



Graphs

Graphs

- Graphs are mathematical structures used to model pairwise relations between abstract objects

- Graphs are a set of Vertices and Edges

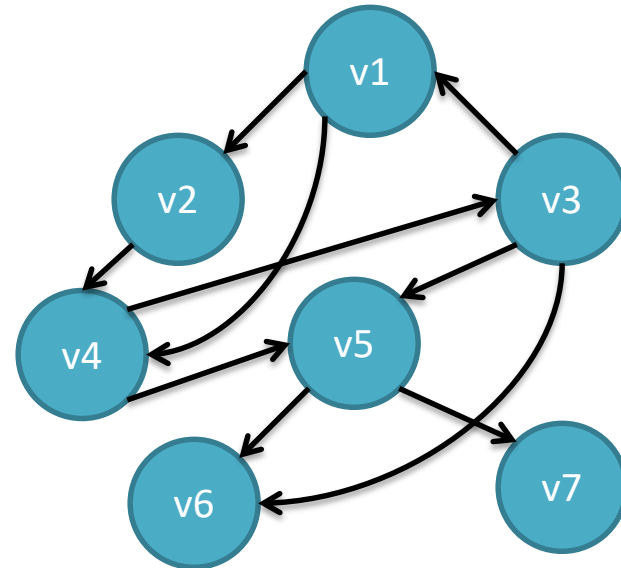
$$G = (V,E)$$

Where $V = \{v_i\}$;

And $E = \{e_k = \langle v_i, v_j \rangle\}$

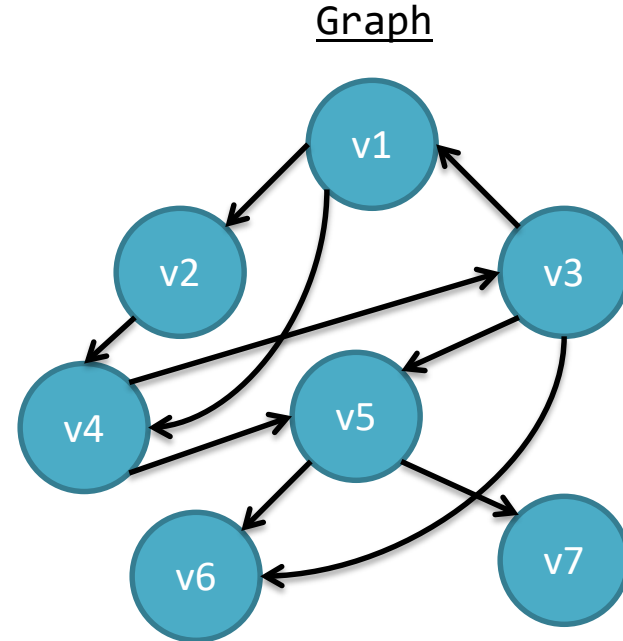
- Graphs can be directed or undirected
 - Directed is one way
 - Undirected is bi-directional

Graph



Graphs

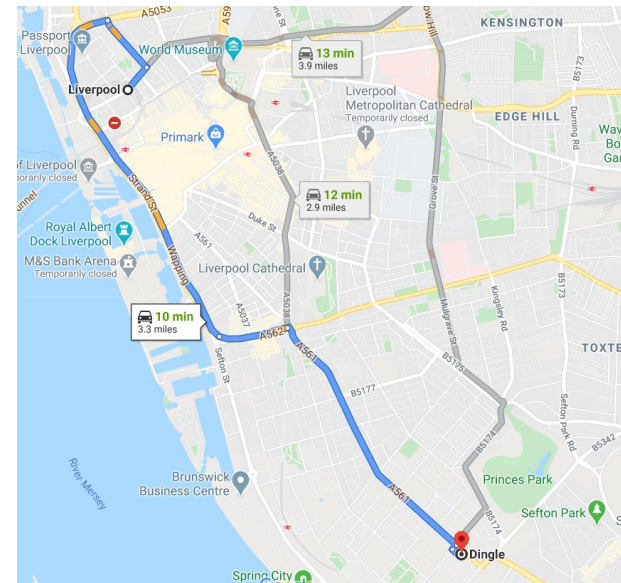
- Graphs do not have a given structure
 - Data may not be comparable
 - They can contain cycles
- The structure of the graph is usually dependent on the problem being solved
- Vertices may be considered as an abstract object
- Edges may be considered the relationship between those objects
- Edges may have associated values called weights



Graphs

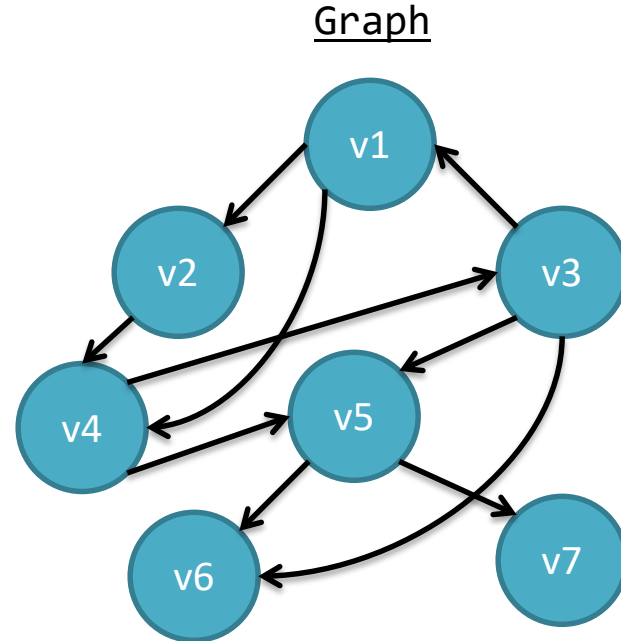
- A map can be considered a graph
 - Cities are Vertices
 - Edge weights are the distance between them
- Finding a shortest path is a classic problem that can be solved with a graph

Graph



Graphs

- Popular ways to implement a graph
 - Adjacency Matrix
 - Adjacency List (Linked Structure)
 - Incident Matrix



Adjacency Matrix

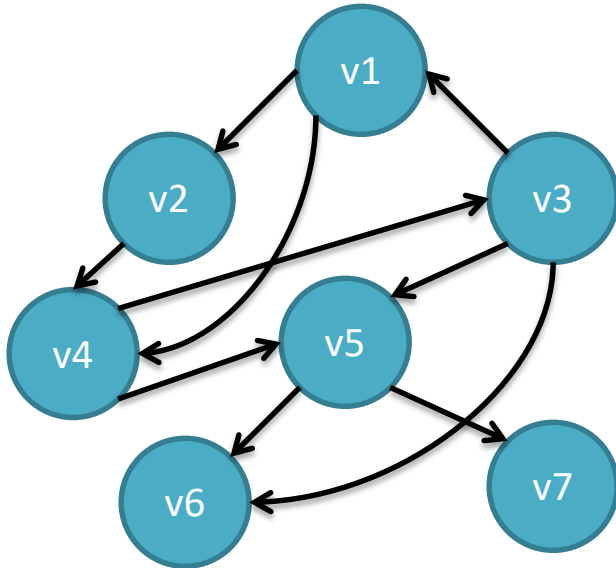
- Rows and Columns represent the Vertices
 - If the graph has n nodes then the matrix will be $n \times n$
- Rows are the “from Vertex”
- Columns are the “to Vertex”
- Non-zero values stored in the matrix represent edges and their weights

Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Adjacency Matrix

Graph

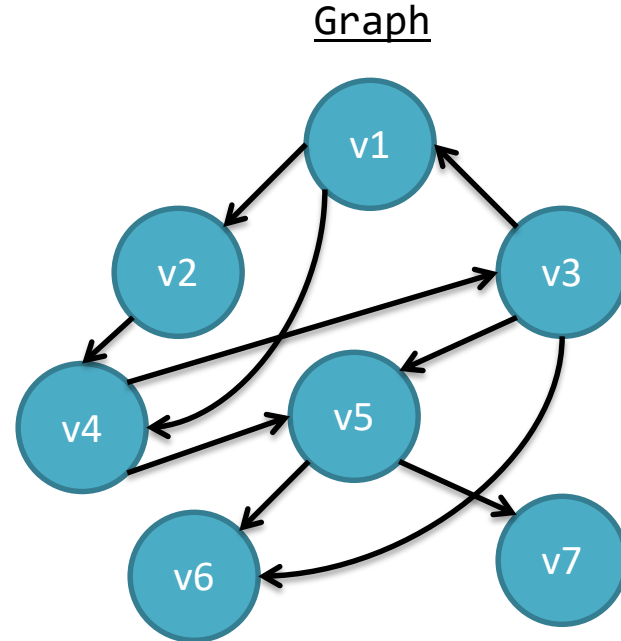


Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

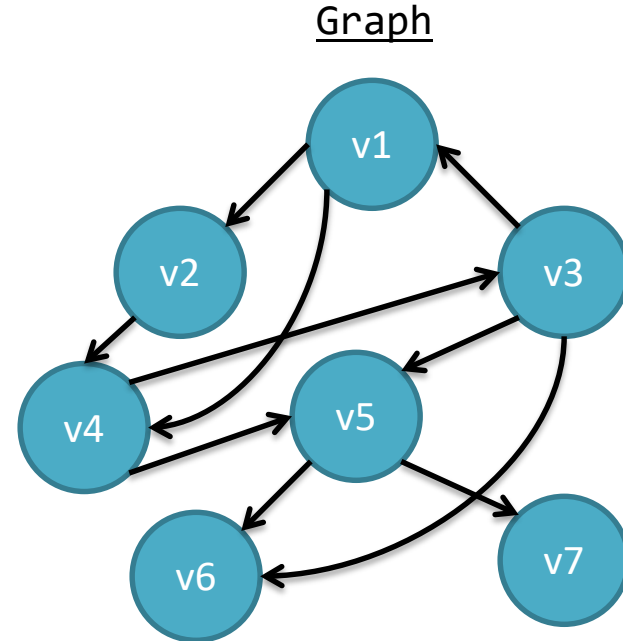
Graph Traversals

- Travel through the graph structure systematically
- Can be used to search the graph, find paths, or detect features of a graph
- Common Graph Traversals
 - Depth First Search (DFS)
 - Stack Based
 - Breadth First Search (BFS)
 - Queue Based



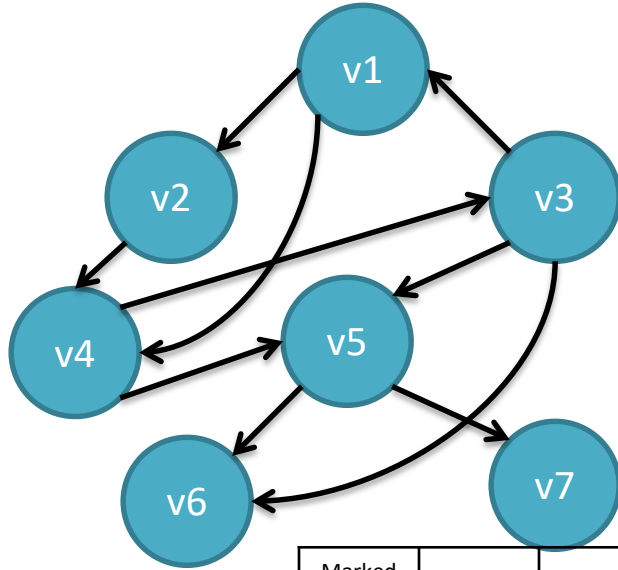
DFS Algorithm

- Assume that each vertex has a unique identifier and we have a List of “marked vertices”
1. Start from an arbitrary vertex
 2. Add the vertex’s identifier to the marked vertices list
 3. Follow an outgoing edge
 4. If that vertex is part of the marked list then backtrack to the previous vertex
 5. Otherwise repeat step 2 until there are no more reachable nodes



DFS Example

Graph



Adjacency Matrix

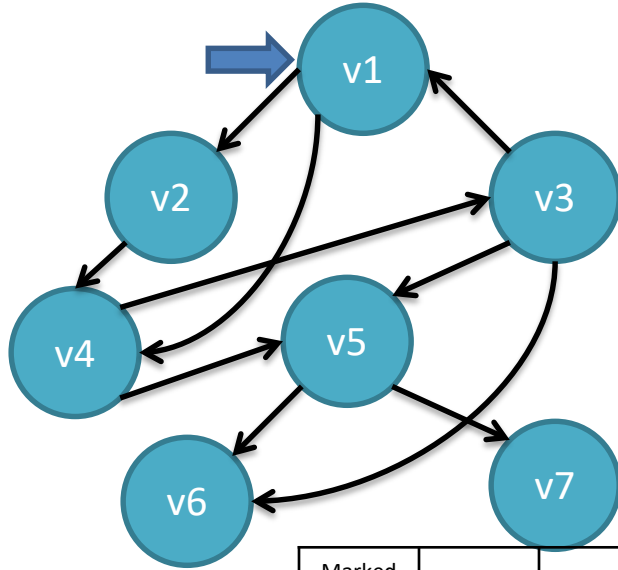
	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List

--	--	--	--	--	--	--	--

DFS Example

Graph



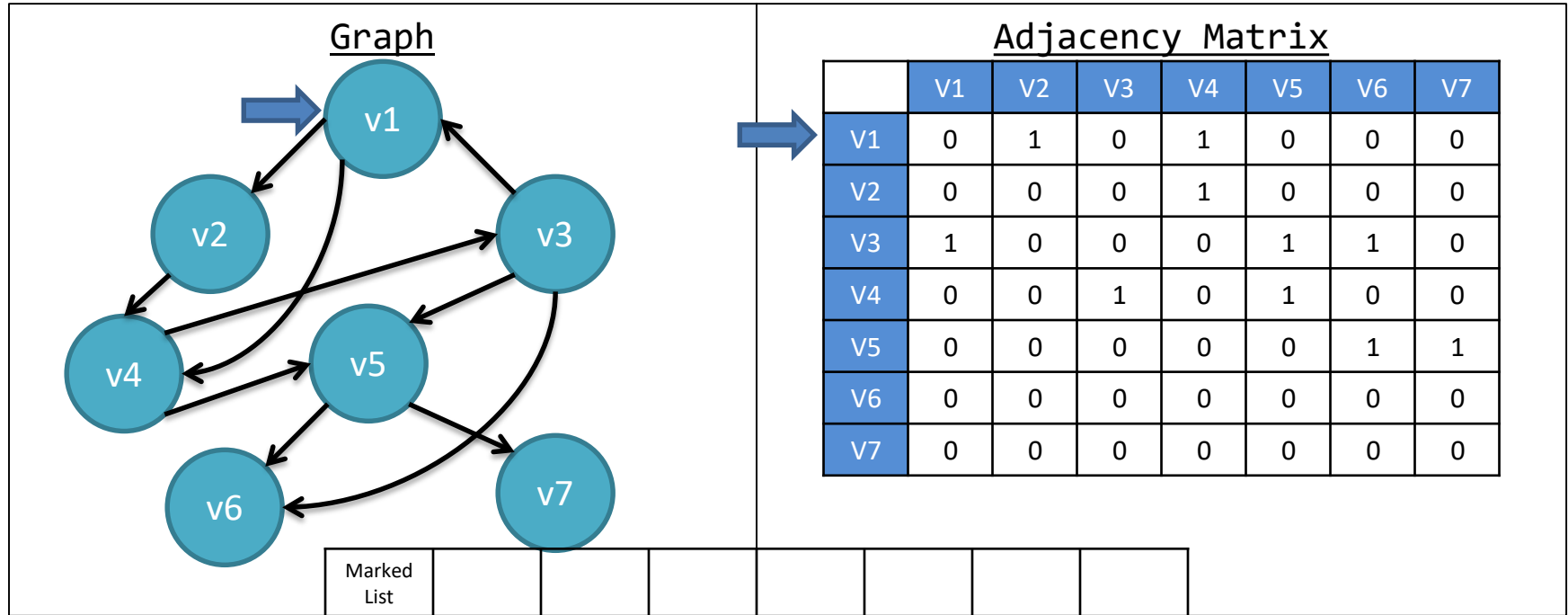
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

