ILOG AMPL CPLEX System

Version 7.0 User's Guide

Standard (Command-line) Version
Including CPLEX Directives

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1 Introduction

1.1 Welcome to AMPL

Welcome to the AMPL Modeling System – a comprehensive, powerful, algebraic modeling language for problems in linear, nonlinear, and integer programming. AMPL is based upon modern modeling principles and utilizes an advanced architecture providing flexibility most other modeling systems lack. AMPL has been proven in commercial applications, and is successfully used in demanding model applications around the world.

AMPL helps you create models with maximum productivity. By using AMPL's natural algebraic notation, even a very large, complex model can often be stated in a concise (often less than one page), understandable form. Since AMPL models are easy to understand, debug, and modify, AMPL also makes maintaining models easy.

1.2 Using This Guide

This brief Guide will take you through starting up AMPL, reading a model and supplying data, and solving (optimizing) the model using CPLEX. The first three sections cover issues such as using command-line options and environment variables and using AMPL on different operating systems. Later sections provide a detailed description of CPLEX directives.

Installation and set-up are not described in this document – consult the accompanying installation instructions. Documentation describing other AMPL-compatible solvers distributed by ILOG is also available separately.

This Guide does not teach you the AMPL language. To learn and effectively use the features of the AMPL language, you should have a copy of the book *AMPL: A Modeling Language for Mathematical Programming* by Robert Fourer, David M. Gay and Brian W. Kernighan (ISBN 0-534-50983-5, first published in 1993 by The Scientific Press, now published by Duxbury Press). This book includes a tutorial on AMPL and optimization modeling; models for many "classical" problems such as production, transportation, blending, and scheduling; discussions of modeling concepts such as linear,

nonlinear and piecewise-linear models, integer linear models, and columnwise formulations; and a reference section.

AMPL is continuously undergoing development, and while we strive to keep users updated on language features and capabilities, the official reference to the language is the AMPL book, which is naturally revised less frequently.

1.3 Installing AMPL

Please read these instructions in their entirety before beginning the installation. Remember that most distributions will operate properly only on the specific platform and operating system version for which they were intended. If you upgrade your operating system you may need to obtain a new distribution.

All AMPL installations include cplexamp (cplexamp.exe on Windows), the CPLEX solver for AMPL. This combined distribution is known as the AMPL/CPLEX system.

Note that cplexamp may not be licensed for a few users with unsupported solvers. However, most AMPL installations will include the use of cplexamp.

1.3.1 Requirements

AMPL may be installed and run on the following configurations:

Computer	*Operating System*	*Release*
DEC Alpha	DEC UNIX	4.0 and higher
HP PA-7xxx	HP-UX	11 and higher
HP PA-8xxx	HP-UX	11 and higher
Intel PC	Linux	2.1 and higher
Intel PC	Windows NT	3.5 and higher
Intel PC	Windows 9x/2000	
RS6000 or PowerPC	AIX	4.3 and higher
SGI 32-bit (MIPS3)	Irix	6.5 and higher
SGI 64-bit (MIPS4)	Irix	6.5 and higher
Sun SPARC	Solaris	2.6 and higher
Sun Ultra	Solaris	2.6 and higher

1.3.2 Unix Installation

On Unix systems AMPL is installed into the current working directory. We recommend that you perform the installation in an empty directory. After installation, make sure the executable files have read and execute privileges turned on for all users and accounts that will use AMPL.

CD-ROM

The ILOG CD contains the AMPL/CPLEX system for several different platforms. First, read the file INFO_UNX.TXT. The section titled, "AMPL Modeling Language" contains information to help you locate the distribution for your platform. Note that the files listed in this section contain the entire AMPL/CPLEX System, not just the AMPL language processor. After you have located the files, read the CD booklet for instructions on extracting the distribution.

FTP

Execute:

```
gzip-dc < /path/ampl.tgz | tar xf -
```

where /path is the full path name into which ampl.tgz was downloaded.

AMPL and Parallel CPLEX (except for Linux)

If you have purchased the AMPL and Parallel CPLEX, follow the above installation instructions for the appropriate media. Then rename cplexamp to cplexamp.ser. And rename parcplexamp to cplexamp.

1.3.3 Windows Installation

On Windows systems AMPL is installed into a directory which you can specify during the installation (the default location is C:\AMPL).

CD-ROM

The ILOG CD contains the AMPL/CPLEX system for several different platforms. First, read the file INFO_PC.TXT. The section titled, "AMPL Modeling Language" contains information to help you locate the distribution for your version of Windows. Note that the files listed in this section contain the entire AMPL/CPLEX System, not just the AMPL language processor. After you have located the files, read the CD booklet for instructions on setting up the distribution.

FTP

After downloading the files, execute SETUP.EXE from the “Run” dialog or in a “Command” (MS-DOS Commands) window. Follow the instructions presented by the setup program. To start the Run dialog box on Windows 95, Windows 98, and Windows NT 4.0, click on the Start button and select “Run…”.

AMPL and Parallel CPLEX

If you have purchased the AMPL and Parallel CPLEX, follow the above installation instructions for the appropriate media. The AMPL/CPLEX System will automatically use the parallel processors available on your computer provided your license configuration includes the parallel option.

1.3.4 Licensing

AMPL is licensed in the same way as CPLEX. If you have already activated a license for the CPLEX Suite on this machine and you are not adding

AMPL as a new feature, then AMPL is already licensed, and you should skip these licensing instructions.

1.3.4.1 Updating an Existing License

If you are upgrading from a previous version of CPLEX, please refer to the CPLEX License Update Instructions (provided separately) or contact the CPLEX License Administrator. You should skip any installation steps that would establish a new license.

1.3.4.2 New Installation

If you are installing CPLEX or AMPL for the first time, you will receive an ILOG License Manager (ILM) manual and a license key that enables the use of AMPL and/or CPLEX. Follow the instructions in that manual for details on how to install the license key.

1.3.5 Usage Notes

The CPLEX solver for AMPL is named cplexamp (cplexamp.exe on Windows). This version of AMPL will use this solver by default. Older versions of the CPLEX solver for AMPL were simply named cplex (cplex.exe on Windows). Some users of that version may have specified the solver in their model or run files like this:

```
option solver cplex;
```

If you have models containing settings like this, you will encounter errors (or the old version of the solver might be invoked). There are two ways to fix this. Ideally, you should change these lines to:

```
option solver cplexamp;
```

If that is not practical, you can copy the file cplexamp to cplex. If you do the latter, you must remember to copy it again the next time you upgrade cplexamp.

Parallel users of the barrier method on the Sun UltraSparc will need to set the PARALLEL system environment variable to a value greater than or equal to the number of licensed threads. For example, from the C shell the command

```
setenv PARALLEL 4
```

will enable the parallel CPLEX solver for AMPL to use 4 threads, subject to license restrictions.

1.3.6 Installed Files

Unix systems

ampl	AMPL
cplexamp	The CPLEX solver for AMPL
examples.txt	Description of the example files (installed with the examples)

Windows systems

ampl.exe	AMPL
cplexamp.exe	The CPLEX solver for AMPL
uninst.isu	File used by the Windows uninstall procedure
ampltabl.dll	ODBC database interface

Examples

/models	Sample AMPL models - Most of these correspond to examples in the AMPL book. More information on some of the examples is provided in the readme file in this directory.
/looping	Advanced sample AMPL models - A description of each is provided in the readme file in this directory.
/compass/finance /compass/invest /compass/logistic /compass/product /compass/purchase /compass/schedule	More samples - The compass directory is sub-divided into industry-specific sub-directories. The models have been brought together from a variety of sources. Together, they provide a palette of AMPL models that you may use as a starting point for your projects.

Parallel systems

On systems that support parallelization, additional files may be present:

Unix

parcplexamp	Parallel CPLEX solver for AMPL

2 Using AMPL

2.1 Running AMPL

If you have added the AMPL installation directory to the search path, you can run AMPL from any directory. Otherwise, run AMPL by moving to the AMPL directory and typing `ampl` at the shell prompt.

At the `ampl:` prompt, you can type any AMPL language statement, or any of the commands described in Section A.13 of the book *AMPL: A Modeling Language for Mathematical Programming.* To end the session, type `quit;` at the AMPL prompt.

2.2 Using a Text Editor

Generally, you will edit your model and data (both expressed using AMPL language statements) in a text editor, and type commands at the `ampl:` prompt to load your model and data, solve a problem, and inspect the results. Although you could type in the statements of a model at the `ampl:` prompt, AMPL does not include a built-in text editor, so you cannot use AMPL commands to edit the statements you have previously entered. Microsoft Windows users (on PCs) and XWindows users (on Unix systems) should open separate windows for editing files and running AMPL.

If you cannot open multiple windows on your desktop, you can use AMPL's `shell` command to invoke an editor from within AMPL. You can use any editor that saves files in ASCII format. Windows command-line (or "DOS") users can use edit or notepad and Unix users vi or emacs. If you are using edit under DOS, for instance, you can type:

```
ampl: shell 'edit steel.dat';
```

Use editor menus and commands to edit your file, then save it and exit the editor. At the `ampl:` prompt you can type new AMPL commands, such as:

```
ampl: reset data;
ampl: data steel.dat;
```

Note that editing a file in a text editor does not affect your AMPL session until you explicitly reload the edited file, as shown above.

2.3 Running AMPL in Batch Mode

If you have previously developed a model and its data, and would like to solve it and display the results automatically, you can create a file containing the commands you would like AMPL to execute, and specify that file at the command line when you run AMPL. For example, you might create a file called steel.run, containing the commands:

```
model steel.mod;
data steel.dat;
option solver cplexamp;
solve;
display Make >steel.ans;
```

(This assumes that steel.run is in the same directory as the model and data files, and that AMPL can be found on the path.) You can then run AMPL as follows:

```
C:\> ampl steel.run
```

A more flexible approach, which is a commonly followed convention among AMPL users, is to put just the AMPL commands (the last three lines in the example above) in a file with the *.run extension. You can then type:

```
C:\> ampl steel.mod steel.dat steel.run
```

which accomplishes the same thing as:

```
C:\> ampl
ampl: include steel.mod;
ampl: include steel.dat;
ampl: include steel.run;
ampl: quit;
C:\>
```

If you intend to load several files and solve a problem, but you want to type a few interactive commands in the middle of the process, type the character `-` in place of a filename. AMPL processes the files on the command line from left to right; when it encounters the `-` symbol it displays the `ampl:` prompt and accepts commands until you type `end;`. For example, you could type:

```
C:\> ampl steel.mod steel.dat - steel.run
ampl: let avail := 50;
ampl: end;
```

This will solve the problem as before, but with the parameter `avail` set to 50 instead of 40 (the value specified in steel.dat). To start AMPL, load and model and data file, and wait for your interactive commands, type:

```
C:\> ampl steel.mod steel.dat -
```

3 AMPL-Solver Interaction

3.1 Choosing a Solver

AMPL's solver interface supports linear, nonlinear and mixed integer models with no built-in size limitations. This interface is rich enough to support many of the features used by advanced solvers to improve performance and solution accuracy, such as piecewise-linear constructs, representation of network problems, and automatic differentiation of nonlinear functions. To take advantage of these features, solvers must be written to utilize AMPL's interface. ILOG provides no support for the usage of AMPL with solvers not distributed by ILOG.

You choose a solver for a particular problem by giving its executable filename (without the EXE suffix) in the AMPL `option solver` command. For example, to use the (AMPL-compatible) CPLEX solver, type:

```
ampl: option solver cplexamp;
```

Most solvers have algorithmic options, such as CPLEX with its Mixed Integer and Barrier options. In cases like these, you give the solver executable name to AMPL (for example, with `option solver cplexamp`); the solver will determine, from the problem characteristics as passed by AMPL (e.g. a quadratic objective or integer variables) as well as solver options you specify, which algorithmic options will be used.

3.2 Specifying Solver Options

You can specify option settings for a particular solver through the AMPL `option` command. (CPLEX-specific directives are described later in this document.) Since all solvers provide default settings for their options, this is necessary only when your problem requires certain non-default settings in order to solve, or when certain settings yield improved performance or solution accuracy.

To specify solver options, you type `option` *`solver`*`_options 'settings';` where *`solver`* is replaced by the name of the solver you

are using. This approach allows you to set up different options for each solver you are using, and switch among them by simply choosing the appropriate solver with the `option solver` command. For example:

```
ampl: option cplex_options 'relax scale=1';
```

Solver options consist of an identifier alone, or an identifier followed by an = sign and a value. Some solvers treat uppercase and lower versions of an option identifer as equivalent, while others are sensitive to upper- and lower-case (so that `RELAX` is not the same as `relax`, for example).

Solver option settings can easily become long enough to stretch over more than one line. In such cases you can either continue a single quoted string by placing a \ (back-slash) character at the end of each line, as in:

```
ampl: option cplex_options 'crash=0 dual \
ampl?   feasibility=1.0e-8 scale=1 \
ampl?   lpiterlim=100';
```

Or you can write a series of individually quoted strings, which will be concatenated automatically by AMPL, as in:

```
ampl: option cplex_options 'crash=0 dual'
ampl? ' feasibility=1.0e-8 scale=1'
ampl? ' lpiterlim=100';
```

If you use the latter approach, be sure to include spaces at the beginning or end of the individual strings, so that the identifiers will be separated by spaces when all of the strings are concatenated.

Often you will want to add solver options to the set you are currently using. If you simply type a command such as `option solver_options 'new options';` however, you will "overwrite" the existing option settings. To avoid this problem, you can use AMPL's $ notation for options: the symbol `$option_name` is replaced by the current value of `option_name`. To *add* an optimality tolerance to the CPLEX options in the above example, you would write:

```
ampl: option cplex_options $cplex_options
ampl? ' optimality=1.0e-8';
```

3.3 Initial Variable Values and Solvers

Some optimizers (including most nonlinear solvers but excluding simplex-based linear solvers) make use of initial values for the decision variables as a starting point in their search for an optimal solution. A good choice of initial values can greatly speed up the solution process in some cases. Moreover, in nonlinear models with multiple local optima, the optimal solution reported by the solver may depend on the initial values for the variables.

