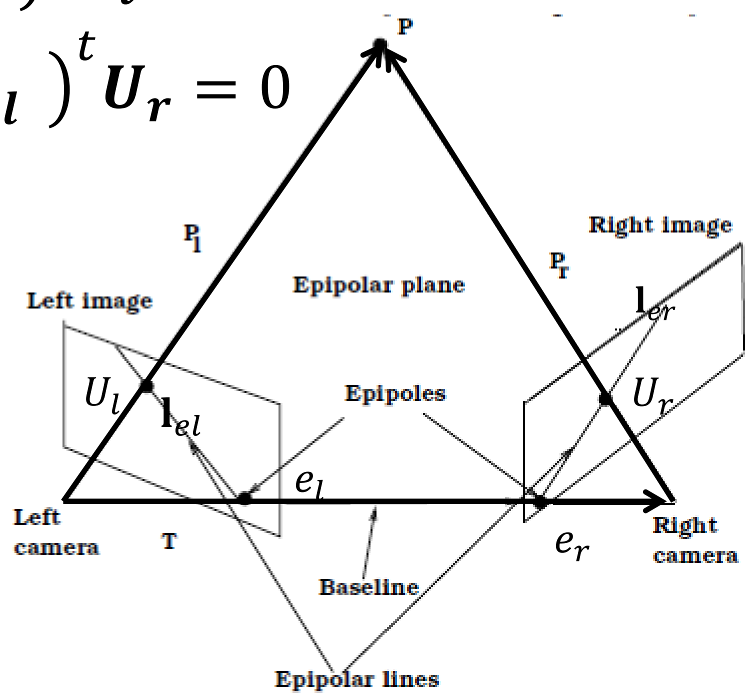
$$U_l^t \, \mathbf{F} U_r = \mathbf{0}$$

Since all epipolar line pass through the epipole,

$$\boldsymbol{e}_r^t \mathbf{l}_{er} = \mathbf{0} \rightarrow \boldsymbol{e}_r^t \mathbf{F}^t U_l = 0 \qquad \rightarrow \left(\mathbf{F}\boldsymbol{e}_r\right)^t U_l = 0$$

$$\boldsymbol{e}_l^t \mathbf{l}_{el} = \mathbf{0} \rightarrow \boldsymbol{e}_l^t \mathbf{F} U_r = 0 \qquad \rightarrow \left(\mathbf{F}^t\boldsymbol{e}_l\right)^t U_r = 0$$

$$\boxed{\begin{aligned} \mathbf{F}\boldsymbol{e}_r &= 0 \\ \mathbf{F}^t\boldsymbol{e}_l &= 0 \end{aligned}}$$



Given F, the epipole of right (left) image is the null eigenvector of **F** ($\mathbf{F}^t$)