Announcement

Quiz #4 is available in Blackboard.

Due date: 11:59pm EST, Wednesday, March 22

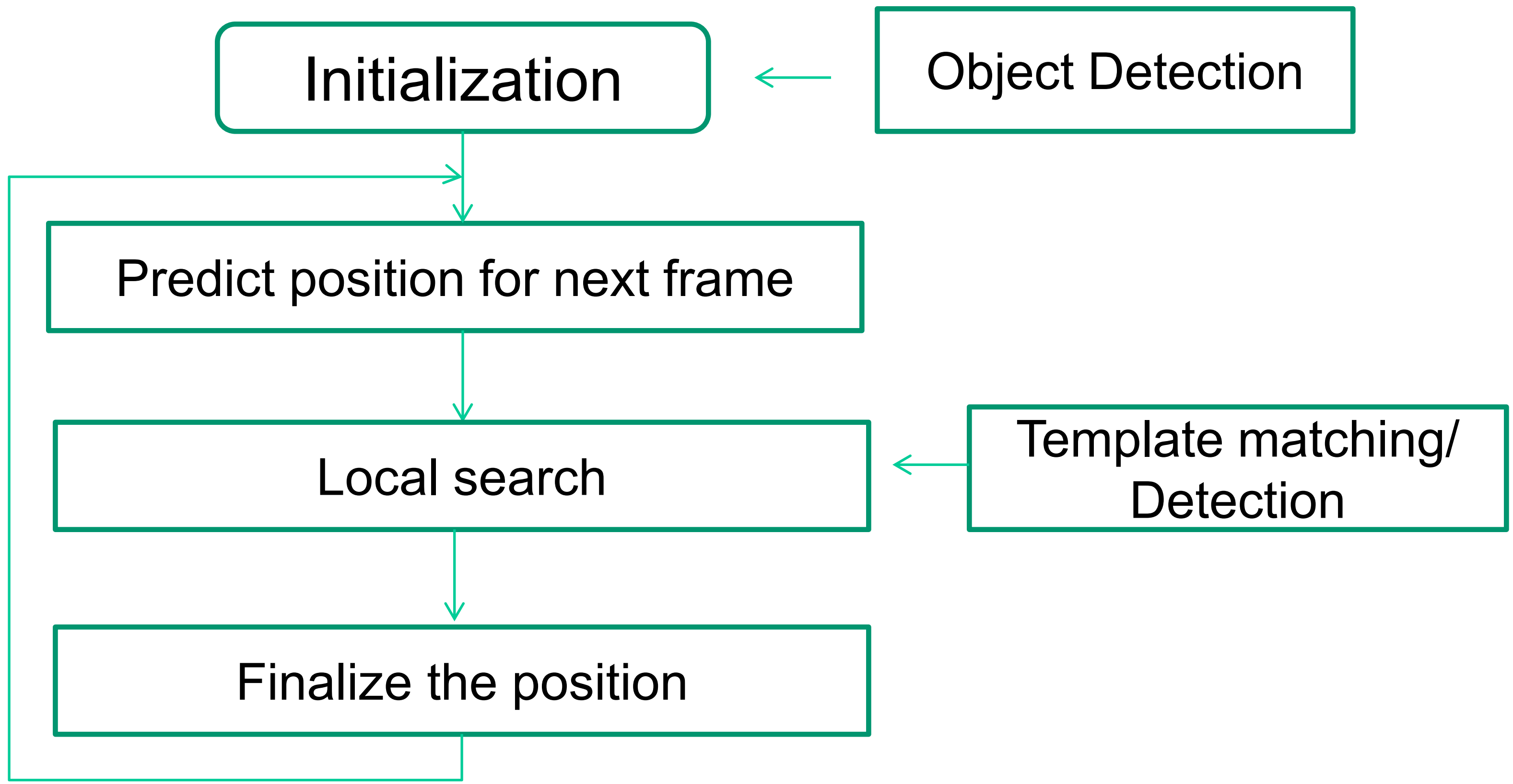
Open book and open notes

Today

Early vision on multiple images

- Object tracking

General Strategy for Object Tracking



A well-known algorithm following the strategy -- Kalman filter

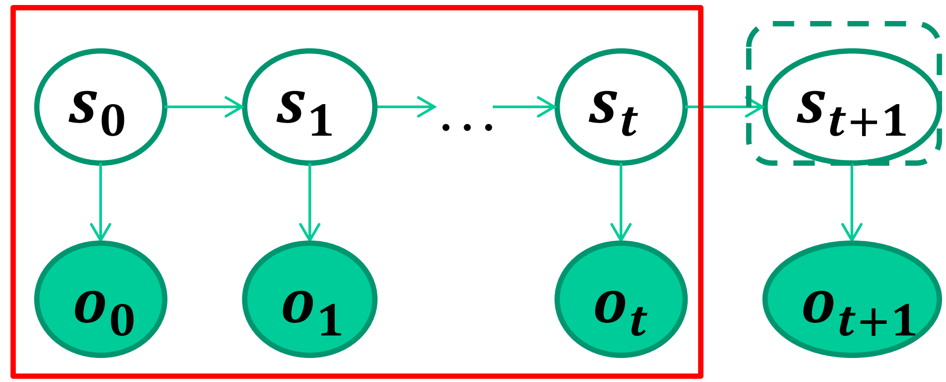
Tracking – General Probabilistic Formulation

Given

- $P(\mathbf{s}_t|\mathbf{o}_0,\cdots,\mathbf{o}_t)$ - "Prior"

We should like to know

- $P(\mathbf{s}_{t+1}|\mathbf{o}_0,\cdots,\mathbf{o}_t)$
  - "Predictive distribution"
- $P(\mathbf{s}_{t+1}|\mathbf{o}_0,\cdots,\mathbf{o}_t,\mathbf{o}_{t+1})$
  – "Posterior"



How to compute them?

$$P(\mathbf{s}_{t+1}|\mathbf{o}_0,\cdots,\mathbf{o}_t) = \sum_{\mathbf{s}_t} P(\mathbf{s}_{t+1},\mathbf{s}_t|\mathbf{o}_0,\cdots,\mathbf{o}_t)$$

$$= \sum_{\mathbf{s}_t} \boxed{P(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{o}_0,\cdots,\mathbf{o}_t)} * P(\mathbf{s}_t|\mathbf{o}_0,\cdots,\mathbf{o}_t)$$

Tracking – General Probabilistic Formulation