AMPL passes initial values for decision variables, and dual values if available, to the solver. You can set initial values using the `:=` syntax in the `var` declaration of your AMPL model.

When you solve a problem two times in a row, the *final* values from the first solver invocation become the *initial* values for the second solver invocation (unless you override this behavior with statements in your AMPL model). In nonlinear models with multiple local optima, this can cause some solvers to report a different solution on the second invocation.

Simplex-based solvers typically discard initial values. However, they can use basis status information, if available. Basis statuses can be set either within AMPL or by a previous optimization. Information on interpreting and setting variable statuses is provided in Section 9.

3.4 Problem and Solution Files

When you type `solve;` AMPL processes your model and data to create a temporary "problem file" such as steel.nl, which will be read by the solver. It then loads and executes the solver program, which is responsible for creating a "solution file" such as steel.sol. AMPL reads the solution file and makes the solution values available through the variable, constraint and objective names you have declared in your AMPL model. Unless you specify otherwise, AMPL then deletes the temporary problem and solution files.

You can display the solution information (e.g. the values of the decision variables and constraints) in your AMPL session with commands such as `display`. For example, if you have just solved a problem created from steel.mod and steel.dat, you could type:

```
ampl: display Make, Time;
```

To save this output in a file, you can use redirection:

```
ampl: display Make, Time > mysol.txt;
```

Note that when you simply mention the name of a constraint in a `display` statement, AMPL will display the dual value (shadow price) of that constraint, not its "left-hand side" value. You can use the AMPL suffix notation to display the "left-hand side" value, as described in the book *AMPL: A Modeling Language for Mathematical Programming*.

3.4.1 Saving Temporary Files

Since AMPL deletes the temporary problem (*.nl) and solution (*.sol) files after a solver is finished, no permanent record of the solution is kept unless you save the output yourself (for example, using `display` with redirection as illustrated above). To override the deletion of temporary files, you can use the AMPL `write` command. For example:

```
C:\> ampl
ampl: model steel.mod; data steel.dat;
ampl: write bsteel;
ampl: solve;
```

```
CPLEX 7.0: optimal solution; objective 192000
2 iterations (0 in phase I)

ampl: quit;
```

The first letter `b` in the "filename" portion of the `write` command is interpreted specially, as explained below. If you now display the files in the current directory with a command such as `dir steel.*`, you will find the problem file steel.nl and the solution file steel.sol.

To later view the solution values, you would use the `solution` command. For example:

```
C:\> ampl
ampl: model steel.mod; data steel.dat;
ampl: solution steel.sol;

CPLEX 7.0: optimal solution; objective 192000
2 iterations (0 in phase I)

ampl: display Make;
Make [*] :=
bands  6000
coils  1400
;
ampl: quit;
```

You must include the model and data statements, as shown above, so that AMPL knows the definitions of symbolic names like `Make`. But `solution` then retrieves the earlier results from steel.sol, without running a solver.

When `b` is used as the first character of the "filename" portion of the `write` command, AMPL uses a compact and efficient binary format for the problem and solution files. When `g` is used instead, AMPL writes the files in an ASCII format which may be easier to transmit electronically over systems like the Internet. (In technical support and consulting situations, ILOG may ask you to send a file using this format.) When `m` is used, AMPL writes the problem in MPS format, and the filename ends in `mps` (e.g., steel.mps). This is described further under "Using MPS File Format."

3.4.2 Creating Auxiliary Files

AMPL can create certain (human- and program-readable) auxiliary files that help relate the various set, variable, constraint and objective names used in your AMPL model to the "column" and "row" indices which are written to the problem file and seen by the solver. This is particularly valuable when the AMPL presolve phase actually eliminates variables (by substitution) and constraints (which are found to be redundant or never binding) before the problem is sent to the solver.

To create the auxiliary files, you set the AMPL option `auxfiles` to a string of letters denoting the combination of auxiliary files you would like produced, and then use the `write` command to create and save the auxiliary files along with the problem (*.nl) file. For example, the command:

```
ampl: option auxfiles 'cr';
```

will cause the `write` command to create auxiliary files containing the names of the variables (columns) and constraints (rows) as sent to the solver. The table below shows the types of auxiliary files that can be created and the letter you use to request them via the AMPL option `auxfiles`.

Letter	*Extension*	*Description*
a	.adj	adjustment to objective, e.g. to compensate for fixed variables eliminated by presolve
c	.col	AMPL names of the variables (columns) sent to the solver
f	.fix	names of variables fixed by presolve, and the values to which they are fixed
r	.row	AMPL names of the constraints (rows) sent to the solver
s	.slc	names of "slack" constraints eliminated by presolve because they can never be binding
u	.unv	names of variables dropped by presolve because they are never used in the problem instance

3.5 Running Solvers Outside AMPL

With the `write` and `solution` commands just described, you can arrange to execute your solver outside the AMPL session. You might want to do this if you receive an "out of memory" message from your *solver* (not from AMPL itself): When the solver is invoked from within AMPL, a fair amount of memory is already used for the AMPL Modeling System program code and for data structures created by AMPL for its own use in memory. If you execute the solver alone, it can use all available memory.

To run your solver separately, first use AMPL to create a problem file:

```
C:\> ampl
ampl: model steel.mod; data steel.dat;
ampl: write bsteel;
ampl: quit;
```

Then run your solver with a command like the one below (for CPLEX):

```
C:\> cplexamp steel -AMPL solver_options
```

In this example, the first argument `steel` matches the filename (after the initial letter `b`) in the AMPL write command. The `-AMPL` argument tells the solver that it is receiving a problem from AMPL. This may optionally be followed by any *solver options* you need for the problem, using the same syntax used with the `option` *`solver`*`_options` command but omitting the outer quotes (for example `crash=1 relax`). Assuming that the solver runs successfully to completion, it will write a solution file (steel.sol in this case). You can then restart AMPL and read in the results with the solution command, as outlined earlier:

```
C:\> ampl
ampl: model steel.mod; data steel.dat;
ampl: solution steel.sol;
```

3.6 Using MPS File Format

MPS file format, originally developed decades ago for IBM's Mathematical Programming System, is a widely recognized format for linear and integer programming problems. Although it is a "standard" supported by many solvers and modeling systems (including AMPL), MPS file format is neither compact nor easy to read and understand; AMPL's binary file format is a much more efficient way for modeling systems and solvers to communicate. Also, MPS file format cannot be used for nonlinear problems, and not all "MPS compatible" solvers support exactly the same format, particularly for mixed integer problems.

AMPL does have the ability to translate a model into MPS file format, as outlined below. With this feature, you may be able to solve AMPL models with a solver which reads its problem input in MPS file format. If you choose to use this feature, you will find AMPL's ability to produce auxiliary files (outlined earlier) very useful, since these files can be used to relate the MPS file format information to the sets, variables, constraints and objectives defined in the AMPL model. However, you will *not* be able to bring the solution (e.g. variable values, dual values, etc.) back into AMPL; further work with the solution must be performed outside of AMPL.

To translate your model into MPS file format, use the `write` command as outlined above with `m` as the first letter of the "filename". To illustrate, the command shown below creates a file named steel.mps:

```
ampl: write msteel;
```

In most cases, you will need to run your solver separately to obtain the solution.

Note that the MPS format does not provide a way to distinguish between objective maximization and minimization. However, CPLEX assumes that the objective is to be minimized. (There is no standardization on this issue; other solvers may assume maximization.) Thus, it is incumbent upon the user of the MPS format to ensure that the objective sense in the AMPL model corresponds to the solver's interpretation.

3.7 Temporary Files Directory

By default – when the AMPL option `TMPDIR` is set to '' (an empty string) – AMPL writes the problem and solution files and other temporary files to the current directory. You can give a specific location for the temporary files by setting option `TMPDIR` to a valid path. On a PC, you might use:

```
ampl: option TMPDIR 'D:\temp';
```

On a Unix machine, a typical choice would be:

```
ampl: option TMPDIR '/tmp';
```

4 Customizing AMPL

4.1 Command Line Switches

Certain AMPL options normally set with the `option` command during an AMPL session can also be set when AMPL is first invoked. This is done using a command-line switch consisting of a hyphen and a single letter, followed in some cases by a numeric or string value. You will find these switches most useful when you have one or more model, data, or "run" files which you want AMPL to process using different option settings at different times, without actually editing the files themselves.

The table below is a summary of the command line switches and their equivalent names when set with the AMPL `option` command:

Switch	***AMPL Option***	***Description***
-C*n*	Cautions *n*	*n* = 0: suppress caution messages *n* = 1: report caution messages (default) *n* = 2: treat cautions as errors
-e*n*	eexit *n*	*n* > 0: abandon command after *n* errors *n* < 0: abort AMPL after |*n*| errors *n* = 0: report any number of errors
-f	funcwarn 1	do not treat unavailable functions of constant arguments as variable
-P	presolve 0	turn off AMPL's presolve phase
-S	substout 1	use "defining" equations to eliminate variables
-L	linelim 1	fully eliminate variables with linear "defining" equations, so model is recognized as linear
-T	gentimes 1	show time to generate each model component
-t	times 1	show time taken in each model translation phase
-o*str*	outopt *str*	set problem file format (b, g, m) and stub name; to display more possibilities use -o?
-s	randseed ''	use current time for random number seed
-s*n*	randseed *n*	use *n* for random number seed
-v	version	display the AMPL software version number

If you type `ampl -?` at the shell prompt, AMPL will display a summary list of all of the command line switches. (On some Unix shells ? is a special character, so you may need to use `"-?"` (with the double quotation marks).

4.2 Persistent Option Settings

If you have many option settings or other commands that you would like performed each time AMPL starts, you may create a text file containing these commands (in AMPL language syntax). Then set the environment variable name `OPTIONS_IN` to the pathname of this text file. For example, on a Windows PC, you should type:

```
C:\> set OPTIONS_IN=c:\amplinit.txt
```

A C shell user on a Unix machine would need to type something like:

```
% setenv OPTIONS_IN ~ijr/amplinit.txt
```

AMPL reads the file referred to by `OPTIONS_IN` and executes any commands therein before it reads any other files mentioned on the command line or prompts for any interactive commands.

If you want AMPL to preserve all of your option settings from one session to the next, you can cause AMPL to write the options into a text file named by setting the AMPL option `OPTIONS_INOUT`:

```
ampl: option OPTIONS_INOUT 'c:\amplopt.txt';
```

Before exiting, AMPL writes a series of `option` commands to the file named by `OPTIONS_INOUT` which, when read, will set all of the options to the values they had at the end of the session. To use this text file, set the corresponding environment variable to the same filename:

```
C:\> set OPTIONS_INOUT=c:\amplopt.txt
```

After you do this, AMPL will read and execute the commands in `amplopt.txt` when it starts up. When you end a session, AMPL will write the current option settings – including any changes you have made during the session – into this file, so that they will be preserved for use in your next session.

If both the `OPTIONS_IN` and `OPTIONS_INOUT` environment variables are defined, the file referred to by `OPTIONS_IN` will be processed first, then the file referred to by `OPTIONS_INOUT`.

5 Using CPLEX with AMPL

5.1 Problems Handled by CPLEX

CPLEX is designed to solve linear programs as described in Chapters 1-8 and 11-12 of *AMPL: A Modeling Language for Mathematical Programming*, as well as the integer programs described in Chapter 15. Integer programs may be pure (all integer variables) or mixed (some integer and some continuous variables); integer variables may be binary (taking values 0 and 1 only) or may have more general lower and upper bounds.

For the network linear programs described in Chapter 12, CPLEX also incorporates an especially fast network optimization algorithm.

The Barrier algorithmic option to CPLEX, though originally designed to handle linear programs, also allows the solution of a special class of nonlinear problems, namely, quadratic programs (QPs), as described later in this section. However, CPLEX does not solve general (non-QP) nonlinear programs. For instance, if you attempt to solve the following nonlinear problem described in Chapter 13 of the AMPL book, CPLEX will generate an error message:

```
ampl: model models\nltransd.mod;
ampl: data models\nltrans.dat;
ampl: option solver cplexamp;
ampl: solve;
at0.nl contains a nonlinear objective.

ampl:
```

This restriction applies if your model uses any function of variables that AMPL identifies as "not linear" – even a function such as abs or min that shares some properties of linear functions.

5.1.1 Piecewise-linear Programs

CPLEX does solve piecewise-linear programs, as described in Chapter 14, if AMPL transforms them to problems that CPLEX's solvers can handle. The

transformation to a linear program can be done if the following conditions are met:

- Any piecewise-linear term in a minimized objective must be convex, its slopes forming an increasing sequence as in:

```
<<–1,1,3,5; –5, –1,0,1.5,3>> x[j]
```

- Any piecewise-linear term in a maximized objective must be concave, its slopes forming a decreasing sequence as in:

```
<<1,3; 1.5,0.5,0.25>> x[j]
```

- Any piecewise-linear term in the constraints must be either convex and on the left-hand side of a ≤ constraint (or equivalently, the right-hand side of a ≥ constraint), or else concave and on the left-hand side of a ≥ constraint (or equivalently, the right-hand side of a ≤ constraint).

In all other cases, the transformation is to a mixed integer program. AMPL automatically performs the appropriate conversion, sends the resulting linear or mixed integer program to CPLEX, and converts the solution into user-defined variables. The conversion has the effect of adding a variable to correspond to each linear piece; when the above rules are not satisfied, additional integer variables and constraints must also be introduced.

5.1.2 Quadratic Programs

This user guide provides but a brief description of quadratic programming. In effect, it is assumed that you are familiar with the area. Interested users may wish to consult a good reference, such as *Practical Optimization*, by Gill, Murray and Wright (Academic Press, 1981) for more details.

A mathematical description of a quadratic program is given as:

$$\begin{array}{ll} \textit{minimize} & \frac{1}{2}x^T Q x + c^T x \\ \textit{subject to} & Ax \sim b \\ & l \le x \le u \end{array}$$

where ~ stands for ≤, ≥, or = operators.