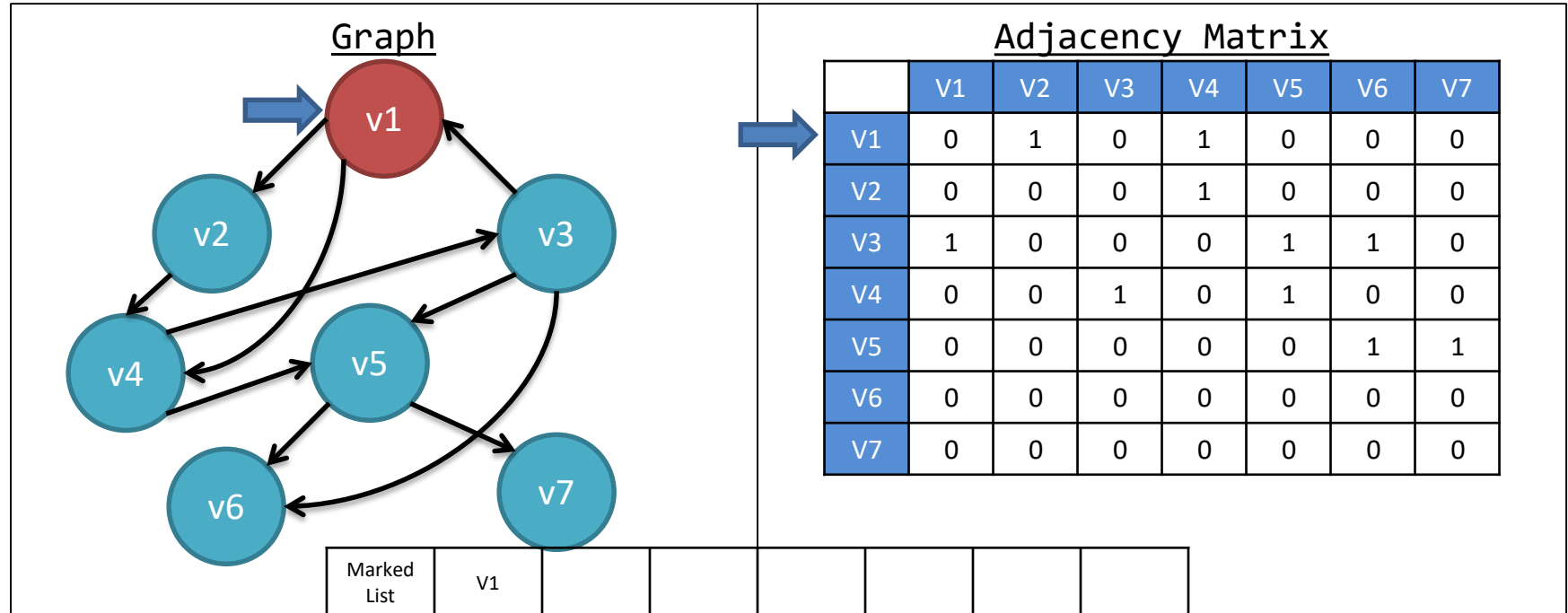
Marked
List

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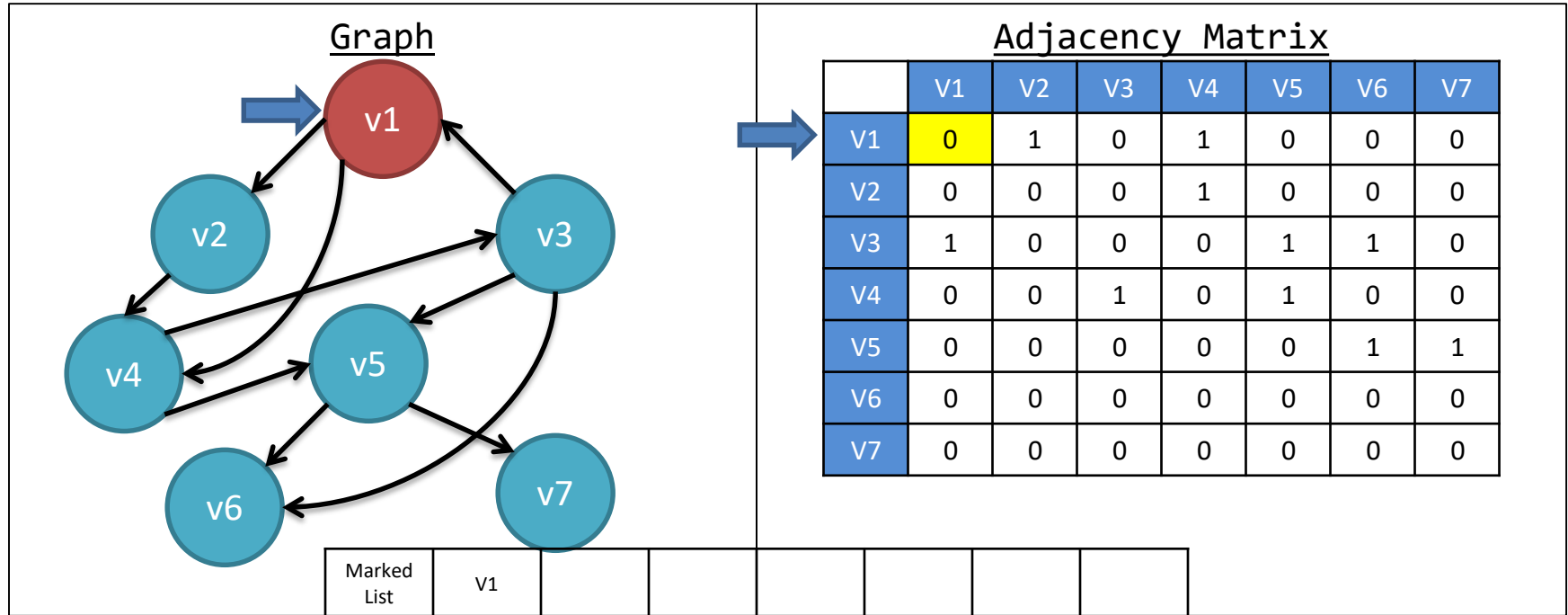
DFS Example



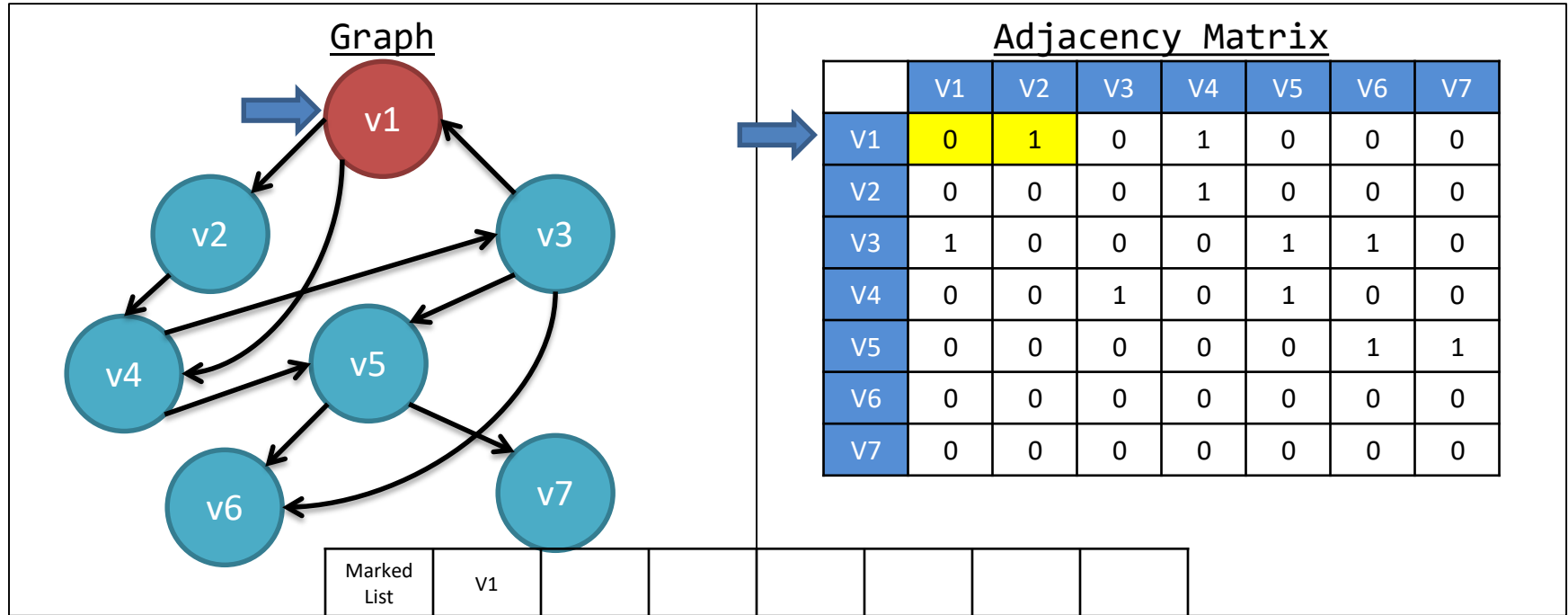
DFS Example



DFS Example

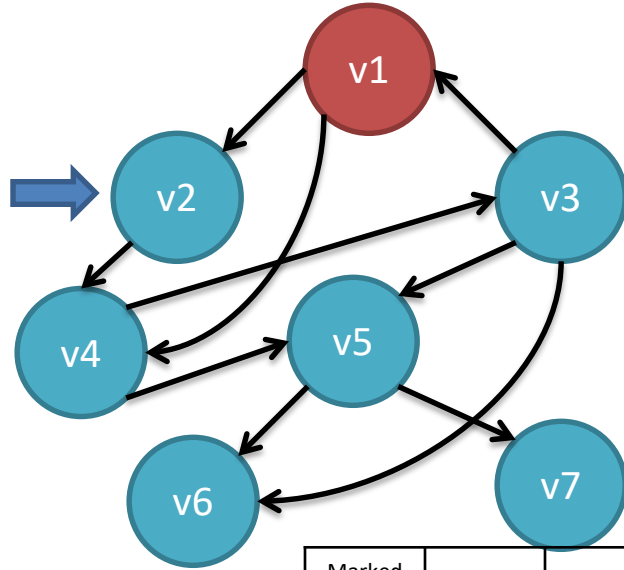


DFS Example



DFS Example

Graph



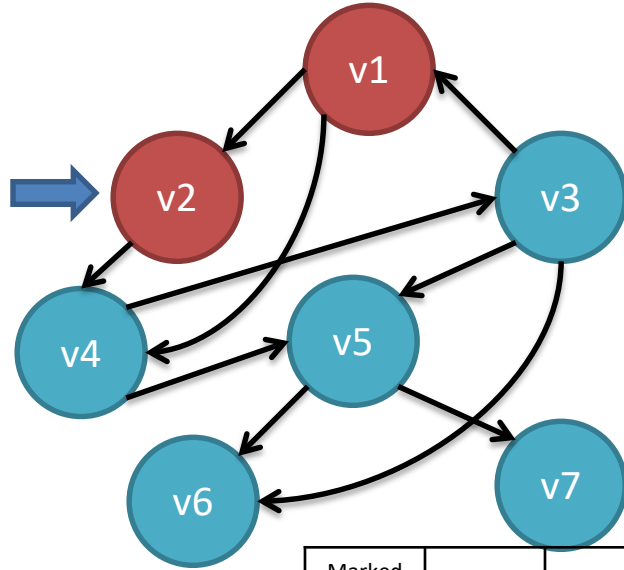
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1						