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- $P(\mathbf{s}_t|\mathbf{o}_0,\cdots,\mathbf{o}_t)$ - "Prior"

We should like to know

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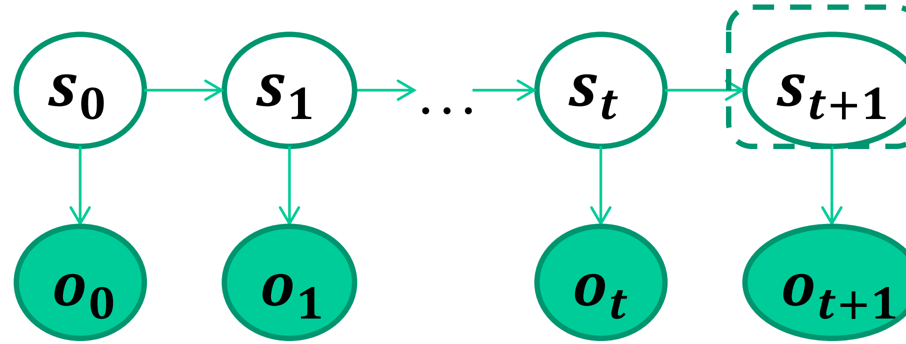


How to compute them?

$$P(\mathbf{s}_{t+1}|\mathbf{o}_0,\cdots,\mathbf{o}_t,\mathbf{o}_{t+1}) = \frac{\boxed{P(\mathbf{o}_{t+1}|\mathbf{s}_{t+1},\mathbf{o}_0,\cdots,\mathbf{o}_t)} * P(\mathbf{s}_{t+1}|\mathbf{o}_0,\cdots,\mathbf{o}_t)}{\sum_{\mathbf{s}_{t+1}} P(\mathbf{o}_{t+1}|\mathbf{s}_{t+1},\mathbf{o}_0,\cdots,\mathbf{o}_t) * P(\mathbf{s}_{t+1}|\mathbf{o}_0,\cdots,\mathbf{o}_t)}$$

Tracking – General Assumptions

- **First-order markov**
  - The current state only depends on the state of the previous time step
  - $P(\mathbf{s}_t|\mathbf{s}_0,\cdots,\mathbf{s}_{t-1}) = P(\mathbf{s}_t|\mathbf{s}_{t-1})$
- **Given the current state, the measurement at current time step is independent of the previous measurements**
  - $P(\mathbf{o}_t|\boldsymbol{s}_t,\mathbf{o}_0,\cdots,\mathbf{o}_{t-1}) = P(\mathbf{o}_t|\mathbf{s}_t)$