In the above formula, Q represents a matrix of quadratic objective function coefficients. Its diagonal elements Q_{ii} are the coefficients of the quadratic terms x_i^2. The non-diagonal elements Q_{ij} and Q_{ji} are added together to be the coefficient of the term $x_i x_j$.

CPLEX's Barrier algorithmic option incorporates an extension for quadratic programming. For a problem to be solvable using this option, the following conditions must hold:

1. All constraints must be linear.
2. The objective must be a sum of terms, each of which is either a linear expression or a product of two linear expressions.

3. For any values of the variables (whether or not they satisfy the constraints), the quadratic part of the objective must have a nonnegative value (if a minimization) or a nonpositive value (if a maximization).

The last condition is known as positive semi-definiteness (for minimization) or negative semi-definiteness (for maximization). CPLEX automatically recognizes nonlinear problems that satisfy these conditions, and invokes the barrier algorithm to solve them. Nonlinear problems of any other kind are rejected with an appropriate message.

5.2 Specifying CPLEX directives

In many instances, you can successfully apply CPLEX by simply specifying a model and data, setting the `solver` option to `cplex`, and typing `solve`. For larger linear programs and especially the more difficult integer programs, however, you may need to pass specific options (also referred to as directives) to CPLEX to obtain the desired results.

To give directives to CPLEX, you must first assign an appropriate character string to the AMPL option called `cplex_options`. When CPLEX is invoked by `solve`, it breaks this string into a series of individual directives. Here is an example:

```
ampl: model diet.mod;
ampl: data diet.dat;
ampl: option solver cplexamp;
ampl: option cplex_options 'crash=0 dual \
ampl?    feasibility=1.0e-8 scale=1 \
ampl?    lpiterlim=100';
ampl: solve;
CPLEX 7.0: crash 0
dual
feasibility 1e-08
scale 1
lpiterlim 100
CPLEX 7.0: optimal solution; objective 88.2
1 iterations (0 in phase I)
```

CPLEX confirms each directive; it will display an error message if it encounters one that it does not recognize.

CPLEX directives consist of an identifier alone, or an identifier followed by an = sign and a value; a space may be used as a separator in place of the =.

You may store any number of concatenated directives in `cplex_options`. The example above shows how to type all the directives in one long string, using the \ character to indicate that the string continues on the next line. Alternatively, you can list several strings, which AMPL will automatically concatenate:

```
ampl: option cplex_options 'crash=0 dual'
ampl?    ' feasibility=1.0e-8 scale=1'
ampl?    ' lpiterlim=100';
```

In this form, you must take care to supply the space that goes between the directives; here we have put it before feasibility and iterations.

If you have specified the directives above, and then want to try setting, say, optimality to 1.0e−8 and changing crash to 1, you might think to type:

```
ampl: option cplex_options
ampl?    'optimality=1.0e-8 crash=1';
```

However, this will replace the previous `cplex_options` string. The other previously specified directives such as feasibility and iterations will revert to their default values. (CPLEX supplies a default value for every directive not explicitly specified; defaults are indicated in the discussion below.) To append new directives to `cplex_options`, use this form:

```
ampl: option cplex_options $cplex_options
ampl?    ' optimality=1.0e-8 crash=1';
```

A $ in front of an option name denotes the current value of that option, so this statement just appends more directives to the current directive string. As a result the string contains two directives for crash, but the new one overrides the earlier one.

6 Using CPLEX for Linear Programming

6.1 CPLEX Linear Programming Algorithms

For linear programs, CPLEX employs either a simplex method or a barrier method to solve the problem. Refer to a linear programming textbook for more information on these algorithms. Four distinct methods of optimization are incorporated in the CPLEX package:

- A primal simplex algorithm that first finds a solution feasible in the constraints (Phase I), then iterates toward optimality (Phase II).
- A dual simplex algorithm that first finds a solution satisfying the optimality conditions (Phase I), then iterates toward feasibility (Phase II).
- A network primal simplex algorithm that uses logic and data structures tailored to the class of pure network linear programs.
- A primal-dual barrier (or interior-point) algorithm that simultaneously iterates toward feasibility and optimality, optionally followed by a primal or dual crossover routine that produces a basic optimal solution (see below).

CPLEX normally chooses one of these algorithms for you, but you can override its choice by the directives described below.

The simplex algorithm maintains a subset of basic variables (or, a basis) equal in size to the number of constraints. A basic solution is obtained by solving for the basic variables, when the remaining nonbasic variables are fixed at appropriate bounds. Each iteration of the algorithm picks a new basic variable from among the nonbasic ones, steps to a new basic solution, and drops some basic variable at a bound.

The coefficients of the variables form a constraint matrix, and the coefficients of the basic variables form a nonsingular square submatrix called the basis matrix. At each iteration, the simplex algorithm must solve certain linear systems involving the basis matrix. For this purpose CPLEX

maintains a factorization of the basis matrix, which is updated during most iterations, and is occasionally recomputed.

The sparsity of a matrix is the proportion of its elements that are not zero. The constraint matrix, basis matrix and factorization are said to be relatively sparse or dense according to their proportion of nonzeros. Most linear programs of practical interest have many zeros in all the relevant matrices, and the larger ones tend also to be the sparser.

The amount of RAM memory required by CPLEX grows with the size of the linear program, which is a function of the numbers of variables and constraints and the sparsity of the coefficient matrix. The factorization of the basis matrix also requires an allocation of memory; the amount is problem-specific, depending on the sparsity of the factorization. When memory is limited, CPLEX automatically makes adjustments that reduce its requirements, but that usually also reduce its optimization speed.

The CPLEX directives in the following subsections apply to the solution of linear programs, including network linear programs. The letters i and r denote integer and real values, respectively.

6.2 Directives for Problem and Algorithm Selection

CPLEX consults several directives to decide how to set up and solve a linear program that it receives. The default is to apply the dual simplex method to the linear program as given, substituting the network variant if the AMPL model contains node and arc declarations. The following discussion indicates situations in which you should consider experimenting with alternatives.

`dualthresh=i` **(default 32000)**

`primal`

`dual`

Every linear program has an equivalent "opposite" linear program; the original is customarily referred to as the primal LP, and the equivalent as the dual. For each variable and each constraint in the primal there are a corresponding constraint and variable, respectively, in the dual. Thus when the number of constraints is much larger than the number of variables in the primal, the dual has a much smaller basis matrix, and CPLEX may be able to solve it more efficiently.

The `primal` and `dual` directives instruct CPLEX to set up the primal or the dual formulation, respectively. The `dualthresh` directive makes a choice: the dual LP if the number of constraints exceeds the number of variables by more than i, and the primal LP otherwise.

`primalopt`

`dualopt`

`baropt`

The `primalopt`, `dualopt` and `baropt` directives instruct CPLEX to apply the primal simplex algorithm, the dual simplex algorithm, or the barrier method respectively. The two simplex variants use similar basis matrices but employ opposite strategies in constructing a path to the optimum. Any of the algorithms can be applied regardless of whether the primal or the dual LP is set up as explained above; in general the six combinations of `primalopt/ dualopt/baropt` and `primal/dual` perform differently.

Unless primalopt or baropt are specified, CPLEX uses dualopt by default. Linear programs that are highly degenerate (many basic variables at their bounds) and that have little variability in the righthand sides (constant terms) of their constraints often solve faster using the dual simplex. Also consider trying the dual simplex if CPLEX's primal simplex reports problems of numerical inaccuracy; few linear programs exhibit poor numerical performance in both the primal and the dual algorithms. In general the barrier method tends to work well when the product of the constraint matrix and its transpose remains sparse.

`netopt=i` **(default 1)**

CPLEX incorporates an optional heuristic procedure that looks for "pure network" constraints in your linear program. If this procedure finds sufficiently many such constraints, CPLEX applies its fast network simplex algorithm to them. Then, if there are also non-network constraints, CPLEX uses the network solution as a start for solving the whole LP by the general primal or dual simplex algorithm, whichever you have chosen.

The default value of i=1 invokes the network-identification procedure if and only if your model uses node and arc declarations, and CPLEX sets up the primal formulation as discussed above. Setting i=0 suppresses the procedure, while i=2 requests its use in all cases. You can have CPLEX display the number of network nodes (constraints) and arcs (variables) that it has extracted, by setting the `prestats` directive (described with the preprocessing options below) to 1.

CPLEX's network simplex algorithm can achieve dramatic reductions in optimization time for "pure" network linear programs defined entirely in terms of node and arc declarations. (For a pure network LP, every arc declaration must contain at most one from and one to phrase, and these phrases may not specify optional coefficients.) In the case of linear programs that are mostly defined in terms of node and arc declarations, but that have some "side" constraints defined by subject to declarations, the benefit is highly dependent on problem structure; it is best to try experimenting with both i=0 and i=1.

`relax`

This directive instructs CPLEX to ignore any integrality restrictions on the variables. The resulting linear program is solved by whatever algorithm the above directives specify.

maximize

minimize

While AMPL completely specifies the problem and its objective sense, it is possible to change the objective sense after specifying the model. The two directives instruct CPLEX to set the objective sense to be minimize or maximize, respectively.

6.3 Directives for Preprocessing

Prior to applying any simplex algorithm, CPLEX modifies the linear program and initial basis in ways that tend to reduce the number of iterations required. The following directives select and control these preprocessing features.

aggregate=i1 **(default 1)**

aggfill=i2 **(default 10)**

When i1 is left at its default value of 1, CPLEX looks for constraints that (possibly after some rearrangement) define a variable x in terms of other variables:

- two-variable constraints of the form $x = y + b$;
- constraints of the form $x = \sum_j y_j$, where x appears in less than i2 other constraints.

Under certain conditions, both x and its defining equation can be eliminated from the linear program by substitution. In CPLEX's terminology, each such elimination is an aggregation of the linear program. When i1 is –1, CPLEX decides how many passes to perform. Set i1 to 0 to prevent any such aggregations. Set i1 to a positive integer to specify the precise number of passes.

Aggregation can yield a substantial reduction in the size of some linear programs, such as network flow LPs in which many nodes have only one incoming or one outgoing arc. If i2 > 2, however, aggregation may also increase the number of nonzero constraint coefficients, resulting in more work at each simplex iteration. The default setting of i2=10 usually makes a good tradeoff between reduction in size and increase in nonzeroes, but you may want to experiment with lower values if CPLEX reports that many aggregations have been made. If CPLEX consistently reports that no aggregations can be performed, on the other hand, you can set i1 to 0 to turn off the aggregation routine and save memory and processing time.

To request a report of the number of aggregations, see the `prestats` directive later in this section.

dependency=i **(default 0)**

By default (i=0), during the presolve phase, CPLEX does not check for or identify dependent rows in the coefficient matrix. Setting i=1 turns on the dependency checker.

predual=i **(default 0)**

By default, after presolving the problem CPLEX decides whether to solve the primal or dual problem based on which problem it determines it can solve faster. Setting i=1 explicitly instructs CPLEX to solve the dual problem, while setting it to -1 explicitly instructs CPLEX to solve the primal problem. Regardless of the problem CPLEX solves internally, it still reports primal solution values. This is often a useful technique for problems with more constraints than variables.

prereduce=i **(default 3)**

This directive determines whether primal reductions, dual reductions or both are performed during preprocessing. By default, CPLEX performs both. Set this directive to 0 to prevent all reductions, 1 to only perform primal reductions, and 2 to only perform dual reductions. While the default usually suffices, performing only one kind or the other may be useful when diagnosing infeasibility or unboundedness.

presolve=i **(default 1)**

Prior to invoking any simplex algorithm, CPLEX applies transformations that reduce the size of the linear program without changing its optimal solution. In this presolve phase, constraints that involve only one non-fixed variable are removed; either the variable is fixed and also dropped (for an equality constraint) or a simple bound for the variable is recorded (for an inequality). Each inequality constraint is subjected to a simple test to determine if there exists any setting of the variables (within their bounds) that can violate it; if not, it is dropped as nonconstraining. Further iterative tests attempt to tighten the bounds on primal and dual variables, possibly causing additional variables to be fixed, and additional constraints to be dropped.

AMPL's presolve phase, as described in Section 10.2 of the AMPL book, also performs many (but not all) of these transformations. To see how many variables and constraints are eliminated by AMPL's presolve, set option `show_stats` to 1. To suppress AMPL's presolver, so that all presolving is done in CPLEX, set option `presolve` to 0.

CPLEX's presolve can be suppressed by changing i to 0 from its default of 1. In rare cases the presolved linear program, although smaller, is actually harder to solve. Thus if CPLEX reports that many variables and constraints have been eliminated by presolve, you may want to compare runs with and without presolve. On the other hand, if CPLEX consistently reports that presolve eliminates no variables or constraints, you can save a little processing time by turning presolve off.

To request a report of the number of eliminations performed by presolve, see the `prestats` directive below.

prestats=i **(default 0)**

When this directive is changed to 1 from its default of 0, CPLEX reports on the activity of the aggregation and presolve routines:

```
Presolve eliminated 1645 rows and 2715 columns
in 3 passes.
Aggregator did 22 substitutions.
Presolve Time =    1.70 sec.
```

During the development of a large or complex model, it is a good idea to monitor this report, and to turn on its AMPL counterpart by setting option `show_stats` to 1. An unexpectedly large number of eliminated variables or constraints may indicate that the formulation is in error or can be substantially simplified.

`scale=i` **(default 0)**

This directive governs the scaling of the coefficient matrix. The default value of i=0 implements an equilibration scaling method, which is generally very effective. You can turn off the default scaling by setting i=-1. A value of 1 invokes a modified, more aggressive scaling method that can produce improvements on some problems. (Since CPLEX has internal logic that determines when it need not scale a problem, setting the scale directive to –1 rarely improves performance.)

6.4 Directives for Controlling the Simplex Algorithm

Several key strategies of the primal and dual simplex algorithms can be changed through CPLEX directives. If you are repeatedly solving a class of linear programs that requires substantial computer time, experimentation with alternative strategies can be worthwhile.

