Homework 2

Due: 1:15 pm, Wednesday, October 5

Note:
1. While discussion on the course material is encouraged, you are required to work on the homework independently.
2. To get the credit, sufficient and appropriate intermediate steps are required in your answers or proofs.
3. Turn in the homework on time. Late homework will not be accepted.
4. Both undergraduate and graduate students must answer the first 4 questions. The graduate students must answer the 5th question.

1. In the class, we showed that any point on the plane can be represented as in its parametric form $T(\alpha, \beta) = P + \alpha u + \beta v$ and also can be represented as in its point-normal form $(T - P) \cdot n = 0$, where $n = u \times v$ is the normal of the plane. Show that these two forms of plane are equivalent.

2. Do the following sequences commute? If yes, prove it mathematically. Otherwise, give a counterexample.
   (1) A translation along the x axis and a translation along the y axis
   (2) A rotation about the x-axis and a rotation about the y-axis
   (3) A rotation about the x-axis and a non-uniform scaling ($s_x \neq s_y$)

3. (1) We have used vertices in three dimensions to define objects such as three dimensional polygons. Given a set of vertices, find a test to determine whether the polygon, which they determine, is planar.
   (2) Three vertices determine a triangle if they do not lie in the same line. Devise a test for collinearity of three vertices.

4. Suppose a triangle consisting of points $P_1 = [0 \ 0 \ 0]^T$, $P_2 = [2 \ 0 \ 0]^T$, and $P_3 = [1 \ 2 \ 3]^T$. Rotate the triangle by 30 degrees about the line passing through $P_1$ (the origin) and $P_3 = [1 \ 2 \ 3]^T$. Give the vertices of the triangle after rotation.

5. (Graduate students only) Show that the shear transformation can be achieved from a sequence of translation, rotation, and scaling transformations.