Tracking – General Probabilistic Formulation

Prediction: $$P(\mathbf{s}_{t+1}|\mathbf{o}_0,\cdots,\mathbf{o}_t) = \sum_{\mathbf{s}_t} P(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{o}_0,\cdots,\mathbf{o}_t) * P(\mathbf{s}_t|\mathbf{o}_0,\cdots,\mathbf{o}_t)$$

$$= \sum_{\mathbf{s}_t} \underbrace{\boxed{P(\mathbf{s}_{t+1}|\mathbf{s}_t)}}_{\text{System model}} * P(\mathbf{s}_t|\mathbf{o}_0,\cdots,\mathbf{o}_t)$$

Update:

$$P(\mathbf{s}_{t+1}|\mathbf{o}_0,\cdots,\mathbf{o}_t,\mathbf{o}_{t+1}) = \frac{P(\mathbf{o}_{t+1}|\mathbf{s}_{t+1},\mathbf{o}_0,\cdots,\mathbf{o}_t) * P(\mathbf{s}_{t+1}|\mathbf{o}_0,\cdots,\mathbf{o}_t)}{\sum_{\mathbf{s}_{t+1}} P(\mathbf{o}_{t+1}|\mathbf{s}_{t+1},\mathbf{o}_0,\cdots,\mathbf{o}_t) * P(\mathbf{s}_{t+1}|\mathbf{o}_0,\cdots,\mathbf{o}_t)}$$

$$= \frac{\boxed{P(\mathbf{o}_{t+1}|\mathbf{s}_{t+1})} * P(\mathbf{s}_{t+1}|\mathbf{o}_0,\cdots,\mathbf{o}_t)}{\sum_{\mathbf{s}_{t+1}} P(\mathbf{o}_{t+1}|\mathbf{s}_{t+1}) * P(\mathbf{s}_{t+1}|\mathbf{o}_0,\cdots,\mathbf{o}_t)}$$

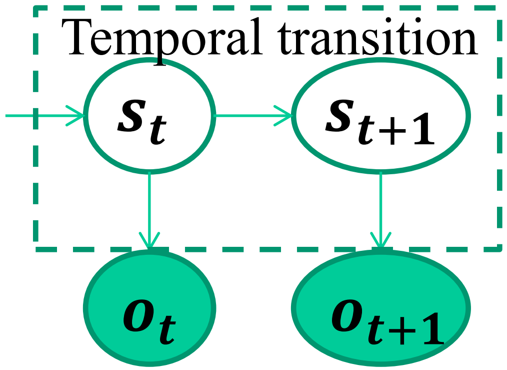
Measurement model

Kalman Filtering

Assumptions

- **Linear state model**
  - The next state $\boldsymbol{s}_{t+1}$ is linearly related to the current state $\boldsymbol{s}_t$
- **Uncertainty satisfy a Gaussian**

Linear State Model



$$\mathbf{s}_{t+1} = \Phi \mathbf{s}_t + \mathbf{w}_t$$

Φ: state transition matrix

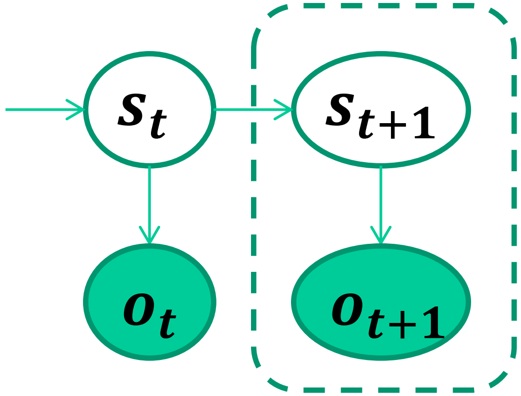
$\mathbf{w}_t$: system perturbation satisfying a Gaussian distribution $\mathbf{w}_t \sim N(0, \boldsymbol{W})$