`advance=i` **(default 1)**

By default (i=1) CPLEX uses existing basis status information (see the discussion on basis statuses in Section 9.2) to begin optimization. To ignore basis statuses, set i=0.

`basisinterval=i` **(default 50000)**

This directive controls the frequency with which CPLEX automatically writes a basis file to disk. This is a safeguard against unanticipated events such as power failures interrupting a run. By using the resulting basis file in conjunction with the `xxxstart` directive (described below), one can resume the run from the most recently written basis files.

`crash=i` **(default 1)**

This directive governs CPLEX's procedure for choosing an initial basis, except when the basis is read from a file as specified by the directive `readbasis` described below. A value of i=0 causes the objective to be ignored in choosing the basis, while values of –1 and 1 select two different heuristics for taking the objective into account. The best setting for your purposes will depend on the specific characteristics of the linear programs you are solving, and must be determined through experimentation.

`pgradient=i` **(default 0)**

This directive governs the primal simplex algorithm's choice of a "pricing" procedure that determines which variable is selected to enter the basis at each iteration. Your choice is likely to make a substantial difference to the tradeoff between computational time per iteration and the number of iterations. As a rule of thumb, if the number of iterations to solve your linear program exceeds three times the number of constraints, you should consider experimenting with alternative pricing procedures.

The recognized values of i are as follows:

−1	Reduced-cost pricing
0	Hybrid reduced-cost/devex pricing
1	Devex pricing
2	Steepest-edge pricing
3	Steepest-edge pricing with slack initial norms
4	Full reduced-cost pricing

The "reduced cost" procedures are sophisticated versions of the pricing rules most often described in textbooks. The "devex" and "steepest edge" alternatives employ more elaborate computations, which can better predict the improvement to the objective offered by each candidate variable for entering the basis.

Compared to the default of i = 0, the less compute-intensive reduced-cost pricing (i= −1) may be preferred if your problems are small or easy, or are unusually dense — say, 20 to 30 nonzeroes per column. Conversely, if you have more difficult problems which take many iterations to complete Phase I, consider using devex pricing (i=1). Each iteration may consume more time, but the lower number of total iterations may lead to a substantial overall reduction in time. Do not use devex pricing if your problem has many variables and relatively few constraints, however, as the number of calculations required per iteration in this situation is usually too large to afford any advantage.

If devex pricing helps, you may wish to try steepest-edge pricing (i=2). This alternative incurs a substantial initialization cost, and is computationally the most expensive per iteration, but may dramatically reduce the number of iterations so as to produce the best results on exceptionally difficult problems. The variant using slack norms (i=3) is a compromise that sidesteps the initialization cost; it is most likely to be advantageous for relatively easy problems that have a low number of iterations or time per iteration.

Full reduced-cost pricing (i=4) is a variant that computes a reduced cost for every variable, and selects as entering variable one having most negative reduced cost (or most positive, as appropriate). Compared to CPLEX's standard reduced-cost pricing (i= −1), full reduced-cost pricing takes more time per iteration, but in rare cases reduces the number of iterations more than enough to compensate. This alternative is supplied mainly for

completeness, as it is proposed in many textbook discussions of the simplex algorithm.

`dgradient=i` **(default 0)**

This directive governs the dual simplex algorithm's choice of a "pricing" procedure that determines which variable is selected to leave the basis at each iteration. Your choice is likely to make a substantial difference to the tradeoff between computational time per iteration and the number of iterations. As a rule of thumb, if the number of iterations to solve your linear program exceeds three times the number of constraints, you should consider experimenting with alternative pricing procedures.

The recognized values of i are as follows:

0 Pricing procedure determined automatically

1 Standard dual pricing

2 Steepest-edge pricing

3 Steepest-edge pricing in slack space

4 Steepest-edge pricing with unit initial norms

Standard dual pricing (i=1), described in many textbooks, selects as leaving variable one that is farthest outside its bounds. The three "steepest edge" alternatives employ more elaborate computations, which can better predict the improvement to the objective offered by each candidate for leaving variable. The default (i=0) lets CPLEX choose a dual pricing procedure through an internal heuristic based on problem characteristics.

Steepest-edge pricing involves an extra initialization cost, but its extra cost per iteration is much less in the dual simplex algorithm than in the primal. Thus if you find that your problems solve faster using the dual simplex, you should consider experimenting with the steepest-edge procedures. The standard procedure (i=2) and the variant "in slack space" (i=3) have similar computational costs; often their overall performance is similar as well, though one or the other can be advantageous for particular applications. The variant using "unit initial norms" (i=4) is a compromise that sidesteps the initialization cost; it is most likely to be advantageous for relatively easy problems that have a low number of iterations or time per iteration.

`pdswitch=i` **(default 0)**

This directive determines whether a switch of algorithms will be made when undoing perturbations or shifts that may occur during execution of the Primal or Dual Simplex algorithm. By default, CPLEX automatically decides whether to switch algorithms. Consider setting this directive to 1 if you observe many cycles of removing perturbations or shifts at the end of the optimization. Set it to -1 to inhibit switching.

`pricing=i` **(default 0)**

To promote efficiency, when using reduced-cost pricing in primal simplex, CPLEX considers only a subset of the nonbasic variables as candidates to enter the basis. The default of i=0 selects a heuristic that dynamically

determines the size of the candidate list, taking problem dimensions into account. You can manually set the size of this list to i>0, but only very rarely will this improve performance.

`refactor=i` **(default 0)**

This directive specifies the number of iterations between refactorizations of the basis matrix.

At the default setting of i=0, CPLEX automatically calculates a refactorization frequency by a heuristic formula. You can determine the frequency that CPLEX is using by setting the display directive (described below) to 1. Since each update to the factorization uses more memory, CPLEX may reduce the factorization frequency if memory is low. In extreme cases, the basis may have to be refactored every few iterations and the algorithm will be very slow.

Given adequate memory, CPLEX's performance is relatively insensitive to changes in refactorization frequency. For a few extremely large, difficult problems you may be able to improve performance by reducing i from the value that CPLEX chooses.

`netfind=i` **(default 1)**

This directives governs the method used by the CPLEX network optimizer to extract a network from the linear program. The value of i influences the size of the network extracted, potentially reducing optimization time. The default value (i=1) extracts only the natural network from the problem. CPLEX then invokes its network simplex method on the extracted network. In some cases, CPLEX can extract a larger network by multiplying rows by –1 (reflection scaling) and rescaling constraints and variables so that more matrix coefficients are plus or minus 1. Setting the netfind directive to 2 enables reflection scaling only, while setting it to 3 allows reflection scaling and general scaling.

`simthreads=i`

This directive only applies to users of parallel CPLEX solvers. It specifies the number of parallel processes used during the simplex method optimization. The default value depends on the number of threads licensed, and the ability of the invoked algorithm to run in parallel. For example, the dual simplex method can exploit parallel processing (on certain platforms), while the primal simplex method cannot.

`xxxstart=i` **(default 0)**

Set this parameter to 1 to instruct CPLEX to start the optimization from one of the .xxx format basis files generated by CPLEX when it last solved this problem (see the description of the `basisinterval` directive above for more information). Note that these .xxx format files are for the presolved problem, so one should not specify .xxx files in `readbasis` and `writebasis` directives.

6.5 Directives for Controlling the Barrier Algorithm

Several key strategies of the barrier algorithm can be changed through CPLEX directives. If you are repeatedly solving a class of linear programs that requires substantial computer time, experimentation with alternative strategies can be worthwhile.

`baralg=i` **(default 0)**

The automatically determined choice of barrier algorithm (i1=0) is usually the fastest. However, on primal- or dual-infeasible problems, the infeasibility-estimate start algorithm (i1=1) or the infeasibility-constant start algorithm (i1=2) may improve numerical stability, possibly at the cost of speed. Setting i1=3 selects the standard barrier algorithm.

`bargrowth=r` **(default 1e+6)**

This directive is used to detect unbounded optimal faces. At higher values, the barrier algorithm will be less likely to conclude that the problem has an unbounded optimal face, but more likely to have numerical difficulties if the problem does have an unbounded face. Any positive number is acceptable input.

`barcorr=i` **(default –1)**

CPLEX may perform centering corrections if it encounters numerical difficulties during the barrier method optimization. By default (i=–1) the Barrier Solver automatically computes an estimate for the maximum number of centering corrections done at each iteration. If the automatic estimate is computed to be 0, setting the value to a positive integer may improve the numerical stability of the algorithm, probably at the expense of computation time.

`barobjrange=r` **(default 1e+15)**

This directive sets the maximum absolute value of the objective function. CPLEX's barrier algorithm looks at this value to detect unbounded problems. Any positive value is acceptable input. However, care should be taken to avoid choosing a value so small that CPLEX will conclude a problem is unbounded when it is not.

`barstart=i` **(default 1)**

This directive controls the starting point CPLEX uses to initiate the barrier method. The default setting of 1 will suffice for most problems. Consider other values (2, 3 and 4) if the barrier method appears to converge slowly, or when the predual directive is specified.

`barthreads=i`

This directive only applies to users of parallel CPLEX solvers. It specifies the number of parallel processes used during the barrier method optimization. The default value depends on the number of threads licensed.

`barvarup=r` **(default 1e+20)**

This directive sets the upper bound for all variables that have infinite upper bounds. It is used to prevent difficulties associated with unbounded optimal faces. Any positive value less then or equal to 1e+20 is acceptable input. However, care should be taken to avoid choosing a value so small that CPLEX concludes that a problem has an unbounded optimal face when it does not.

`comptol=r` **(default 1e–8)**

This directive specifies the complementarity tolerance used by the barrier algorithm to test convergence. The barrier algorithm will terminate with an optimal solution if the relative complementarity is smaller than this value. Any positive number larger than 1e–10 is acceptable input.

`crossover=i` **(default 1)**

On a linear problem, by default (i=1) CPLEX initiates the "crossover" algorithm to convert the barrier solution to a basic (or vertex) solution using a primal simplex-like method. If i=2, a dual simplex-like method is used for the crossover. The crossover algorithm can be turned off by setting i=0.

`densecol=i` **(default 0)**

CPLEX uses this directive to distinguish dense columns in the constraint matrix. Because barrier algorithm performance can improve dramatically if dense columns are treated separately, changing this value may improve optimization time. Columns with more nonzeros than this setting are considered to be dense. If left at the default value, CPLEX will automatically determine a value, considering factors such as the size of the problem. Any nonnegative integer is acceptable input.

`ordering=i` **(default 0)**

This directive selects the method used to permute the rows of the constraint matrix in order to reduce fill in the Cholesky factor. There is a trade-off between ordering speed and sparsity of the Cholesky factor. The automatic default setting usually chooses the best ordering for the problem.

The approximate minimum degree (AMD) algorithm (i=1) balances speed and fill. The approximate minimum fill (AMF) algorithm (i=2) usually generates slightly better orderings than AMD, at the cost of more ordering run-time. The nested dissection (ND) algorithm, triggered by using i=3 sometimes reduces Barrier run-time dramatically – ten-fold reductions have been observed for some problems. This option sometimes produces worse orderings, though, and it requires much more ordering run-time.

6.6 Directives for Improving Stability

CPLEX is highly robust and has been designed to avoid problems such as degenerate stalling and numerical inaccuracy that can occur in the simplex algorithm. However, some linear programs can benefit from adjustments to the following directives if difficulties are encountered.

doperturb=i1 **(default 0)**

perturbation=r **(default 1.0e–6)**

perturblimit=i2 **(default 0)**

The simplex algorithm tends to make very slow progress when it encounters solutions that are highly degenerate (in the sense of having many basic variables lying at one of their bounds, rather than between them). When CPLEX detects degenerate stalling, it automatically introduces a perturbation that expands the bounds on every variable by a small amount, thereby creating a different but closely related problem. Generally, CPLEX can make faster progress on this less constrained problem; once optimality is indicated, the perturbation is removed by resetting the bounds to their original values.

The value of r determines the size of the perturbation. If you receive messages from CPLEX indicating that the linear program has been perturbed more than once, r is probably too large; reduce it to a level where only one perturbation is required.

The default `doperturb` value of i1=0 selects CPLEX's automatic perturbation strategy. If an automatic perturbation occurs early in the solution process, consider setting i1=1 to select perturbation at the outset. This alternative will save the time of first allowing the optimization to stall before activating the perturbation mechanism, but is useful only rarely, for extremely degenerate problems.

The perturblimit parameter governs the number of stalled iterations CPLEX allows before perturbing the problem. The default value of i2=0 causes CPLEX to determine this number based on the characteristics of the particular problem being solved. Setting i2 to a positive integer value identifies a specific number of stalled iterations to tolerate before perturbing the problem.

feasibility=r1 **(default 1.0e–6)**

markowitz=r2 **(default 0.01)**

optimality=r3 **(default 1.0e–6)**

If a problem is making slow progress through Phase I, or repeatedly becomes infeasible during Phase II, numerical difficulties have arisen. Adjusting the algorithmic tolerances controlled by these directives may help. Decreasing the feasibility tolerance, increasing the optimality tolerance and/or increasing the Markowitz tolerance will typically improve numerical behavior.

The feasibility tolerance r1>0 specifies the degree to which a linear program's basic variables may violate their bounds. You may wish to lower r1 after finding an optimal solution if there is any doubt that the solution is truly optimal; but if it is set too low, CPLEX may falsely conclude that the problem has no feasible solution. Valid values for r1 lie between 1e–9 and 0.1.

The Markowitz threshold r2<1 influences the order in which variables are eliminated during basis factorization. Increasing r2 may yield a more accurate factorization, and consequently more accurate computations during iterations of the simplex algorithm. Too large a value may produce an inefficiently dense factorization, however. Valid values for r2 lie between 0.0001 and 0.99999.

The optimality tolerance r3>0 specifies how closely the optimality (or dual feasibility) conditions must be satisfied for CPLEX to declare an optimal solution. Valid values for r3 lie between 1e–9 and 0.01.