DFS Example

Graph



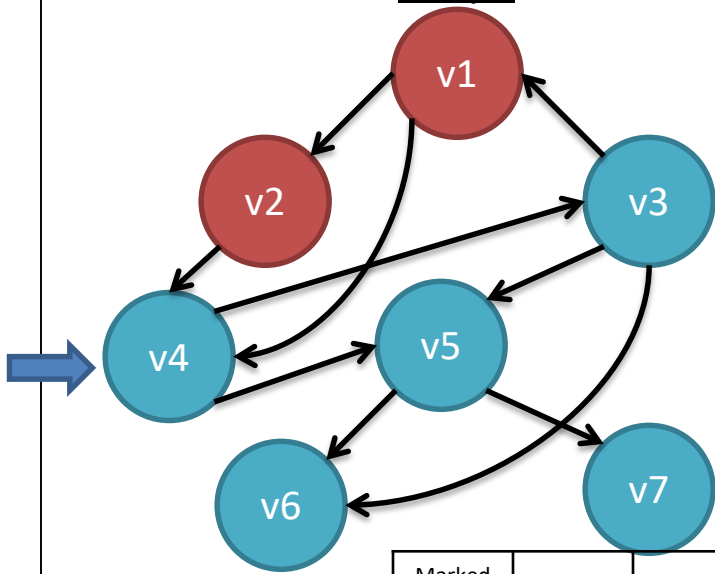
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2					

DFS Example

Graph



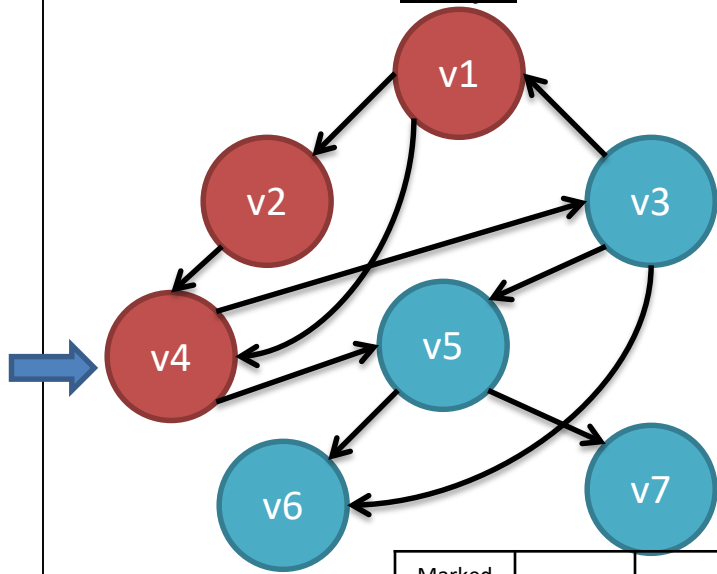
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2					

DFS Example

Graph



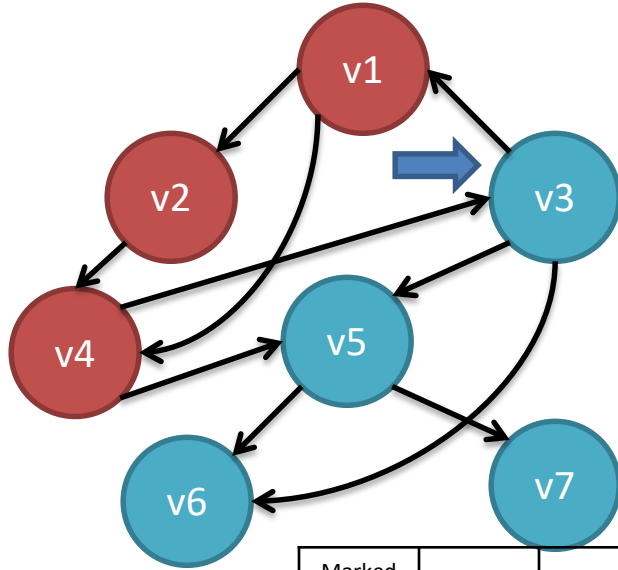
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4				

DFS Example

Graph



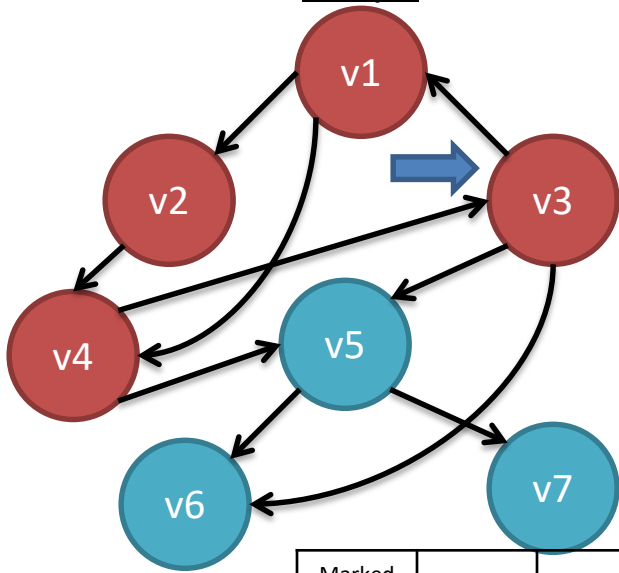
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4				

DFS Example

Graph



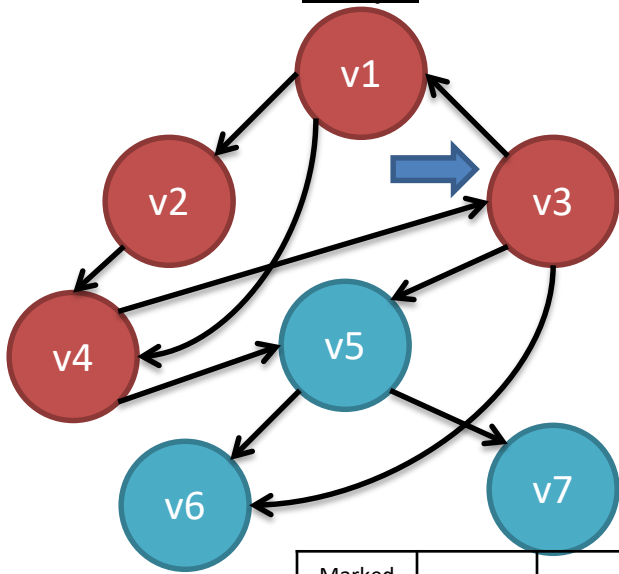
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3			

DFS Example

Graph



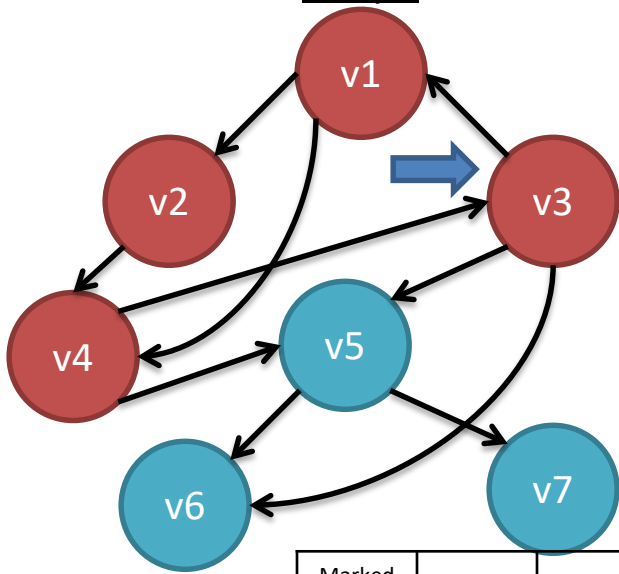
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3			