If motion between two frames is small,

$$\Phi = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Measurement Model

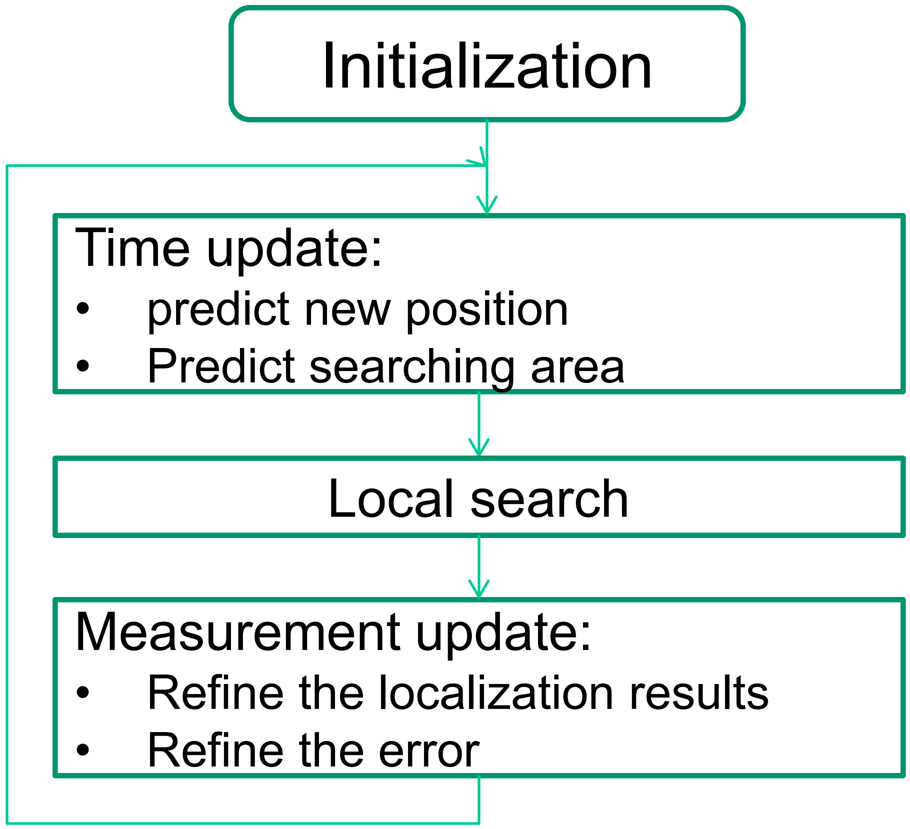
$$\mathbf{o}_t = \mathbf{H}\mathbf{s}_t + \mathbf{r}_t$$



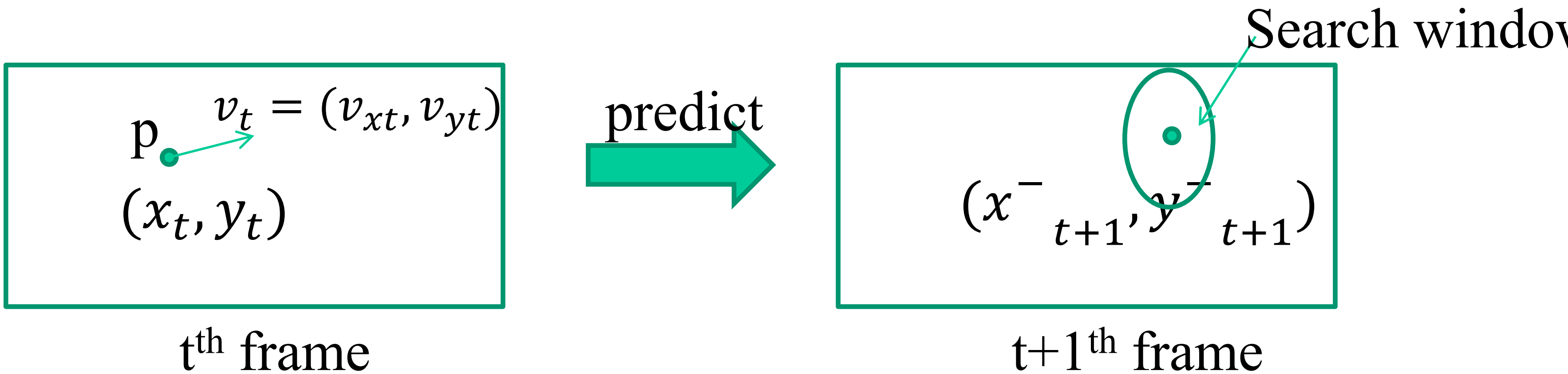
- $\mathbf{H}$ describes how the measurement relates to the state vector
- **For example,** $\mathbf{o}_t = (x_t, y_t)$ is the measured feature position

  The simplest case $H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$
- $\mathbf{r}_t$ is the measurement uncertainty satisfying a Gaussian distribution $\mathbf{r}_t \sim N(0, \boldsymbol{R})$