6.7 Directives for Starting and Stopping

Normally CPLEX uses an internal procedure to determine a starting point for the simplex algorithm, then iterates to optimality. The following directives override these conventions so that you can start from a saved basis, and can stop when a certain criterion is satisfied.

Command-line versions of CPLEX for AMPL can also be stopped by using "break" (typically, by pressing the "Control" and "C" keys simultaneously). The best solution found so far is returned.

`bariterlim=i` **(default problem dependent)**

CPLEX stops after i barrier method iterations and returns its current solution, whether or not it has determined that the solution is optimal.

`lpiterlim=i` **(default 2.1e+9 or larger)**

CPLEX stops after i simplex method iterations and returns its current solution, whether or not it has determined that the solution is optimal.

`lowerobj=r1` **(default –1.0e+75)**

`upperobj=r2` **(default +1.0e+75)**

CPLEX stops at the first iteration where the solution is feasible in the constraints, and the objective value is below r1 or above r2. At their default values these directives have no practical effect. Setting r1 (for a minimization) or r2 (for a maximization) to a "good" value for the objective will cause CPLEX to stop as soon as it achieves this value.

`readbasis=f1`

`writebasis=f2`

Current versions do not require you to explicitly save the basis to hot-start CPLEX – variable status is automatically stored and used between CPLEX invocations. The `readbasis` and `writebasis` directives are included for backward compatibility with previous versions of CPLEX for AMPL, which did not use variable status information.

If the `readbasis` directive is specified, then the initial basis is instead read from the file f1, which must also be in the standard MPS basis format. This basis determines the initial solution.

If the `writebasis` directive is specified, CPLEX writes a record of the final simplex basis to the file named f2, in the standard MPS basis format. Normally this is an optimal basis, but it may be otherwise if an optimum does not exist or could not be found by the chosen algorithm, or if the iterations were terminated prematurely by one of the directives described below.

`readvector=f1`

`writevector=f2`

These directives are used to take a barrier algorithm solution and write it to or read it from a CPLEX .vec file. Because AMPL always instructs CPLEX to take its barrier method solution and apply a hybrid method to obtain a basic solution, this feature can only be used if a barrier iteration limit is exceeded.

If the `readvector` directive is specified, CPLEX will read in a .vec file named f2 and use it to initiate the hybrid crossover method that results in an optimal basic solution. Note that CPLEX will not perform additional barrier iterations after reading in the .vec file. Similarly, if the `writeector` directive is specified, CPLEX will write out .vec file named f2.

`singular=i` **(default 10)**

CPLEX will attempt to repair the basis matrix up to i times when it finds evidence that the matrix is singular. Once this limit is exceeded, CPLEX terminates with the current basis set to the best factorizable basis that has been found.

`timelimit=r` **(default 1.0e+75)**

CPLEX stops after r seconds of computation time and returns its current solution, whether or not it has determined that the solution is optimal.

6.8 Directives for Controlling Output

When invoked by `solve`, CPLEX normally returns just a few lines to your screen to summarize its performance. The following directive lets you choose a greater amount of output, which may be useful for monitoring the progress of a long run, or for comparing the effects of other directives on the detailed behavior on CPLEX's algorithms. Output normally comes to the screen, but may be redirected to a file by specifying `solve >`*`filename`*.

`bardisplay=i` **(default 0)**

The default choice of i=0 produces a minimal few lines of output from CPLEX, summarizing the results of a barrier method run.

When i=1, a "log line" recording the barrier iteration number, primal and dual objective values, and infeasibility information is displayed after each barrier iteration.

When i=2, additional information about the barrier run is provided. This level of output is occasionally useful for diagnosing problems of degeneracy or instability in the barrier algorithm.

`iisfind=i` **(default 0)**

When i=1 for an infeasible problem, CPLEX returns an irreducible infeasible subset (IIS) of the constraints and variable bounds. By definition, members of an IIS have no feasible solution, but dropping any one of them permits a solution to be found to the remaining ones. Clearly, knowing the composition of an IIS can help localize the source of the infeasibility.

Setting i=2 generates a potentially smaller IIS, at the cost of greater computation time. When `iisfind` is used, CPLEX uses the `.iis` suffix to specify which variables and constraints are in the IIS, as explained in Section 8.3.

`logfile=f1`

This directive instructs CPLEX to create a log file named f1 that will contain output from the optimization. The amount of output in the log file will depend on other directives, such as the `display` directive described above.

`lpdisplay=i` **(default 0)**

The default choice of i=0 produces a minimal few lines of output from CPLEX, summarizing the results of the run.

When i=1, a "log line" recording the iteration number and the scaled infeasibility or objective value is displayed after each refactorization of the basis matrix. Additional information on the operation of the network simplex algorithm is also provided, if applicable. This is often the appropriate setting for tracking the progress of a long run.

When i=2, a log line is displayed after each iteration. This level of output is occasionally useful for diagnosing problems of degeneracy or instability in the simplex algorithm.

`sensitivity`

When specified, this directive instructs CPLEX to output sensitivity ranges corresponding to the optimal solution. For variables, the suffix `.current` provides the corresponding objective function coefficient in the current problem, and `.down` and `.up` specify the smallest and largest values for which the current basis remains optimal. For constraints, the suffixes apply to the constant value, or right-hand-side. Details on CPLEX-defined suffixes are provided in Section 8.

`timing=i` **(default 0)**

When this directive is changed to 1 from its default value of 0, a summary of processing times is displayed to "standard output":

```
Input =  0.06 CPU  0.06 Wall
Solve =  6.42 CPU 6.42 Wall
Output = 0.05 CPU 0.05 Wall
```

Input is the time that CPLEX takes to read the problem from a file that has been written by AMPL. Solve is the time that CPLEX spends trying to solve the problem. Output is the time that CPLEX takes to write the solution to a file for AMPL to read. CPU values provide processor time, whereas Wall values provide elapsed time.

Setting i=2 writes the timing information to "standard error", and setting i=3 directs the information to both the standard output and the standard error. (The latter two options are only interesting for Unix CPLEX for AMPL users.)

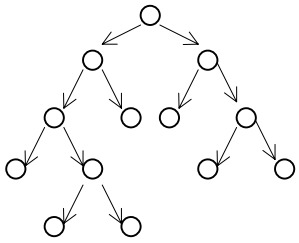
`version`

This directive causes the display of the CPLEX version being used to solve the problem.

7 Using CPLEX for Integer Programming

7.1 CPLEX Mixed Integer Algorithm

For problems that contain integer variables, CPLEX uses a branch-and-bound approach. The optimizing algorithm maintains a hierarchy of related linear programming subproblems, referred to as the search tree, and usually visualized as branching downward:



There is a subproblem at each node of the tree, and each node is explored by solving the associated subproblem.

The algorithm starts with just a top (or root) node, whose associated subproblem is the relaxation of the integer program – the LP that results when all integrality restrictions are dropped. If this relaxation happened to have an integer solution, then it would provide an optimal solution to the integer program. Normally, however, the optimum for the relaxation has some fractional-valued integer variables. A fractional variable is then chosen for branching, and two new subproblems are generated, each with more restrictive bounds for the branching variable. For example, if the branching variable is binary (or 0-1), one subproblem will have the variable fixed at zero, the other node will have it fixed at one. In the search tree, the two new subproblems are represented by two new nodes connected to the root. Most likely each of these subproblems also has fractional-valued integer variables, in which case the branching process must be repeated; successive branchings produce the tree structure shown above.

If there are more than a few integer variables, the branching process has the potential to create more nodes than any computer can hold. There are two

key circumstances, however, in which branching from a particular node can be discontinued:

- *The node's subproblem has no fractional-valued integer variables.* It thus provides a feasible solution to the original integer program. If this solution yields a better objective value than any other feasible solution found so far, it becomes the incumbent, and is saved for future comparison.
- *The node's subproblem has no feasible solution, or has an optimum that is worse than a certain cutoff value.* Since any subproblems under this node would be more restricted, they would also either be infeasible or have an optimum value worse than the cutoff. Thus none of these subproblems need be considered.

In these cases the node is said to be fathomed. Because subproblems become more restricted with each branching, the likelihood of fathoming a node becomes greater as the algorithm gets deeper into the tree. So long as nodes are not created by branching much faster than they are inactivated by fathoming, the tree can be kept to a reasonable size.

When no active nodes are left, CPLEX is finished, and it reports the final incumbent solution back to AMPL. If the cutoff value has been set throughout the algorithm to the objective value of the current incumbent – CPLEX's default strategy – then the reported solution is declared optimal. Other cutoff options, described below, cannot provide a provably optimal solution, but may allow the algorithm to finish much faster.

CPLEX's memory requirement for solving linear subproblems is about the same as its requirement for linear programs discussed in the previous section. In the branch-and-bound algorithm, however, each active node of the tree requires additional memory. The total memory that CPLEX needs to prove optimality for an integer program can thus be much larger and less predictable than for a linear program of comparable size.

Because a single integer program generates many LP subproblems, even small instances can be very compute-intensive and require significant amounts of memory. In contrast to solving linear programming problems, where little user intervention is required to obtain optimal results, you may have to set some of the following directives to get satisfactory results on integer programs. You can either change the way that the branch-and-bound algorithms work, or relax the conditions for optimality, as explained in the two corresponding subsections below. When experimenting with these possibilities, it is also a good idea to include directives that set stopping criteria and display informative output; these are described in the next two subsections. If you consistently fail to receive any useful solution in response to the "solve" command after a reasonable amount of time, and are in doubt as to how to proceed, consult the troubleshooting tips at the end of this section.

7.2 Directives for Preprocessing

All of the preprocessing directives described in *Using CPLEX for Linear Programming* are also applicable to problems that specify integer-valued variables. The following directives control additional preprocessing steps that are applicable to certain mixed integer programs only.

`aggcutlim=i` **(default 3)**

This directive controls the number of constraints that can be aggregated for generating flow cover and mixed integer rounding cuts. In most cases the default setting of 3 will be satisfactory. Set it to 0 to prevent any aggregation.

`boundstr=i` **(default –1)**

Bound strengthening tightens the bounds on variables in mixed integer programs. This may enable CPLEX to fix the variable and remove it from consideration during the branch and bound algorithm. By default (i=–1), CPLEX automatically decides whether to perform bound strengthening. This reduction usually improves performance, but occasionally, due to its iterative nature, takes a long time. In cases where the time required for bound strengthening outweighs any subsequent reduction in run time, disable this feature by setting i=0. To turn on bound strengthening, set i=1.

`cliquecuts=i1` **(default 0)**

`covercuts=i2` **(default 0)**

`disjcuts=i3` **(default 0)**

`flowcuts=i4` **(default 0)**

`flowpathcuts=i5` **(default 0)**

`fraccuts=i6` **(default 0)**

`gubcuts=i7` **(default 0)**

`impliedcuts=i8` **(default 0)**

`mircuts=i9` **(default 0)**

Integer programming solve times can often be improved by generating new constraints (or cuts) based on polyhedral considerations. These additional constraints tighten the feasible region, reducing the number of fractional variables to choose from when CPLEX needs to select a branching variable. CPLEX can generate cuts based on different combinatorial constructs corresponding to the directives listed above.

By default, CPLEX decides whether to generate cuts. Typically the default setting yields the best performance. To disable a particular family of cuts, set its directive to -1. To enable moderate cut generation, set the appropriate directive to 1. To enable aggressive cut generation, set it to 2. Using more aggressive cut generation causes CPLEX to make more passes through the problem to generate cuts. The disjcuts directive also supports a setting of 3 for very aggressive cut generation.

coeffreduce=i **(default 2)**

Coefficient reduction during the presolve phase typically improves CPLEX's performance on integer programs by speeding up the solve times of the LP subproblems solved in the branch-and-bound algorithm. However, while coefficient reduction will tighten the LP subproblems, occasionally it makes them more difficult to solve. So, if CPLEX solves an integer program in a modest number of nodes, but the LP subproblem at each node consumes significant amounts of time, solve time may improve by setting i=0 to disable this feature. The node count may increase, but the savings in time per node may compensate for the increased node count. The default setting of i=2 causes CPLEX to perform coefficient reduction whenever possible, while i=1 will only reduce coefficients to integer values.

cutpass=i **(default 0)**

This directive controls the number of passes CPLEX performs when generating cutting planes for a MIP model. By default, CPLEX automatically determines the number of passes to perform. This setting should suffice for most problems. Set the cutpass directive to -1 to stop all cut generation. Set it to a positive integer to specify a particular number of passes.

cutsfactor=r **(default 4.0)**

The `cutsfactor` directive controls the number of additional cuts generated by CPLEX. While the constraints generated by CPLEX improve performance in most cases, in some problems the additional memory to store them and time required to solve the larger LP subproblems may outweigh the performance gains from the tighter problem formulation. In such cases, use this directive to limit the number of cuts that CPLEX generates. CPLEX will generate no more than r times the number of rows in the problem.

fraccand=i **(default 200)**

This directive limits the number of candidate variables CPLEX will examine when generating fractional cuts on a MIP model. For most purposes the default of 200 will be satisfactory.

fracpass=i **(default 0)**

This directive controls the number of passes CPLEX performs when generating fractional cuts on a MIP model. The default of 0 instructs CPLEX to automatically determine the number of passes and should suffice for most problems. Set it to a positive integer to specify a particular number of passes.

mipstartstatus=i1 **(default 1)**

mipstartvalue=i2 **(default 1)**

These directives control how existing MIP solution information is used by CPLEX. The default value of i1=1 tells CPLEX to use incoming variable and constraint statuses. Incoming statuses can be ignored by setting i1=0.

Note however, that `mipstartstatus` is normally overridden by the AMPL option `send_status`, which can take on the following values:

0	⇒ send no solver status values
1 (default)	⇒ send statuses if there are no integer variables
2	⇒ send statuses even if there are integer variables.

By default (i2=1) variable values are checked to see if they provide an integer feasible solution before the problem is optimized. If an integer feasible solution is found, the objective function value is used as a cutoff during branch-and-bound. To ignore existing values, set i2=0.