DFS Example

Graph

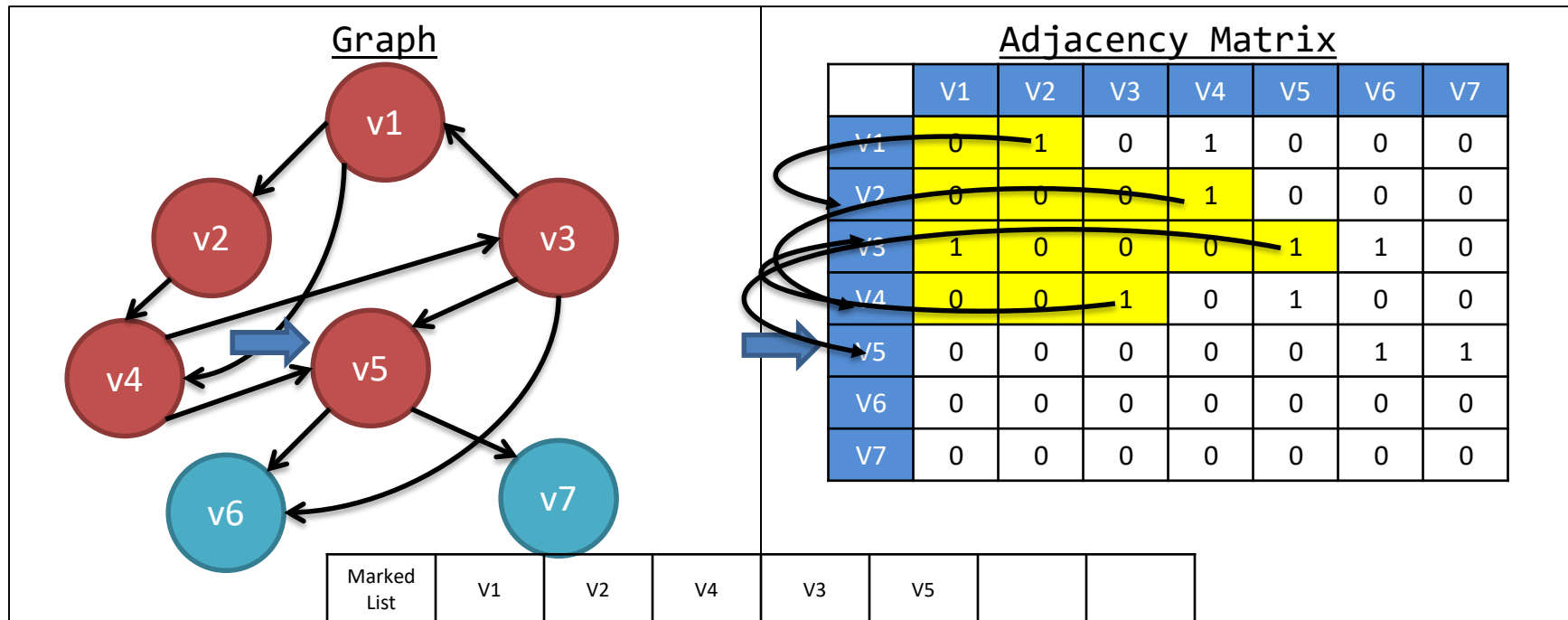


Adjacency Matrix

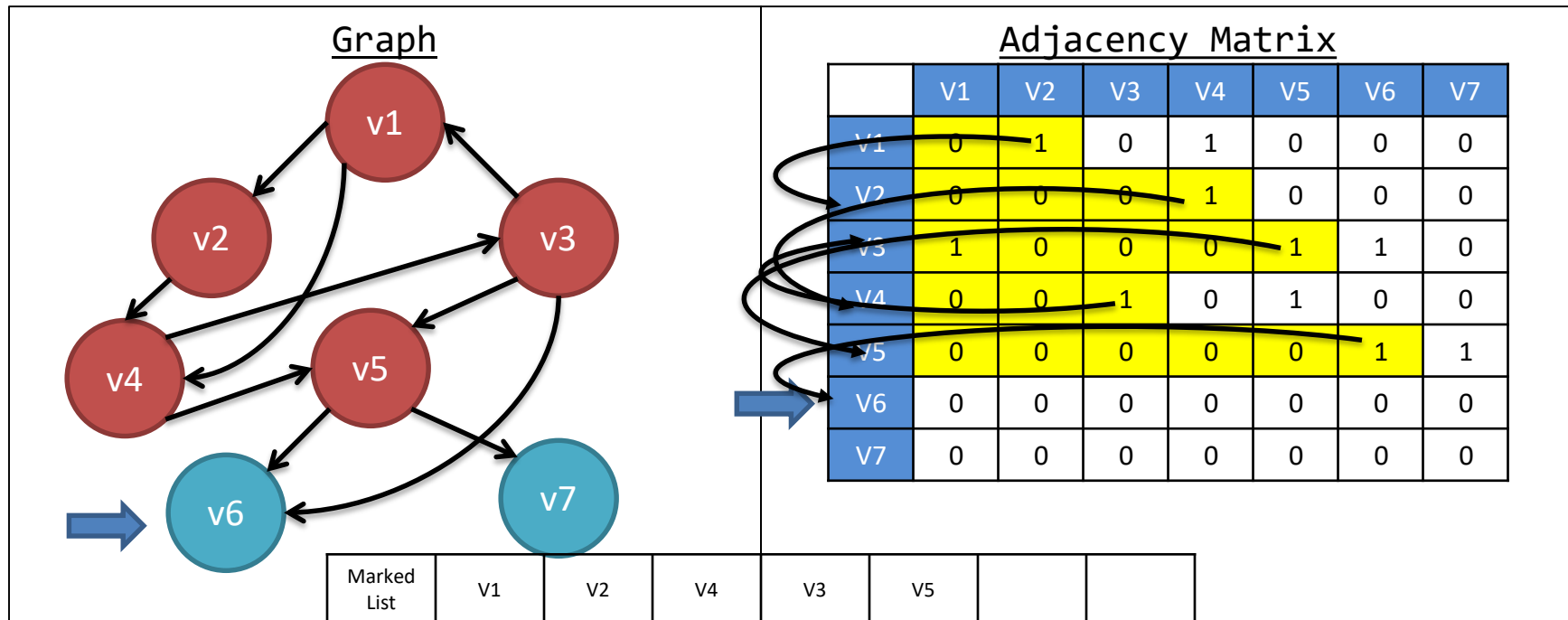
	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3			