Kalman Filtering



Kalman Filtering – Prediction



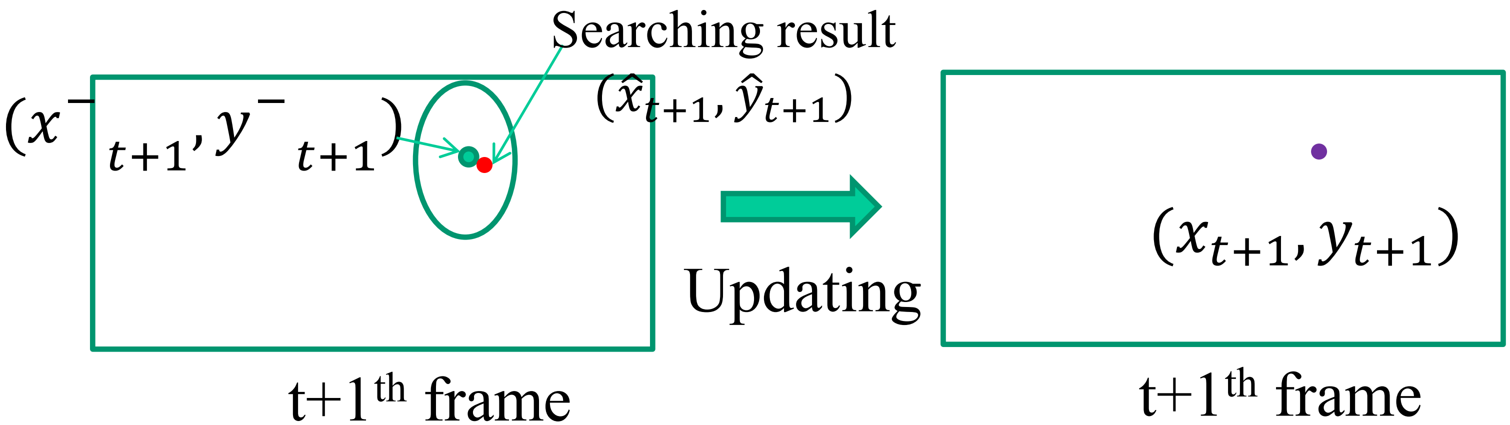
State prediction: $\mathbf{s}^-_{t+1} = \Phi \mathbf{s}_t$

Confidence of the prediction: $\mathbf{\Sigma}^-_{t+1} = \Phi \mathbf{\Sigma}_t \Phi^T + W$

Determine the size of search window

$3\lambda_x \times 3\lambda_y$

Kalman Filtering – Updating

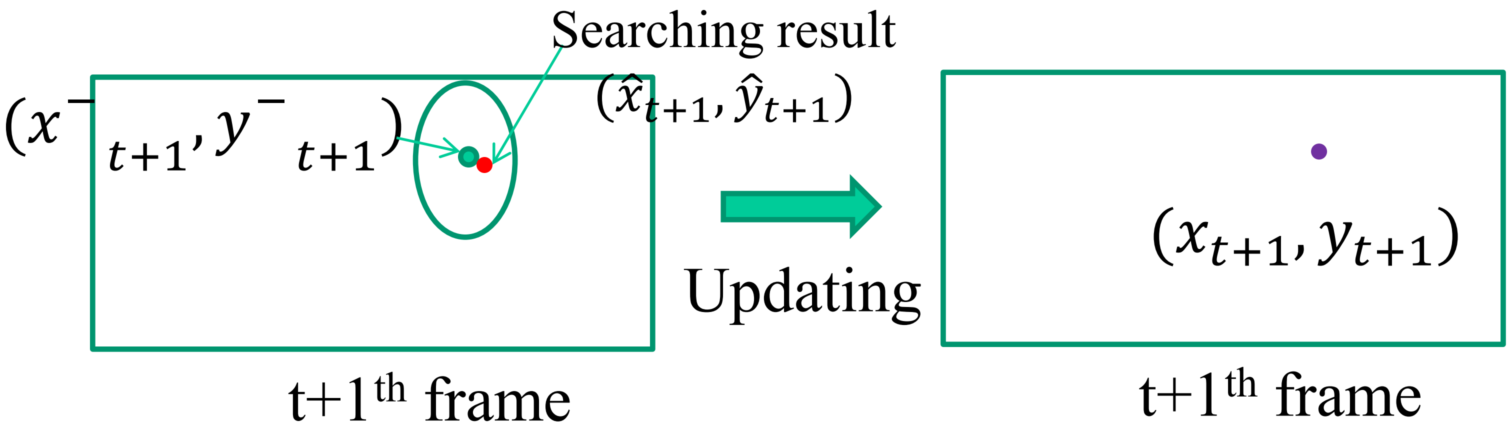


Kalman gain: $K_{t+1} = \sum_{t+1}^{-} H^T \left(H \sum_{t+1}^{-} H^T + R\right)^{-1}$