`prerelax=i` **(default 0)**

Setting i=1 invokes the CPLEX presolve for the linear program associated with the initial relaxation of a mixed integer program. All other presolve settings apply. Sometimes additional reductions can be made beyond any previously performed MIP presolve reductions.

`presolvenode=i` **(default 0)**

The presolvenode directive determines how CPLEX applies its presolve to the LP subproblems at the nodes of the branch and bound tree. By default, CPLEX decides automatically. Set i=1 to force node presolve. Set i=-1 to prevent it. The default setting usually works best.

`probe=i` **(default 0)**

This directive controls whether CPLEX should perform probing before solving the MIP. Probing can lead to dramatic reductions in the problem size, but can also consume large amounts of time. By default (i=0) CPLEX automatically decides whether to perform probing. To disable probing, set i=–1. To enable probing, set it to a value of 1, 2 or 3. A larger value results in an increased level of probing. More probing can lead to greater reductions in problem size, but also significant increases in probing time.

`sos2=i` **(default 1)**

An optimization problem containing piecewise-linear terms may have to be converted to an equivalent mixed integer program, as explained in section 5.1.1. When i is at its default value of 1, this conversion results in only one extra variable per piecewise-linear breakpoint. All of the extra variables associated with a particular piecewise-linear term are marked as belonging together, so that CPLEX's branch-and-bound procedure knows to treat them specially. Variables so marked have come to be known as a "special ordered set of type 2," whence the name sos2 for this directive.

When i is changed to 0 from its default of 1, the conversion creates a larger number of variables, but does not employ the special ordered set feature. This alternative has no known advantages, and is supplied for completeness only.

7.3 Directives for Algorithmic Control

CPLEX has default values for the algorithmic control directives that often work well for solving a wide range of mixed integer programs. However, it is sometimes necessary to specify alternative values for one or more of the following directives to improve solution times.

You can view each of these directives as corresponding to a particular decision faced at each step in the branch-and-bound procedure. To be specific, imagine that an LP subproblem has just been solved. The sequence of decisions and the corresponding directives are then as follows:

- Branch next from which node in the tree? (`backtrack`, `nodesel`)
- Branch by constraining which fractional variable at the selected node? (`mip_priorities, ordertype, varselect`; also see discussion on setting priorities by variable in Section 8.1.)
- Investigate which of a fractional variable's two resulting branches first? (`branch, rootheuristic, heuristicfreq;` also see discussion on setting branching preference by variable in Section 8.1.)
- Solve the resulting new subproblem by which LP algorithm? (`mipalgorithm`)

It is often hard to predict which combination of directives will work best. Some experimentation is usually required; your knowledge of the problem structure may also suggest certain choices of branch-and-bound strategy.

`backtrack=r`	**(default 0.01)**
`bbinterval=i1`	**(default 7)**
`nodeselect=i2`	**(default 1)**

These directives determine the criterion for choosing the next node from which to branch, once a feasible integer solution has been found.

Depending on whether i2 is set to 1, 2 or 3, CPLEX associates a value with each node, and chooses a node based on these values. For i2=1, a node's value is the bound on the integer optimum that is given by solving the LP subproblem at that node. For i2=2 or 3, a node's value is an estimate of the best integer objective that can be achieved by branching from that node; estimates of node objective values are derived from so-called pseudocosts, which are in turn derived from the solutions to the LP subproblems. Settings 2 and 3 differ regarding the exact nature of the estimated objective. Depending on the value at the current (most recently created) active node, CPLEX either branches from that node, or else backtracks to the node that has the best bound (i2=1) or best estimate (i2=2 or 3) among all active nodes.

When used in conjunction with best estimate node selection (i2=2), the `bbinterval` setting (i1) controls the interval for selecting the best bound node. Decreasing this interval may be useful when best estimate finds good

solutions but makes little progress moving the bound. Conversely, increasing i1 may help when the best estimate node selection does not find any good integer solutions.

The backtracking decision is made by comparing the value (bound or estimate) at the current node with the values at parent nodes in the tree. If the value of the current node has degraded (increased for a minimization, decreased for a maximization) by at least a certain amount relative to the values at parent nodes, then a backtrack is performed. The cutoff for degradation is determined by an internal heuristic that is regulated by the value of r.

Lower values of r (which can range from 0 to 1) favor backtracking, resulting in a strategy that is more nearly "breadth first". The search jumps around fairly high in the tree, solving somewhat dissimilar subproblems. Good solutions are likely to be found sooner through this strategy, but the processing time per node is also greater.

Higher values of r discourage backtracking, yielding a strategy that is more nearly "depth first". Successive subproblems are more similar, nodes are processed faster, and integer solutions are often quickly found deep in the search tree. Considerable time may be wasted in searching the descendants of one node, however, before backtracking to a better part of the tree.

The default value of .01 gives a moderately breadth-first search and represents a good compromise. Lower values often pay off when the LP subproblems are expensive to solve.

Setting i2 to 0 chooses a pure depth-first strategy, regardless of r. CPLEX automatically uses this strategy to search for an initial feasible integer solution at the outset of the branch-and-bound procedure.

`branch=i1` **(default 0)**

`heuristicfreq=i3` **(default 0)**

The `branch` directive determines the direction in which CPLEX branches on the selected fractional variable. When branching on a variable x that has fractional value r, CPLEX creates one subproblem that has the constraint x>ceil(r) and one that has the constraint x<floor(r); these are the "up branch" and "down branch" respectively. By default (i1=0) CPLEX uses an internal heuristic to decide whether it should first process the subproblem on the up branch or on the down branch. You may instead specify consistent selection of the up branch (i1=1) or down branch (i1= –1). Sometimes one of these settings leads the algorithm to examine and discard the "poorer" branches high in the tree, reducing the tree size and overall solution time. (Branching control can also be exercised using the `.direction` suffix described in Section 8.1.)

Similarly, CPLEX can apply a rounding heuristic at nodes other than the root node. Use the `heuristicfreq` directive to specify the frequency with which CPLEX applies it. This can help find solutions missed using other settings. The default value (i3=0) instructs CPLEX to use internal logic to decide when to apply the heuristic. To suppress application of the heuristic

at all nodes, let i3= −1. To specify the node frequency with which CPLEX applies the heuristic, set i3 to a positive integer.

`mipalgorithm=i1` **(default 2)**

`mipcrossover=i2` **(default 1)**

This directive specifies the algorithm, or combination of algorithms, that CPLEX will apply to solve the LP subproblem at each branch-and-bound node. The recognized values of i1 are:

0 Dual simplex for a limited number of iterations, then primal simplex

1 Primal simplex

2 Dual simplex, then primal if dual fails

3 Network simplex on the network part of the problem, then dual simplex (see `netopt` in section 6.2).

4 Barrier algorithm

5 Dual simplex until the iteration limit, then Barrier

6 Barrier algorithm without crossover

The default strategy uses an internal heuristic to determine when to switch from dual to primal simplex. It usually performs well, but the other strategies can significantly reduce the time per node. These settings do not significantly affect the number of nodes that must be visited during the search.

When the Barrier algorithm with crossover is used to solve subproblems (i1=4 or 5), by default (i2=1) CPLEX uses primal simplex for the crossover. In certain cases, dual simplex (i2=2) may be faster.

```
option mip_priorities 'v1 i1 v2 i2 ...' ;
```

From CPLEX 7.0 onwards, the `mip_priorities` option has been superceded by the `.priority` suffix. Please see Section 8.1 for a discussion of setting priorities by individual variable.

`mipemphasis=i` **(default 0)**

This directive determines whether CPLEX emphasizes searching for an optimal solution or trying to find a good solution quickly. By default, CPLEX tries to find the true optimal solution to the problem. However, for some problems searching for the optimal solution may reduce the number of integer feasible solutions found. For problems where finding good solutions as quickly as possible is more important than finding the optimal one, set the mipemphasis directive to one. Otherwise, leave this directive at its default value of 0.

`mipthreads=i`

This directive only applies to users of parallel CPLEX solvers. It specifies the number of parallel processes used during the branch-and-bound optimization. The default value depends on the number of threads licensed.

nodefile=i **(default 1)**

nodefilelim=r **(default 1e+75)**

nodefiledir=f

The list of unprocessed nodes in the branch and bound tree typically dominates CPLEX's memory usage when solving integer programs. A setting of 0 for the nodefile directive causes CPLEX to store all nodes in physical memory. The default value of 1 creates a compressed version of the node file in memory.

Writing nodes to disk (i=2,3) enables CPLEX to process more nodes before running out of memory. This is typically more efficient than relying on the operating system's generic swapping procedure. If i=2, an uncompressed node file is written to disk. Compressing the file (i=3) adds computation time, but allows more efficient use of memory.

When the `nodefile` directive instructs CPLEX to write nodes to a node file, the `nodefilelim` directive specifies the maximum file size. The default is infinite, which means CPLEX will continue to write nodes to disk if physical memory is not available until it either exhausts available disk space, solves the problem, or encounters some other limit.

Disk node files are created in the temporary directory specified by the value of the `nodefiledir` directive. If no value is specified, the directory specified by the TMPDIR (on Unix) or TMP (on Windows) environment variable is used. If TMPDIR or TMP are not set either, the current working directory is used. Node files are deleted automatically when CPLEX terminates normally.

ordertype=i **(default 0)**

CPLEX can automatically generate certain priority orders – which determine the choice of branching variable – based on specific problem features. Use `ordertype` to specify the type of priority order. The default value (i=0) bypasses order generation. Setting i=1 generates a priority order where variables with larger costs receive higher priority. Setting i=2 generates a priority order where variables with smaller bound ranges receive higher priority. This setting tends to be useful for models with binary variables that represent a logical decision and associated general integer variables that represent resource levels enabled by the outcome of the decision.

Setting i=3 tends to help set covering problems. In such problems setting a binary variable to 1 covers a group of rows, but incurs a cost. Binary variables with smaller costs per row covered are good choices to set to 1. An i value of 3 gives higher priority to variables with smaller cost per coefficient count. This tends to identify such binary variables quickly.

plconpri=i1 **(default 1)**

plobjpri=i2 **(default 2)**

Certain piecewise-linear expressions in AMPL models give rise to auxiliary CPLEX variables in groups known as "special ordered sets of type 2". Sos2 variables were discussed in the entry for the sos2 directive above.

CPLEX takes i1 to be the branching priority for all sos2 variables that arise from piecewise-linearities in the constraints, and i2 to be the branching priority for all sos2 variables that arise from piecewise-linearities in the objective. A higher number indicates a higher priority.

`startalgorithm=i` **(default 2)**

This directive specifies the algorithm that CPLEX will apply to solve the initial LP relaxation. The recognized values of i are:

1 Primal simplex

2 Dual simplex

3 Network simplex on the network part of the problem, then dual simplex (see `netopt` in the section on *Directives for Problem and Algorithm Selection*)

4 Barrier algorithm

5 Dual simplex until the iteration limit is reached, then Barrier

6 Barrier algorithm without crossover

`strongcand=i1` **(default 10)**

`strongit=i2` **(default 0)**

`strongthreads=i3`

These three directives impact strong branching (see `varsel` directive below).

The `strongcand` directive controls the size of the candidate list for strong branching. The `strongit` parameter limits the number of iterations performed on each branch in strong branching. Normally the default setting (i2=0), which allows CPLEX to determine the iteration parameter, will suffice. You can use low values of i1 and i2 if the time per strong branching node appears excessive; you may reduce the time per node yet still maintain the performance. Conversely, if the time per node is reasonable but CPLEX makes limited progress, consider increasing the values.

Users of parallel CPLEX can control the number of threads used in strong branching using the strongthreads directive. The default value depends on the number of threads licensed and the ability of the invoked algorithm to run in parallel.

`varselect=i` **(default 0)**

Once a node has been selected for branching, this directive determines how CPLEX chooses a fractional-valued variable to branch on. By default (i=0) the choice is made by an internal heuristic based on the problem and its progress.

The maximum infeasibility rule (i=1) chooses the variable with the largest fractional part. This forces larger changes earlier in the tree, and tends to produce faster overall times to reach the optimal integer solution.

The minimum infeasibility rule (i= –1) chooses the variable with the smallest fractional part. This may lead more quickly to a first integer feasible solution, but will usually be slower overall to reach the optimal integer solution.

A pseudocost rule (i=2) estimates the worsening of the objective that will result by forcing each fractional variable to an adjacent integer, and uses these "degradations" in an internal heuristic for choosing a variable to branch on. This setting tends to be most effective when the problem embodies complex tradeoffs, and the dual variables have an economic interpretation.

Strong branching (i=3) considers several different branches by actually solving subproblems for different choices of branching variable. The variable yielding the best results is then chosen. Strong branching requires more time for each node, but usually fewer nodes to solve the problem. This strategy works especially well on binary problems where the number of binary variables is significantly greater than the number of rows. It is also useful when memory is limited: creating fewer nodes requires less memory.

7.4 Directives for Relaxing Optimality

In dealing with a difficult integer program, you may need to settle for a "good" solution rather than a provably optimal one. The following directives offer various ways of weakening the optimality criterion for CPLEX's branch-and-bound algorithm.

`absmipgap=r1` **(default 0.0)**

`mipgap=r2` **(default 1.0e–4)**

The optimal value of your integer program is bounded on one side by the best integer objective value found so far, and on the other side by a value deduced from all the node subproblems solved so far. The search is terminated when either

$$| \text{best node} - \text{best integer} | < r1$$

or

$$| \text{best node} - \text{best integer} | \ / (1.0 + | \text{best node} |) < r2.$$

Thus the returned objective value will be no more than r1 from the optimum, and will also be within about 100 r2 percent of the optimum if the optimal value is significantly greater than 1 in magnitude.

Increasing r1 or r2 allows a solution further from optimum to be accepted. The search may be significantly shortened as a result. Valid values for r2 lie between 1e–9 and 1.0.

`integrality=r` **(default 1.0e–5)**

In the optimal solution to a subproblem, a variable is considered to have an integral value if it lies within r of an integer. For some problems, increasing r (it has to be at least 1e–9) may give an acceptable solution faster.