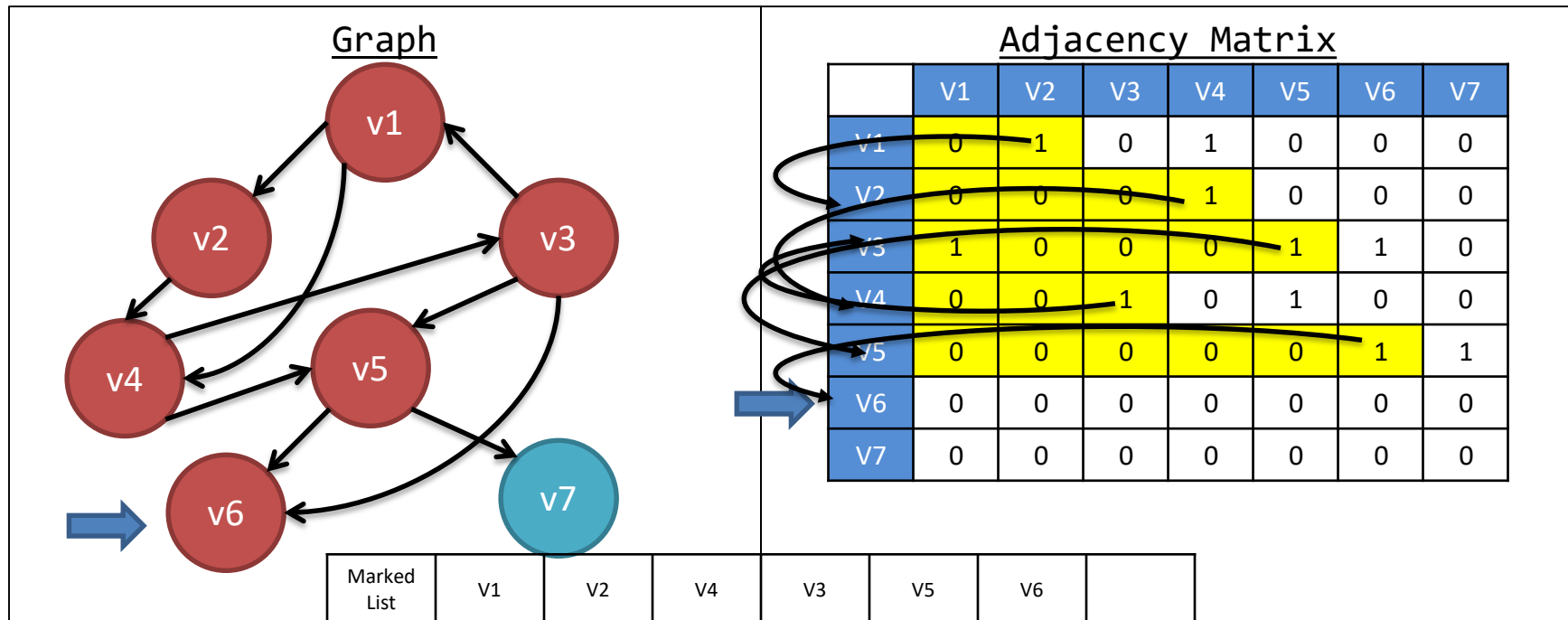
DFS Example



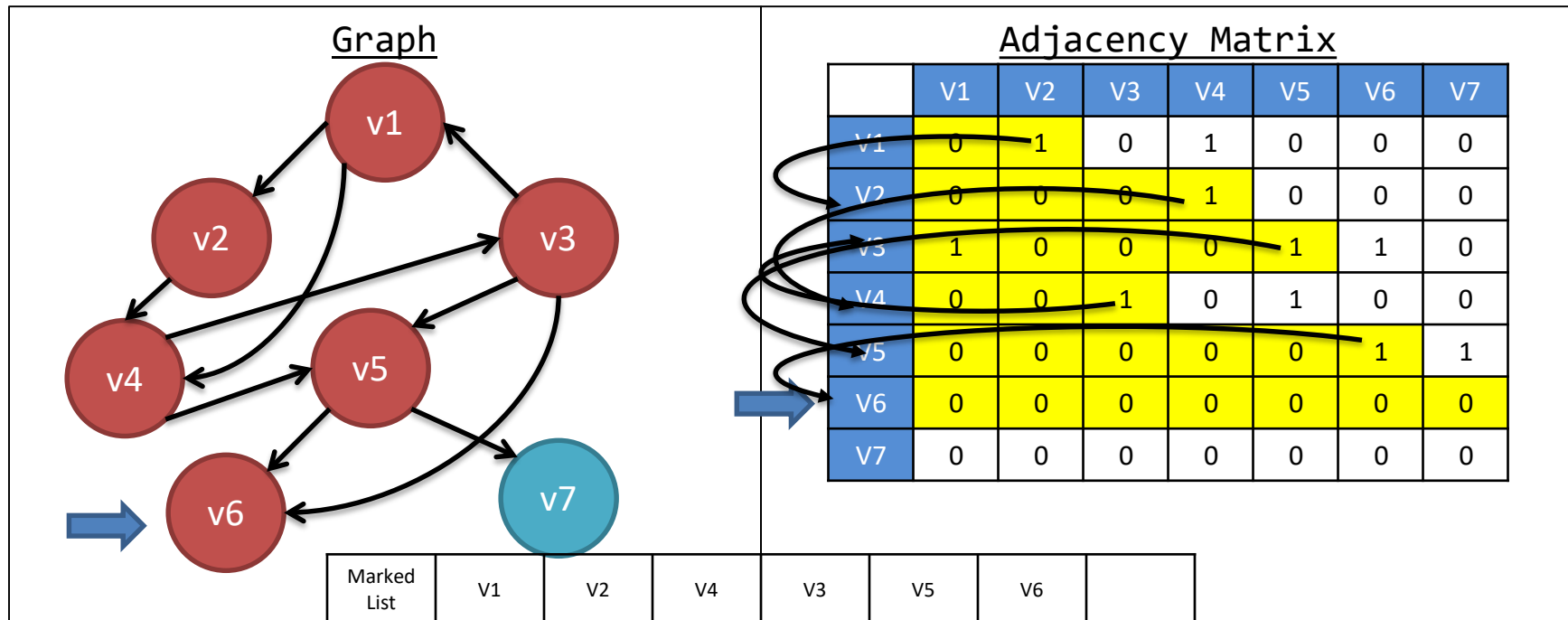
DFS Example



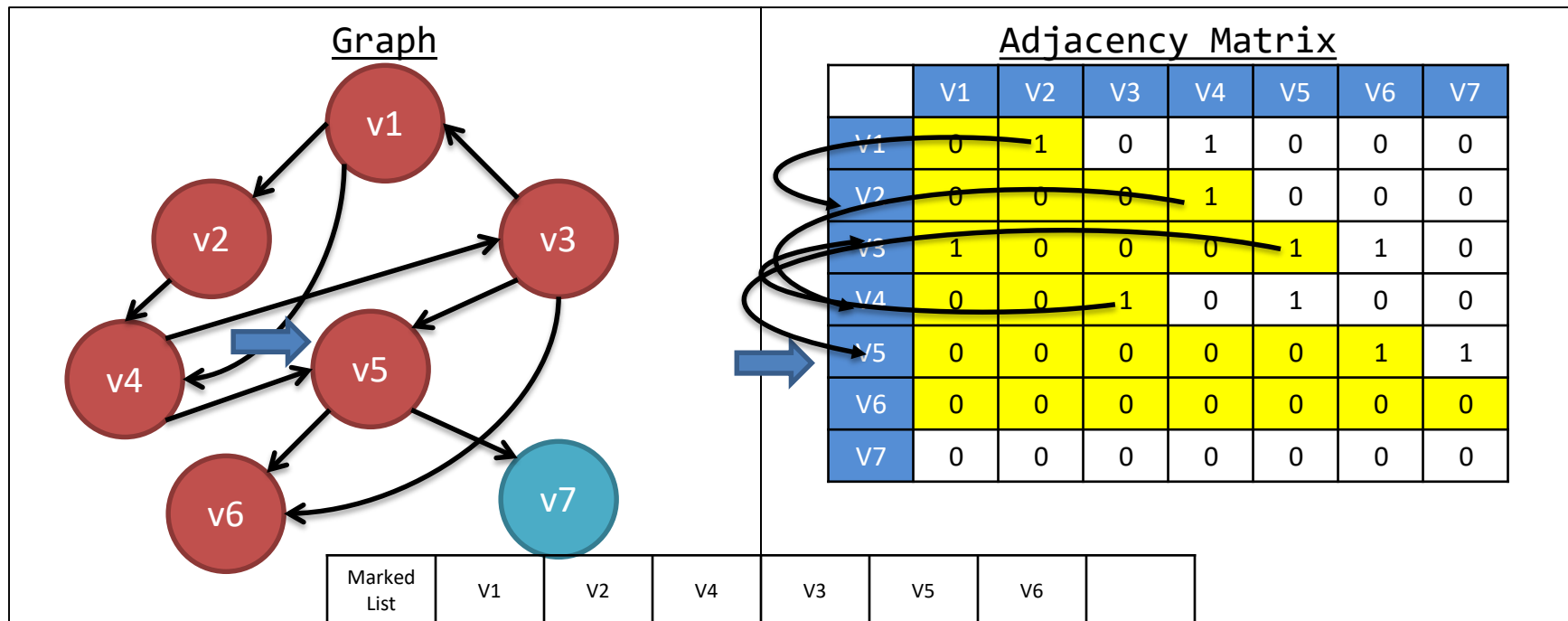
DFS Example



DFS Example

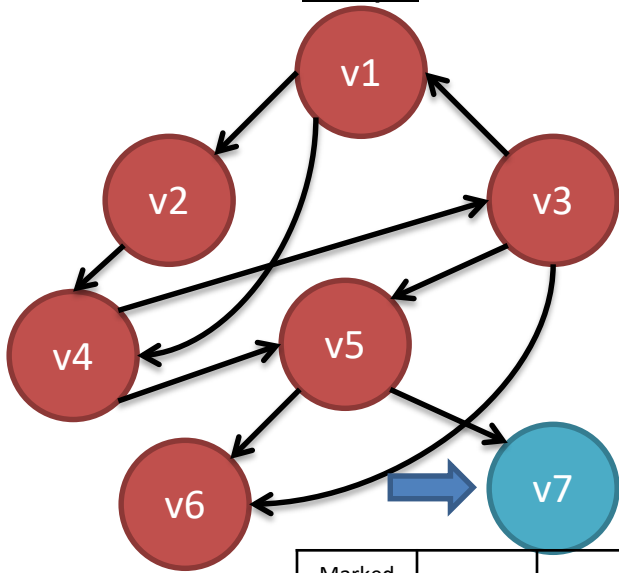


DFS Example



DFS Example

Graph



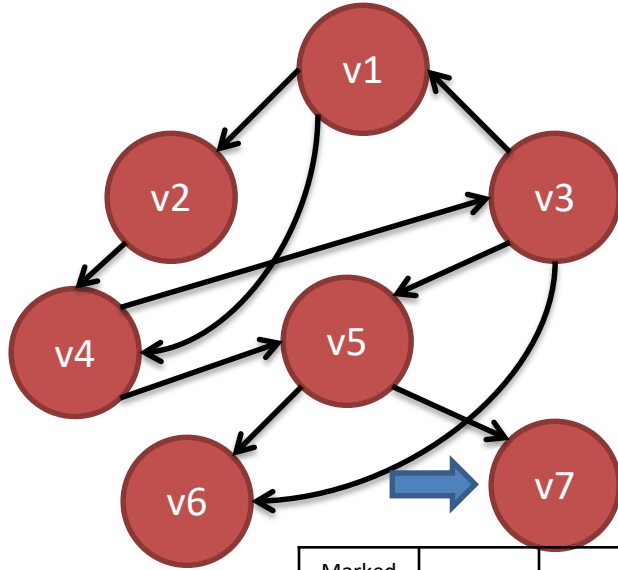
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3	v5	v6	