A weighting factor determines

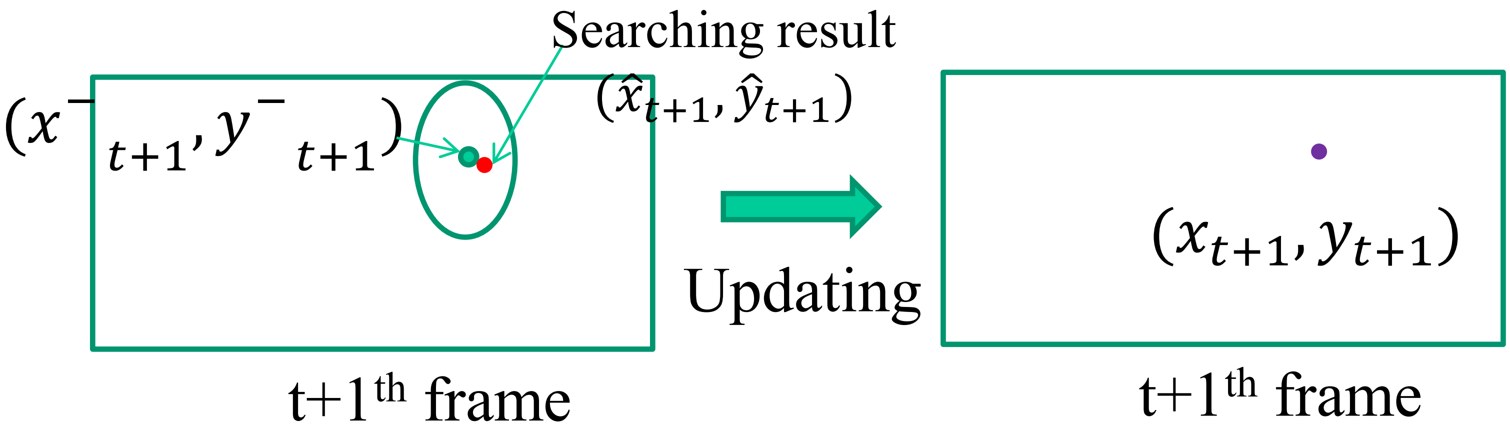
- the contribution of the measurement and the prediction in the final estimation -- which one is more reliable

Kalman Filtering – Updating



State update: $\mathbf{s}_{t+1} = \mathbf{s}^-_{t+1} + K_{t+1}\left(\underbrace{\mathbf{o}_{t+1} - \mathbf{H}\mathbf{s}^-_{t+1}}_{\text{Measurement residual: difference between the measurement and the prediction}}\right)$

Kalman Filtering – Updating



State update: $\mathbf{s}_{t+1} = \mathbf{s}^-_{t+1} + K_{t+1}\left(\mathbf{o}_{t+1} - \mathbf{H}\mathbf{s}^-_{t+1}\right)$

Error covariance update: $\Sigma_{t+1} = (I - K_{t+1}H)\,\Sigma^-_{t+1}$

Kalman Filtering

Initialization

Time update:
- predict new position
- Predict searching area

Local search

Measurement update:
- Refine the localization results
- Refine the error

$\mathbf{s}_0$

$$W = \begin{bmatrix} 9 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$R = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\Sigma_0 = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 36 & 0 \\ 0 & 0 & 0 & 36 \end{bmatrix}$$

Limitation of Kalman Filtering

Failed cases:

- **State transition is not linear**
  - Sudden motion direction/velocity changes