`lowercutoff=r1` **(default –1.0e75)**

`uppercutoff=r2` **(default 1.0e75)**

These directives specify alternative cutoff values; a node is fathomed if its subproblem has an objective less than r1 (for maximization problems) or greater than r2 (for minimization problems). As a result any solution returned by CPLEX will have an optimal value at least as large as r1 or as small as r2. This feature can be useful in conjunction with other limits on the search; but too high a value of r1 or too low a value of r2 may result in no integer solution being found.

`objdifference=r1` **(default 0.0)**

`relobjdiff=r2` **(default 0.0)**

This directive automatically updates the cutoff to more restrictive values. Normally the incumbent integer solution's objective value is used as the cutoff for subsequent nodes. When r1 > 0, the cutoff is instead the incumbent's value –r1 (if minimizing) or +r1 (if maximizing). This forces the mixed integer optimization to ignore integer solutions that are not at least r1 better than the one found so far. As a result there tend to be fewer nodes generated, and the algorithm terminates more quickly; but the true integer optimum may be missed if its objective value is within r1 of the best integer objective found.

If r1=0, r2 is used to adjust the objective function value during the optimization. For a maximization problem, r2 times the absolute value of the objective function value is added to the best feasible objective value obtained so far. Similarly, if the objective is to be minimized, r2 times the absolute value is subtracted from the best-so-far feasible objective value. Subsequent nodes are ignored if their linear relaxations have optimal values worse that this adjusted value. Positive values of r2 usually speed the search, but may cause the true optimum to be missed.

7.5 Directives for Halting and Resuming the Search

There is usually no need to make exhaustive runs to determine the impact of different search strategies or optimality criteria. While you are experimenting, consider using one of the directives below to set a stopping criterion in advance. In each case, the best solution found so far is returned to AMPL. (As mentioned earlier, using "break" on command-line versions of CPLEX for AMPL will return the best known solution – for integer programs, that means the current incumbent.)

You can arrange to save the entire search tree when CPLEX halts, so that the search may be resumed from where it left off. Directives for this purpose are also listed below.

`endtree=f1`

`starttree=f2`

CPLEX progressively allocates more memory for the search tree as the branch-and-bound procedure creates new nodes; it frees all this memory at termination. If the `endtree` directive is specified, CPLEX also writes a record of the final tree to the file named f1, in a compact binary format.

CPLEX normally starts the branch-and-bound procedure from a tree that consists only of the root node, as explained at the beginning of this section. If the `starttree` directive is specified, then CPLEX instead starts from the search tree stored in the file named f2. This file must be one that was previously written, for the same problem, by the `endtree` directive.

These directives are particularly useful for large and difficult problems that may take hours or days to solve to optimality. If you would like to look at the first integer solution that CPLEX finds, for example, you can set `solutionlim=1` together with `endtree` and any other directives you like:

```
ampl: model multmip3.mod;
ampl: data multmip3.dat;
ampl: option solver cplexamp;
ampl: option cplex_options
ampl?   'solutionlim=1 varselect=-1'
ampl?   ' endtree=multmip.tre';
ampl: solve;

CPLEX 7.0: solutionlim 1
varselect -1
endtree multmip.tre
CPLEX 7.0: mixed-integer solutions limit;
objective 238225
251 simplex iterations
64 branch-and-bound nodes

ampl: display Trans > multmip.so1;
```

A display of the Trans variables, at the values they take in the first integer solution, has been directed to the file `multmip.sol` for future examination. You could also browse through the values interactively at this point. When you are ready to continue, you need only set `starttree` to the same file as `endtree`, and make any other changes to the branch-and-bound directives that you wish. Then give the `solve` command again:

```
ampl: option cplex_options
ampl?   'solutionlim=1 varselect=1'
ampl?   ' starttree=multmip.tre'
ampl?   ' endtree=multmip.tre';
ampl: solve;

CPLEX 7.0: solutionlim 1
varselect 1
starttree multmip.tre
endtree multmip.tre
CPLEX 7.0: mixed-integer solutions limit;
objective 235625
```

```
596 simplex iterations
124 branch-and-bound nodes
ampl: display Trans > multmip.so2;
```

CPLEX's counts of the numbers of iterations and nodes are cumulative. Since this is a minimization problem, the objective at this second solution is lower than at the first. To continue past this point with all the same CPLEX directives, you need only type `solve`:

```
ampl: solve;
CPLEX 7.0: solutionlim 1
varselect 1
starttree multmip.tre
endtree multmip.tre
CPLEX 7.0: optimal integer solution; objective
235625
601 simplex iterations
142 branch-and-bound nodes
```

We see here that the objective value from the second solution (235625) was optimal, but that CPLEX had to process an additional 18 nodes to prove optimality. (This is not a fluke. The branch-and-bound procedure must often examine many nodes to prove optimality after it has found an optimal solution.)

`nodelim=i` **(default 2.1e9)**

The search is terminated after i linear programming subproblems have been solved. (The default value can vary depending on the hardware.)

`solutionlim=i` **(default 2.1e9)**

The search is terminated after i feasible solutions satisfying the integrality requirements have been found.

`timelimit=r` **(default 1.0e75)**

The search is terminated after r seconds of computing time.

`treememlim=r` **(default 128)**

CPLEX switches to compressed node storage when the branch-and-bound search tree reaches a size of r megabytes. You can use the treememlim and nodefile directives to control when and how CPLEX stores nodes of the branch-and-bound search tree in physical memory or disk. When used in conjunction with the nodefilelim directive, you can use this directive to assure that CPLEX will return with a (possibly less than optimal) solution rather than terminating with an out-of-memory error.

In general, the default setting should suffice. However, for solving problems that generate large branch-and-bound trees on computers with more than 128 MB of memory, you may wish to increase the default setting. Note that r should be set to a few megabytes less than the amount of memory available; this allows for memory used by the operating system, by other programs, and by CPLEX and its other data structures. For example, if your computer has 256 megabytes of real memory, you might start by setting r to 248. Using the

system's virtual memory typically is much slower than CPLEX's own node file storage scheme, so setting the tree memory limit to a value that uses system virtual memory only slows performance.

Set the tree memory limit to 1e+75 if you only want CPLEX to store nodes in physical memory. If you want CPLEX to stop the optimization rather than compress the nodes after hitting the tree memory limit, specify a value of 0 for the nodefilelim directive. Use the nodefile directive for additional control of how CPLEX stores the nodes.

7.6 Directives for Controlling Output

When invoked by `solve`, CPLEX normally returns just a few lines to your screen to summarize its performance. The following directives let you choose more output, which may be useful for monitoring the progress of a long run, or for comparing the effects of other directives on the behavior of the branch-and-bound algorithm. Output normally comes to the screen, but may be redirected to a file by specifying `solve >`*`filename`*.

`mipdisplay=i1` **(default 0)**

`mipinterval=i2` **(default 1)**

The default of i1=0 produces a minimal few lines of output from CPLEX, summarizing the results of the run.

When i1=1, a single "log line" is displayed for every integer solution found. The information includes the number of nodes processed, and the objective values of the best integer solution found so far and of the best unprocessed node subproblem. (The optimal value lies between these two.)

When i1=2, a more detailed log line is displayed once every i2 nodes, as well as for each node where an integer solution is found. A * indicates lines of the latter type. The default of i2=1 gives a complete picture of the branch-and-bound process, which may be instructive for small examples. With a larger choice of i2, this setting can be very useful for evaluating the progress of long runs; the log line includes a count of the number of active nodes, which gives an indication of the rate at which the search tree is growing or shrinking in memory.

When i1=3, CPLEX also prints information on node cut and node presolve. The LP iteration log for the root node (i1=4) and for all subproblems (i1=5) can also be displayed.

`timing=i` **(default 0)**

This directive can be used to display a summary of processing times. It works the same for integer programming as for linear programming, as described in *Using CPLEX for Linear Programming*.

7.7 Common Difficulties

The following discussion addresses the difficulties most often encountered in solving integer programs with CPLEX.

7.7.1 Running Out of Memory

The most common difficulty when solving MIP problems is running out of memory. This problem arises when the branch-and-bound tree becomes so large that insufficient memory is available to solve an LP subproblem. As memory gets tight, you may observe warning messages while CPLEX attempts to navigate through various operations within limited memory. If a solution is not found shortly, the solutions process will be terminated with an error termination message.

The tree information saved in memory can be substantial. CPLEX saves a basis for every unexplored node. When utilizing the best-bound or best estimate method of node selection, the list of unexplored nodes can become very long for large or difficult problems. How large the unexplored node list can become is entirely dependent on the actual amount of physical memory available, the size of the problem, and the solution algorithm selected. Certainly increasing the amount of memory available extends the problem solving capability. Unfortunately, once a problem has failed because of insufficient memory you cannot project how much further the process needed to go or how much memory would be required to ultimately solve it.

To avoid out-of-memory failures, we recommend resetting the `treememlim` parameter to stop the solution process prior to consuming all available memory. This limit parameter value should be set to a number slightly less than the total available memory, which can include the swap file. Remember that not all installed memory is available - the operating system and other active processes can reduce the amount of memory available to CPLEX.

In some cases, even though the current tree size is within system resource limits, it may be that there is considerable memory fragmentation and as a result, poor performance because of the way in which the tree was built. To combat that fragmentation, it can be helpful to write a tree file and resolve, reading in the tree file.

Some parts of the branch-and-bound tree can be stored in compressed files when the `nodefile` directive is used. Storing part of each node in files will allow more nodes to be explored in a given `treememlim` limit, but file access may be slower than physical memory access. This feature may be especially useful when steepest edge pricing used for subproblem simplex pricing strategy, because the pricing information consumes a lot of memory.

The best approach to reduce memory usage is to modify the solution process. Switching to a higher `backtrack` parameter value and best estimate node selection strategy (or depth-first search node selection which is even more extreme) often works. Depth-first search rarely generates a large unexplored node list since CPLEX will be diving deep into the branch-and-bound tree

rather than jumping around within it. This narrowly focused search also often results in faster individual node processing times. Overall efficiency is sometimes worse than with best-bound node selection since each branch is exhaustively searched to the deepest level before fathoming it in favor of better branches.

Another memory conserving strategy is to use strong branching variable selection (using the `varselect` directive). When using strong branching substantial computational effort is made at each node to determine the best branching variable. As a result, many fewer nodes are generated reducing the overall demand on memory. Often, strong branching is faster as well as using less memory.

On some problems, the automatic generation of cuts results in excessive use of memory with little benefit in speed. In such cases it is expedient to turn off cut generation by setting the `covers` and `cliques` directives to –1.

7.7.2 Failure To Prove Optimality

One frustrating aspect of the branch-and bound technique for solving MIP problems is that the solution process can continue long after the best solution has been found. In these situations the branch-and-bound tree is being exhaustively searched in an effort to guarantee that the current integer feasible solution is indeed optimal. Remember that the branch-and-bound tree may be as large as 2^n nodes, where n equals the number of binary variables. A problem containing only 30 binary variables could produce a tree having over one billion nodes! If no other stopping criteria have been set, the process might continue until the search is complete or your computer's memory is exhausted.

In general you should set at least one limit on the number of nodes processed, number of improved solutions found, or total processing time, using the CPLEX directives given above. Setting limits ensures that the tree search will terminate in reasonable time. You can then inspect the solution and, if necessary, re-run the problem using different directive settings. Consider some of the shortcuts described above for improving performance, particularly those for relaxing optimality. They may provide you with an optimal or very nearly optimal solution, even though a proof of optimality would require more computer resources than you have available.

7.7.3 Difficult MIP Subproblems

Certain classes of MIP problems occasionally produce very difficult subproblems. The subproblems may be dual degenerate. Or an alternative algorithm such as primal simplex or barrier may perform better with the particular model.

If the subproblems are dual degenerate, consider setting `mipalgorithm` to choose primal simplex for solving subproblems.

If the subproblems are taking many iterations per node to solve, consider setting `dgradient` to use a stronger dual-pricing algorithm. Most often, one would use dual steepest edge pricing.

In cases where the barrier algorithm is selected to solve the initial LP relaxation, it may be useful to apply it on the subproblems using one of two options. The first is to use Barrier on all subproblems. Since the Barrier algorithm can not utilize a basis, often a better choice is to allow the dual simplex algorithm to run for a predetermined number of iterations before switching to Barrier. Set the simplex iteration limit to a reasonably low number of dual iterations and then invoke this hybrid solutions strategy by setting the `mipalgorithm` directive to 5. Remember that setting a simplex iteration limit apples to all invocations of the simplex solvers. If the iteration limit is set too low, it might prematurely terminate cleanup iterations sometimes needed at the conclusion of a crossover operation. Since the dual simplex solver will most often be the best method, specify a sufficient number of iterations before forcing a switch to Barrier.

For either of the above `mipalgorithm` strategies, it is beneficial to set the barrier algorithm option to settings 1 or 2. Either of these non-default choices is better at detecting infeasibility, a frequent characteristic of MIP subproblems.

8 Defined Suffixes for CPLEX

The most common use of AMPL suffixes is to represent solver result values that correspond to variables, constraints, and other model components. Yet only the most standard kinds of results, such as reduced costs (given by `X.rc`, where `X` is a variable name) and slacks (given by `C.slack`, where `C` is a constraint name), are covered by the built-in suffixes.

To allow for solver-specific optimization results, AMPL permits solvers to define new suffixes and to associate solution result information with them. Similarly, users can also define suffixes to control the solver. User-defined suffixes understood by CPLEX and suffixes defined by CPLEX are described in this section.

8.1 Algorithmic Control

For each integer variable in a problem, CPLEX recognizes a preferred branching direction and a branching priority, specified by the following two suffixes:

`.direction`

`.priority`

Branching direction preference can be specified for each variable by setting its `.direction` suffix to a value between –1 and 1. (Variables not assigned a suffix value get a default value of zero.) A negative value indicates a preference for branching "down", and a positive value indicates a preference for branching "up". For variables with `.direction` at zero, the branching direction is determined by the branching-related directives described in Section 7.3. (See explanation of branch-and-bound in Section 7 for a description of branching.)