DFS Example

Graph

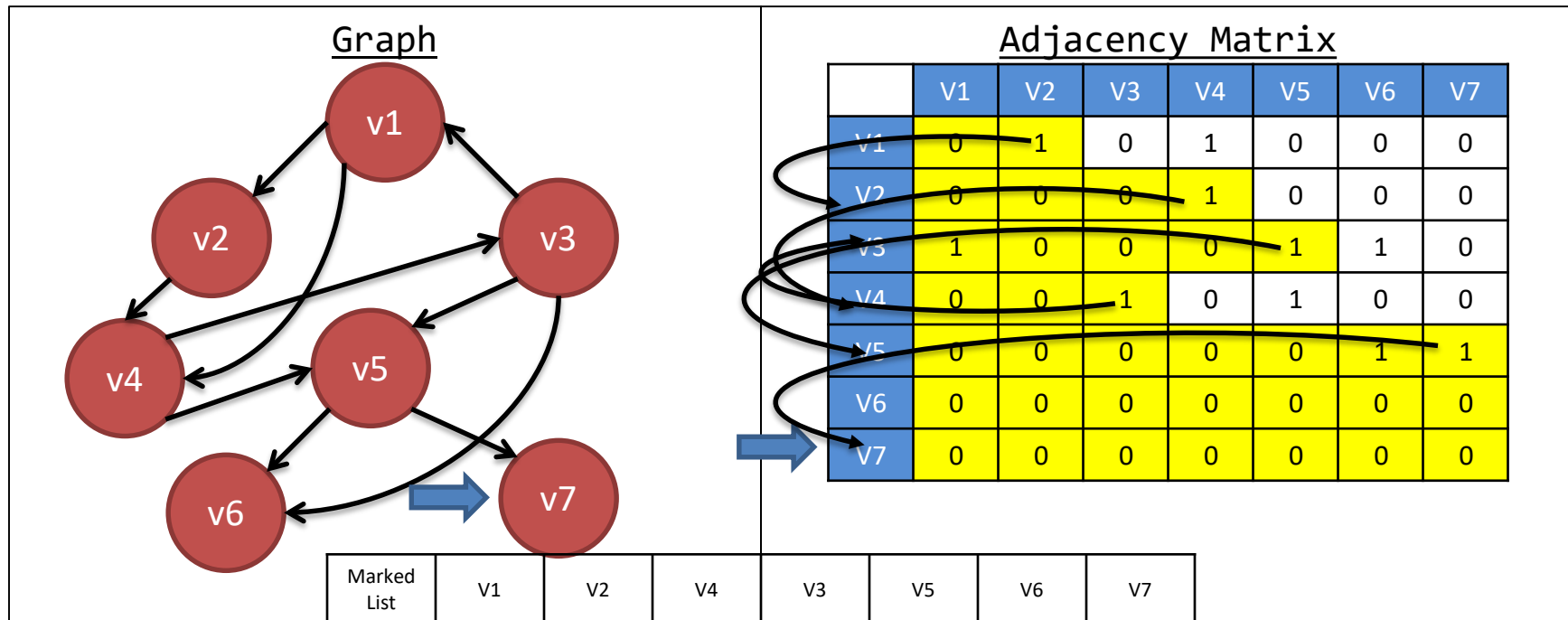


Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

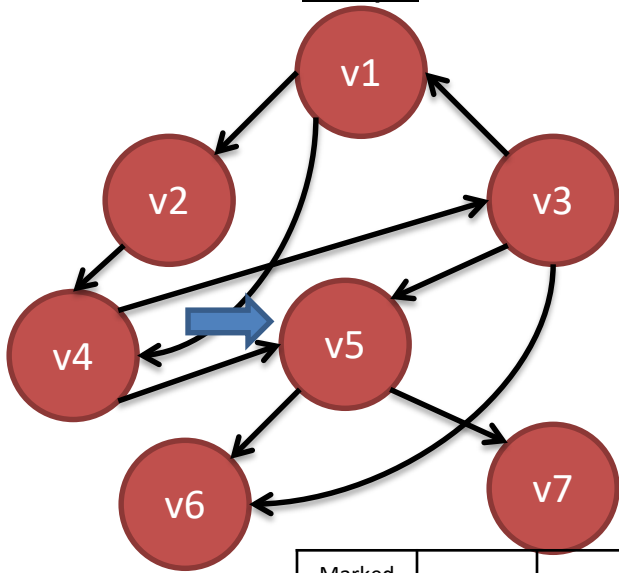
Marked List	v1	v2	v4	v3	v5	v6	v7

DFS Example



DFS Example

Graph



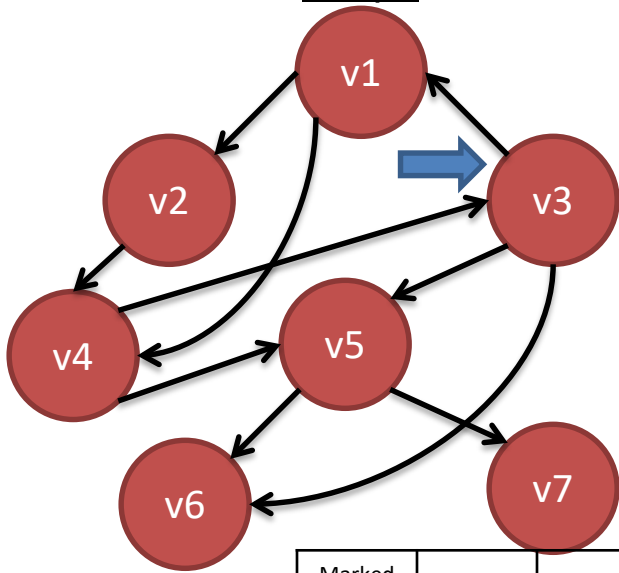
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3	v5	v6	v7

DFS Example

Graph



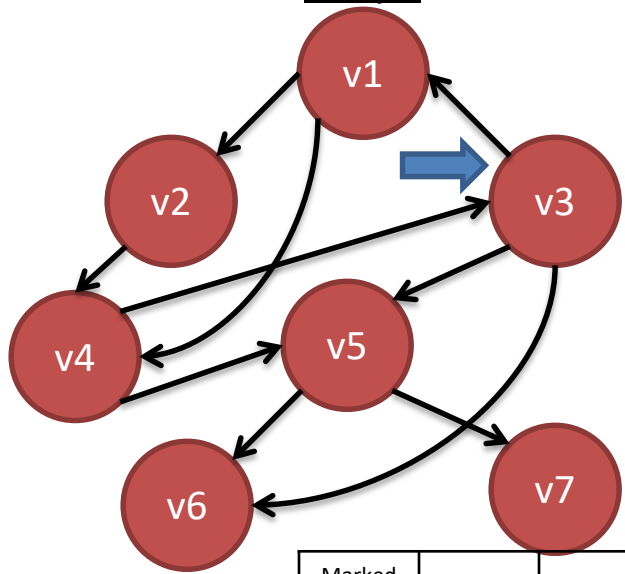
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3	v5	v6	v7

DFS Example

Graph



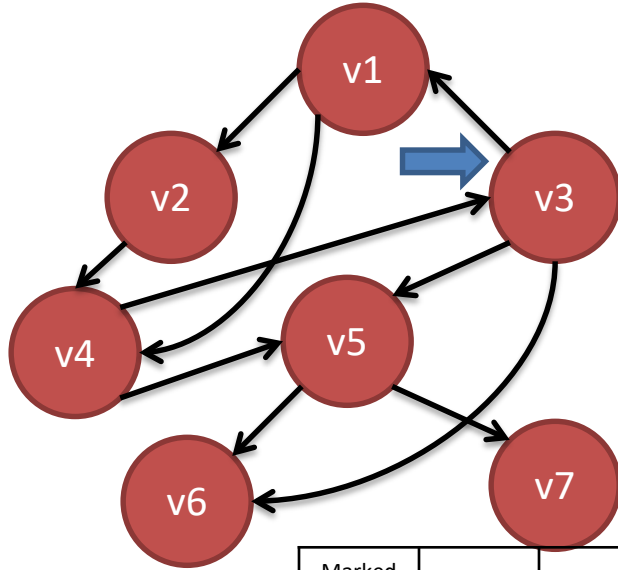
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
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V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3	v5	v6	v7