→ Extended Kalman filtering

- **Uncertainty is not Gaussian**

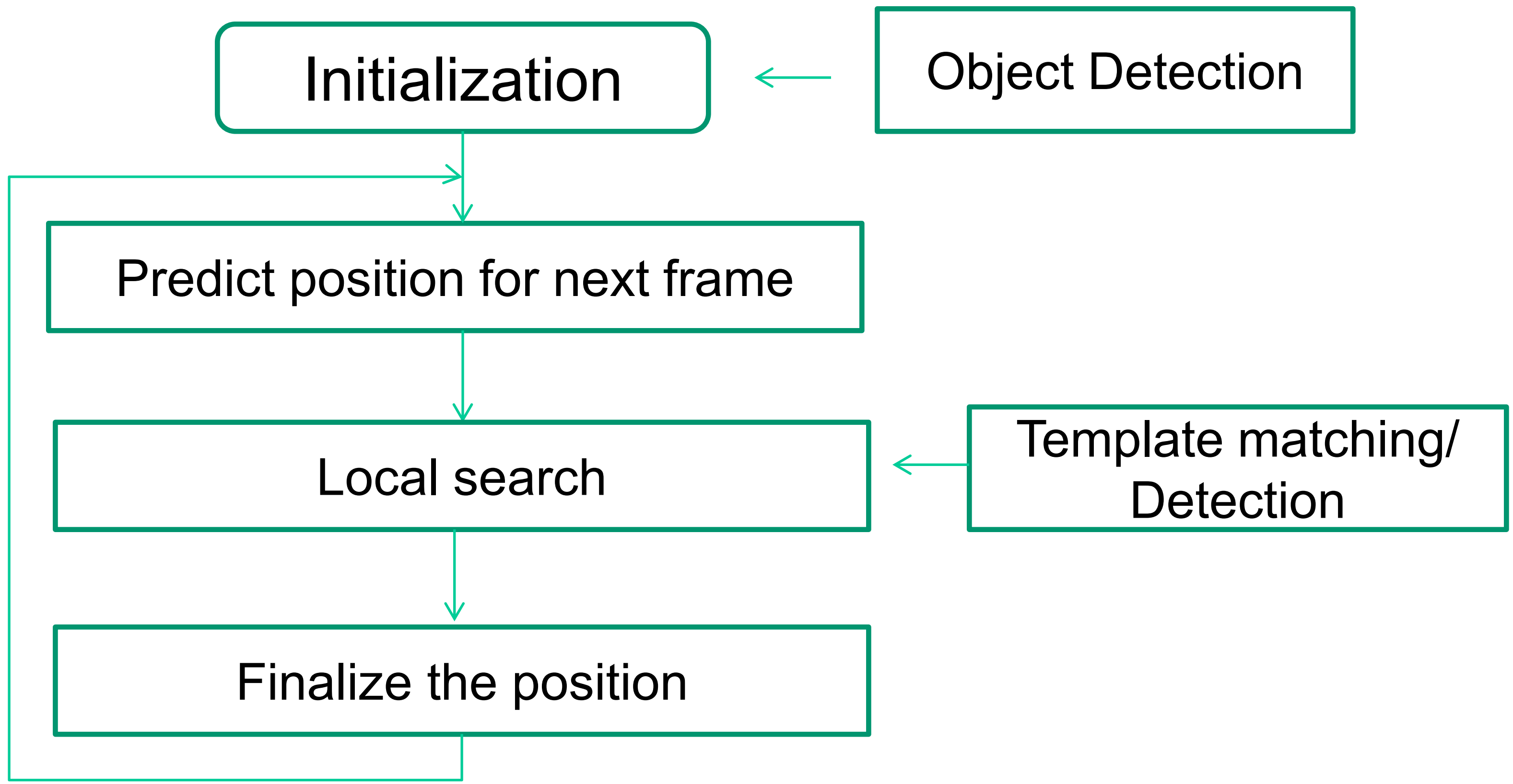
→ Unscented Kalman filtering

Reading Assignments

An Introduction to the Kalman Filter by Welch and Bishop

http://www.cs.unc.edu/~welch/media/pdf/kalman_intro.pdf

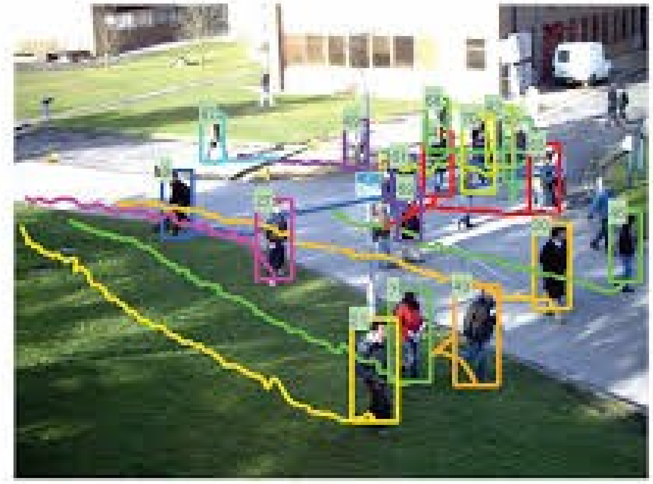
General Strategy for Object Tracking



Tracking – How to Obtain Measurements

- Tracking by detection
- Tracking by template matching

Tracking by Detection



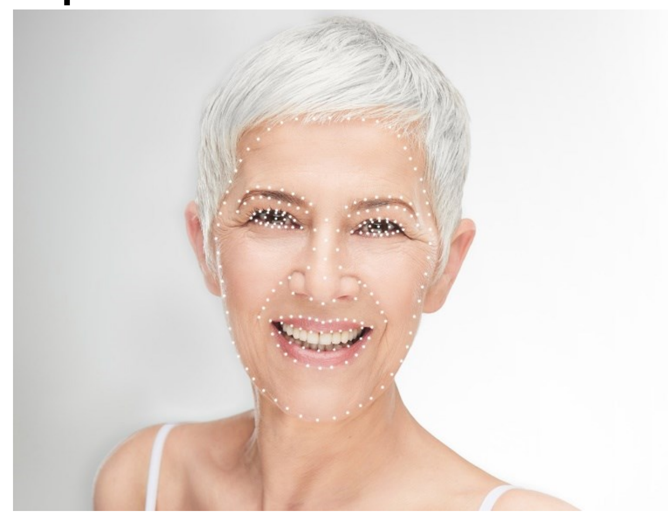
xiang_iccv15.pdf (stanford.edu)

Assume

- Specific type of objects
- a very reliable detector (e.g. faces; back of heads)
- Sparse targets with distinctive properties

Link detects across time

- Only one - easy
- Multiple – challenging
  - Similar appearance
  - Occlusion
  - Crossing



Facial Landmark Tracking | Facetrace | AlgoFace

Tracking by Template Matching

Without prior knowledge of the target – no detectors

- Moving foreground objects
- Tracking a target among multiple objects with similar appearance

Problem: tracking the object in the sequence by matching with a template

Tracking by Template Matching

Template: a region (mostly a rectangular region) with known appearance

Template matching: searching a region that minimizes an SSD error (Sum of Squared Differences)