Each time that CPLEX must choose a fractional-valued integer variable on which to branch, it gives preference to the fractional variables that have the highest `.priority` value. A judicious choice of priorities (any number between 0 and 9999 is valid) can guide the search in a way that reduces the number of nodes generated. For example, let us consider a model drawn from pages 300-301 of the AMPL book:

```
ampl: model models\multmip3.mod;
ampl: data models\multmip3.dat;
ampl: solve;

CPLEX 7.0: optimal integer solution;
objective 235625
601 simplex iterations
91 branch-and-bound nodes
```

Note that CPLEX takes 91 nodes and 601 simplex iterations to find the optimal integer solution. Now, let us provide CPLEX with branching priorities for all variables as well as a preferred branching direction for a single variable. Note that before we re-run CPLEX, we set `mipstartvalue` to discard the existing solution.

```
ampl: option cplex_options 'mipstartvalue 0';
ampl: suffix priority IN,integer,>=0,<=9999;
ampl: suffix direction IN,integer,>=-1,<= 1;
ampl: let {i in ORIG, j in DEST}
ampl?    Use[i,j].priority :=
ampl?    sum {p in PROD} demand[j,p];
ampl: let Use["GARY","FRE"].direction := -1;
ampl: solve;

CPLEX 7.0: optimal integer solution; objective
235625
446 simplex iterations
64 branch-and-bound nodes
```

Indeed, CPLEX now requires fewer nodes (64) and fewer simplex iterations (446) to reach optimality. While this is not a dramatic improvement, larger cases where directing branch-and-bound in this manner makes the difference between unsolvability and finding the solution in a few minutes are well-known.

8.2 Sensitivity Ranging

When the `sensitivity` directive (described in Section 6.8) is included in CPLEX's option list, classical sensitivity ranges are computed and are returned in the following three suffixes:

`.current`

`.down`

`.up`

Let us illustrate the use of these suffixes using an example model from Section 4.3 of the AMPL book:

```
ampl: model steelT.mod;
ampl: data steelT.dat;
ampl: option solver cplexamp;
ampl: option cplex_options 'sensitivity';
ampl: solve;
```

```
CPLEX 7.0: sensitivity
CPLEX 7.0: optimal solution; objective 515033
18 iterations (1 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;
```

The three lines at the end show the suffix commands executed by AMPL in response to the results from CPLEX. These commands are invoked automatically; you do not need to type them.

For variables, suffix `.current` indicates the objective function coefficient in the current problem, while `.down` and `.up` give the smallest and largest values of the objective coefficient for which the current simplex basis remains optimal. (CPLEX returns –1e+20 for `.down` and 1e+20 for `.up` to indicate minus infinity and plus infinity, respectively.)

```
ampl: display Sell.down, Sell.current,
ampl? Sell.up;

:       Sell.down Sell.current    Sell.up
:=
bands 1   23.3          25       1e+20
bands 2   25.4          26       1e+20
bands 3   24.9          27          27.5
bands 4   10            27          29.1
coils 1   29.2857       30          30.8571
coils 2   33            35       1e+20
coils 3   35.2857       37       1e+20
coils 4   35.2857       39       1e+20
;
```

For constraints, the interpretation is similar except that it applies to a constraint's constant term (the so-called right-hand side value).

```
ampl: display time.down, time.current,
ampl? time.up;

: time.down time.current   time.up    :=
1   37.8071       40       66.3786
2   37.8071       40       47.8571
3   25            32       45
4   30            40       62.5
;
```

8.3 Diagnosing Infeasibilities

For a linear program that has no feasible solution, you can ask CPLEX to find an irreducible infeasible subset (or IIS) of the constraints and variable bounds. By definition, members of an IIS have no feasible solution, but dropping any one of them permits a solution to be found to the remaining ones. Clearly, knowing the composition of an IIS can help localize the source of the infeasibility.

The associated suffix is:

```
.iis
```

You turn on the IIS finder using the `iisfind` option described in Section 6.8. An associated option `iis_table`, set up and displayed automatically by CPLEX, shows the strings that may be associated with `.iis` and gives brief descriptions of what they mean.

The following example shows how IIS finding might be applied to the infeasible diet problem from chapter 2 of the AMPL book. After solve detects that there is no feasible solution, it is repeated with the `iisfind` directive:

```
ampl: model diet.mod;
ampl: data diet2.dat;
ampl: option solver cplexamp;
ampl: solve;

CPLEX 7.0: infeasible problem
7 iterations (7 in phase I)

ampl: option cplex_options 'iisfind 1';
ampl: solve;

CPLEX 7.0: iisfind 1
CPLEX 7.0: infeasible problem
0 iterations

Returning iis of 7 variables and 2
constraints.

suffix iis symbolic OUT;

option iis_table '\
0    non   not in the iis\
1    low   at lower bound\
2    fix   fixed\
3    upp   at upper bound\
';
```

You can use display to look at the .iis values that have been returned:

```
ampl: display _varname, _var.iis, _conname,
ampl? _con.iis;

:  _varname     _var.iis   _conname    con.iis
:=
1   "Buy['BEEF']"   upp    "diet['A']"     non
2   "Buy['CHK']"    low    "diet['B1']"    non
3   "Buy['FISH']"   low    "diet['B2']"    low
4   "Buy['HAM']"    upp    "diet['C']"     non
5   "Buy['MCH']"    non    "diet['NA']"    upp
6   "Buy['MTL']"    upp    "diet['CAL']"   non
7   "Buy['SPG']"    low          .          .
8   "Buy['TUR']"    low          .          .
;
```

This information indicates that the IIS consists of four lower and three upper bounds on the variables, plus the constraints providing the lower bound on

B2 and the upper bound on NA in the diet. Together these restrictions have no feasible solution, but dropping any one of them will permit a solution to be found to the remaining ones. (Of course, in our example, we shouldn't actully drop the lower bounds on the Buy variable – we could end up with negative values. However, we could reduce certain lower bounds to zero.)

8.4 Direction of Unboundedness

For an unbounded linear program – one that has in effect a minimum objective value of –Infinity or a maximum of +Infinity – the "solution" is characterized by a feasible point together with a direction of unboundedness from that point. On return from CPLEX, the values of the variables define the feasible point. The direction of unboundedness is given by an additional value associated with each variable through the associated solver-defined suffix:

`.unbdd`

An application of the direction of unboundedness can be found in our example of Benders decomposition applied to a transportation-location problem. One part of the decomposition scheme is a subproblem obtained by fixing the variables Build[i], which indicate the warehouses that are to be built, to trial values build[i].

When all values build[i] are set to zero, no warehouses are built, and the primal subproblem is infeasible. As a result, the dual formulation of the subproblem – which always has a feasible solution – is unbounded. When this dual problem is solved from the AMPL command line, CPLEX returns the direction of unboundedness in the expected way:

```
ampl: model trnloc1d.mod;
ampl: data trnloc1.dat;
ampl: problem Sub: Supply_Price, Demand_Price,
ampl? Dual_Ship_Cost, Dual_Ship;
ampl: let {i in ORIG} build[i] := 0;
ampl: option solver cplexamp;
ampl: option cplex_options 'presolve 0';
ampl: solve;

CPLEX 7.0: presolve 0
CPLEX 7.0: unbounded problem
30 iterations (0 in phase I)
variable.unbdd returned

suffix unbdd OUT;
```

The suffix message indicates that `.unbdd` has been created automatically. You can use this suffix to display the direction of unboundedness, which is quite simple in this case:

```
ampl: display Supply_Price.unbdd;

Supply_Price.unbdd [*] :=
 1 -1    6 -1   11 -1   16 -1   21 –1
 2 -1    7 -1   12 -1   17 -1   22 –1
```

```
 3 -1    8 -1   13 -1   18 -1   23 –1
 4 -1    9 -1   14 -1   19 -1   24 –1
 5 -1   10 -1   15 -1   20 -1   25 -1
;
ampl: display Demand_Price.unbdd;
Demand_Price.unbdd [*] :=
A3 1   A6 1   A8 1   A9 1   B2 1   B4 1
;
```

9 CPLEX Status Codes in AMPL

9.1 Solve Codes

When CPLEX returns control to AMPL following a solve command, built-in AMPL parameters and an AMPL option provide information about the outcome of the optimization, as shown below:

```
ampl: model oil.mod;
ampl: data oil.dat;
ampl: option solver cplexamp;
ampl: display solve_result_num, solve_result;

solve_result_num = -1
solve_result = '?'

ampl: solve;
CPLEX 7.0: optimal solution; objective
12.20834324
37 iterations (0 in phase I)

ampl: display solve_result_num, solve_result;

solve_result_num = 0
solve_result = solved

ampl: option solve_result_table;

option solve_result_table '\
0       solved\
100     solved?\
200     infeasible\
300     unbounded\
400     limit\
500     failure\
';
```

As shown by the session log above, initially the built-in AMPL parameter `solve_result_num` is –1 and parameter `solve_result` is '?'. The solve invocation resets these parameters, however, so that they describe CPLEX's status at the end of its run – the `solve_result_num` parameter by a numeric code and `solve_result` by a message string. In the

example shown, `solve_result_num` is set to 0 and `solve_result` to 'solved', indicating normal termination.

The AMPL option `solve_result_table` lists the valid combinations of `solve_result_num` and `solve_result`, for CPLEX. These combinations should be interpreted as shown below.

Number	*String*	*Interpretation*
0 - 99	solved	optimal solution found
100 – 199	solved?	optimal solution indicated, but error likely
200 – 299	infeasible	constraints cannot be satisfied
300 – 399	unbounded	objective can be improved without limit
400 – 499	limit	stopped by a limit (such as on iterations)
500 – 599	failure	stopped due to solver error

Status ranges are normally used to control algorithmic flow in AMPL scripts, where `solve_result_num` can be tested to distinguish among cases that must be handled in different ways. It is occasionally useful, however, to make fine distinctions among different solver termination conditions. All valid solve codes, with the corresponding termination message from CPLEX, are listed in the table below.

Number	*Message at termination*
0	optimal solution
1	primal has unbounded optimal face
2	optimal integer solution
3	optimal integer solution within mipgap or absmipgap
100	best solution found, primal-dual feasible
200	infeasible problem
201	infeasible with phase II singularities
202	infeasible with phase I singularities
203	optimal with unscaled infeasibilities
204	converged, dual feasible, primal infeasible
205	converged, primal and dual infeasible
206	best solution found, primal infeasible
207	best solution found, primal-dual infeasible
208	infeasible or unbounded in presolve
220	integer infeasible
300	unbounded problem
301	converged, primal feasible, dual infeasible
302	best solution found, dual infeasible

400	phase II objective limit exceeded
401	phase II iteration limit
402	phase I iteration limit
403	phase II time limit
404	phase I time limit
405	primal objective limit reached
406	dual objective limit reached
410	node limit with no integer solution
411	time limit with no integer solution
412	treememory limit with no integer solution
420	mixed integer solutions limit
421	node limit with integer solution
422	time limit with integer solution
423	treememory limit with integer solution
500	unrecoverable failure
501	aborted in phase II
502	aborted in phase I
503	aborted in barrier, dual infeasible
504	aborted in barrier, primal infeasible
505	aborted in barrier, primal and dual infeasible
506	aborted in barrier, primal and dual feasible
507	aborted in crossover
510	unrecoverable failure with no integer solution
511	aborted, no integer solution
520	unrecoverable failure with integer solution
521	aborted, integer solution exists
522	integer optimal with unscaled infeasibilities
523	out of memory, no tree; solution may exist

9.2 Basis Status

Following optimization, CPLEX also returns an individual status for each variable and constraint. This feature is intended mainly for reporting the basis status of variables after a linear program is solved either by the simplex method, or by an interior-point (barrier) method followed by a “crossover” routine. In addition to the variables declared by var statements in an AMPL model, solvers also define "slack" or "artificial" variables that are associated

with constraints. Solver statuses for these latter variables are defined in a similar way.

The major use of solver status values from an optimal basic solution is to provide a good starting point for the next optimization run, possibly after a data change.

You can refer to a variable's solver status by appending `.sstatus` to its name. Initially, when no problem has yet been solved, all variables have the status none. After an invocation of a simplex solver, the same display lists the statuses of the variables at the optimal basic solution. For example, consider the following:

```
ampl: model oil.mod;
ampl: data oil.dat;
ampl: option solver cplex;
ampl: solve;

CPLEX 7.0: optimal solution; objective
12.20834324
37 iterations (0 in phase I)

ampl: option sstatus_table;

option sstatus_table '\
0  none  no status assigned\
1  bas   basic\
2  sup   superbasic\
3  low   nonbasic <= (normally =) lower bound\
4  upp   nonbasic >= (normally =) upper bound\
5  equ   nonbasic at equal lower and upper bounds\
6  btw   nonbasic between bounds\
';

ampl: display InCr.sstatus;

InCr.sstatus [*] :=
MID_C  bas
W_TEX  low
;
```

A table of the recognized CPLEX status values is stored in the AMPL option `sstatus_table` displayed above. Numbers and short strings representing status values are given in the first two columns. (The numbers are mainly for communication between AMPL and CPLEX, though you can access them by using the suffix `.sstatus_num` in place of `.sstatus`.) The entries in the third column are comments.

The output of the display command shows that variable InCr['MID_C'] is in the basis and InCr['W_TEX'] at its lower bound at optimality.

You can change a variable's basis status using AMPL's "let" command. This may be useful in instances where you want to provide an initial basis to jump-start CPLEX.

Appendix A CPLEX Synonyms

The following list contains alternative names for certain CPLEX directives. The use of primary names is recommended.

Synonym	Primary Directive
agglim	aggfill
dense	densecol
display	lpdisplay
doperturb	perturb
endbasis	writebasis
endvector	writevector
growth	bargrowth
heuristic	rootheuristic
iterations	lpiterlim
mipsolutions	solutionlim
nodes	nodelim
nodesel	nodeselect
startbasis	readbasis
startvector	readvector
subalgorithm	mipalgorithm
time	timelimit
treememory	treememlim
varsel	Varselect

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