$$SSD = \sum_{i,j} (I_t\,(x+i, y+j) - T(i,j))^2$$

Tracking by Template Matching

Potential issues:

- **Template updating**
  - When and how
- **Template matching metrics**
  - SSD vs. histogram-based matching

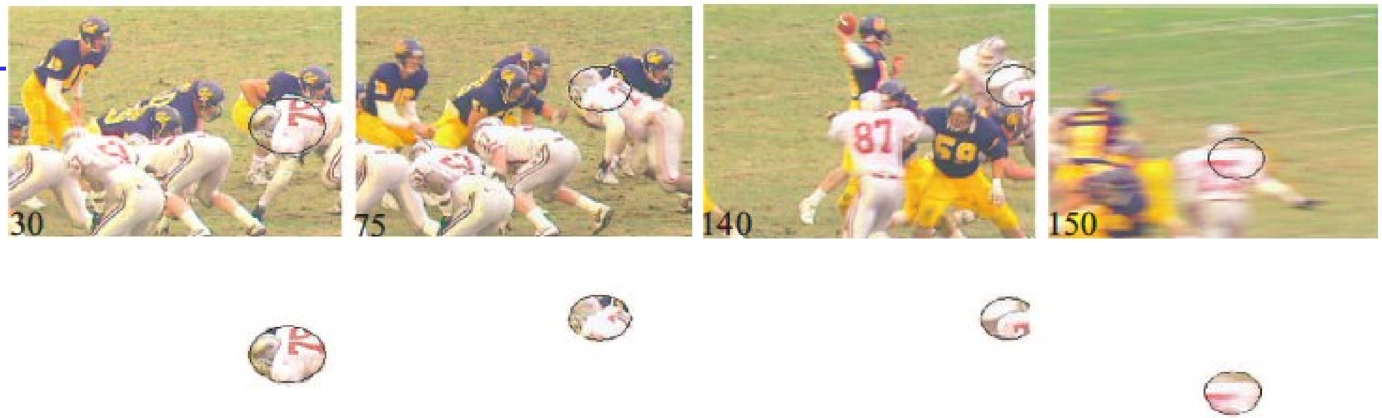


FIGURE 11.5: Four frames from a sequence depicting football players, with superimposed domains. The object to be tracked is the blob on top of player 78 (at the center right in frame 30). We have masked off these blobs (**below**) to emphasize just how strongly the pixels move around in the domain. Notice the motion blur in the final frame. These blobs can be matched to one another, and this is done by comparing histograms (in this case, color histograms), which are less affected by deformation than individual pixel values. *This figure was originally published as Figure 1 of "Kernel-Based Object Tracking" by D. Comaniciu, V. Ramesh, and P. Meer, IEEE Transactions on Pattern Analysis and Machine Intelligence, 2003, © IEEE 2003.*

Forsyth and Ponce, "Computer Vision – A Modern Approach 2e"

Reading Assignments

- Chapter 11 (Tracking) of Forsyth & Ponce

- CVonline: Motion and Time Sequence Analysis

https://sites.google.com/site/cvonlinewiki/home/motion-and-time-sequence-analysis-related-concepts