DFS Example

Graph



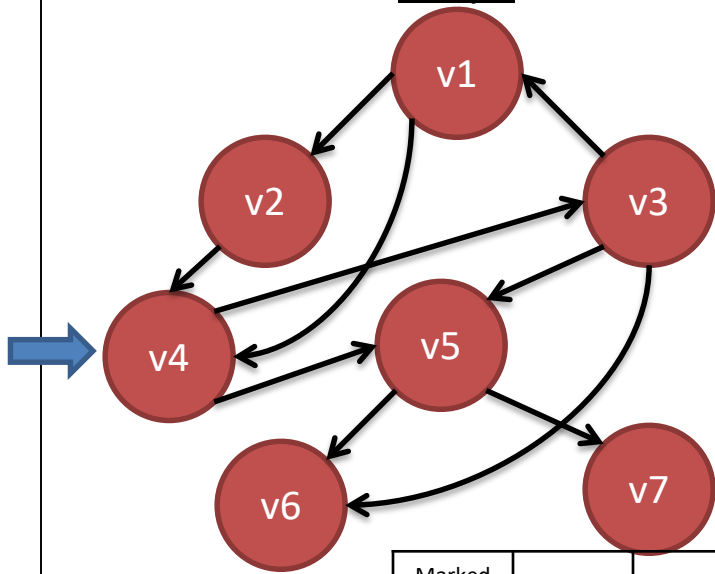
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3	v5	v6	v7

DFS Example

Graph



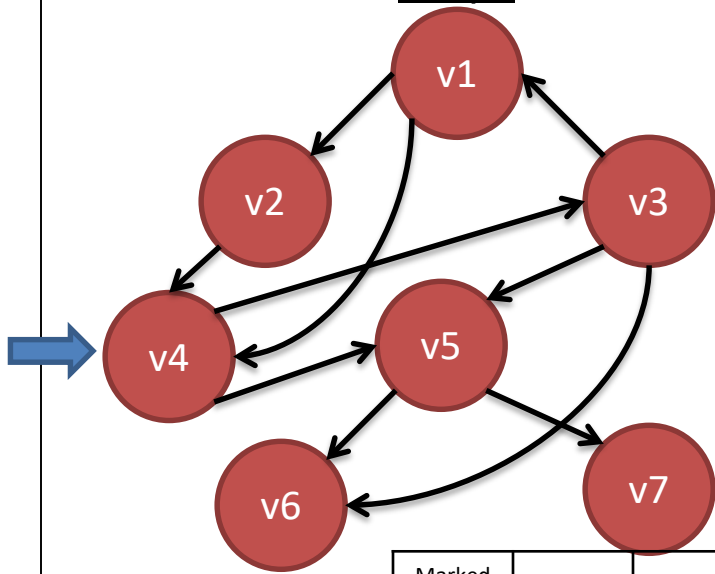
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3	v5	v6	v7

DFS Example

Graph

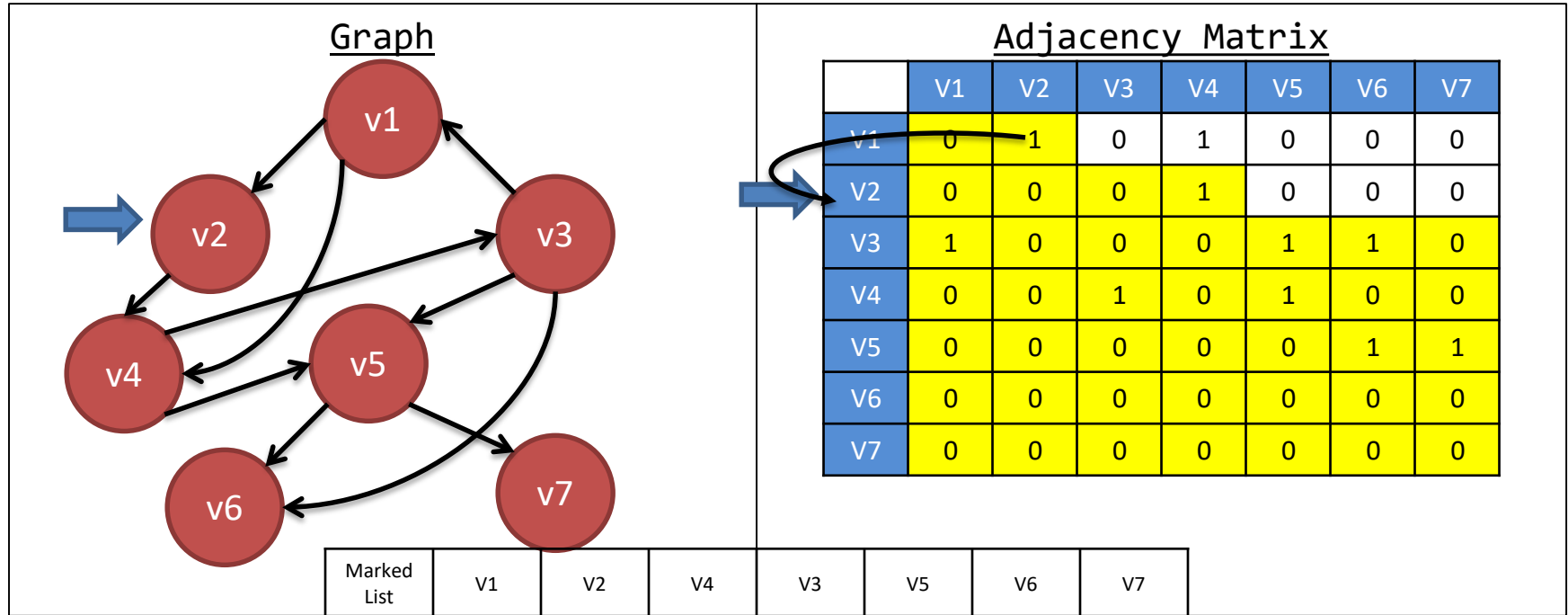


Adjacency Matrix

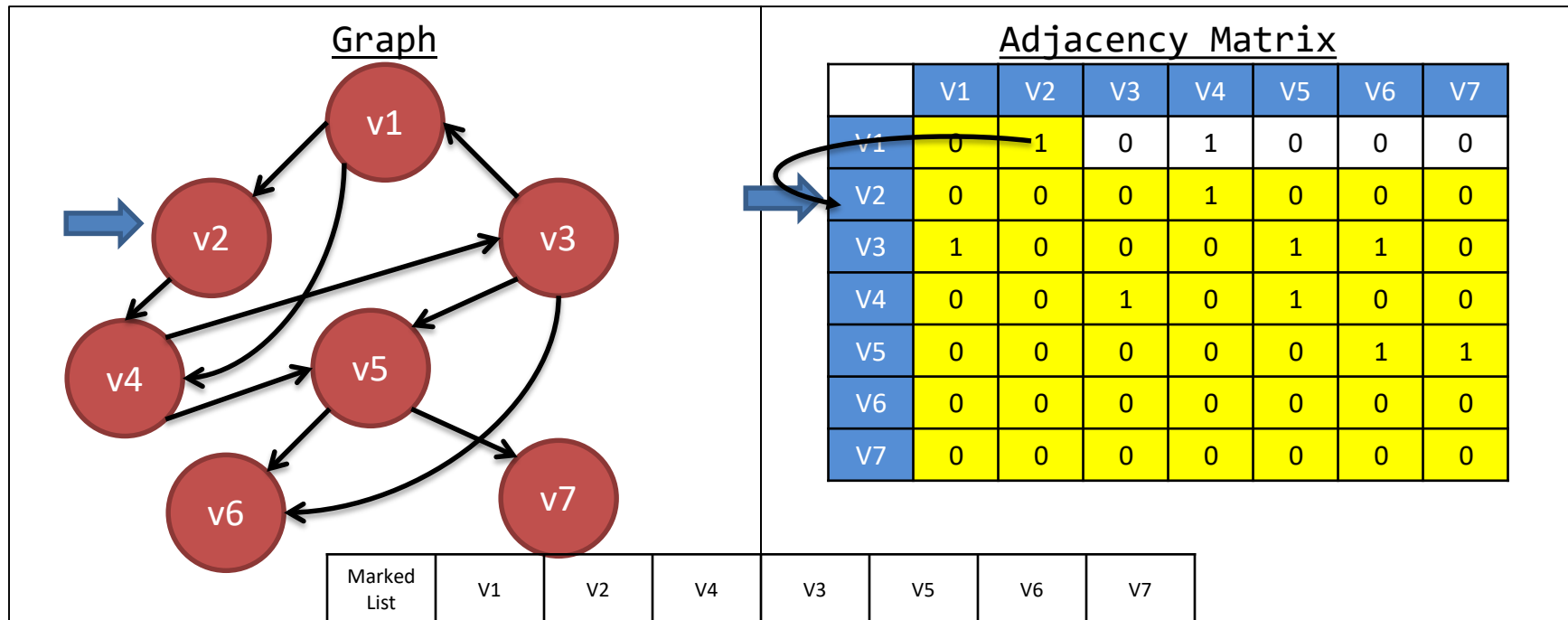
	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3	v5	v6	v7

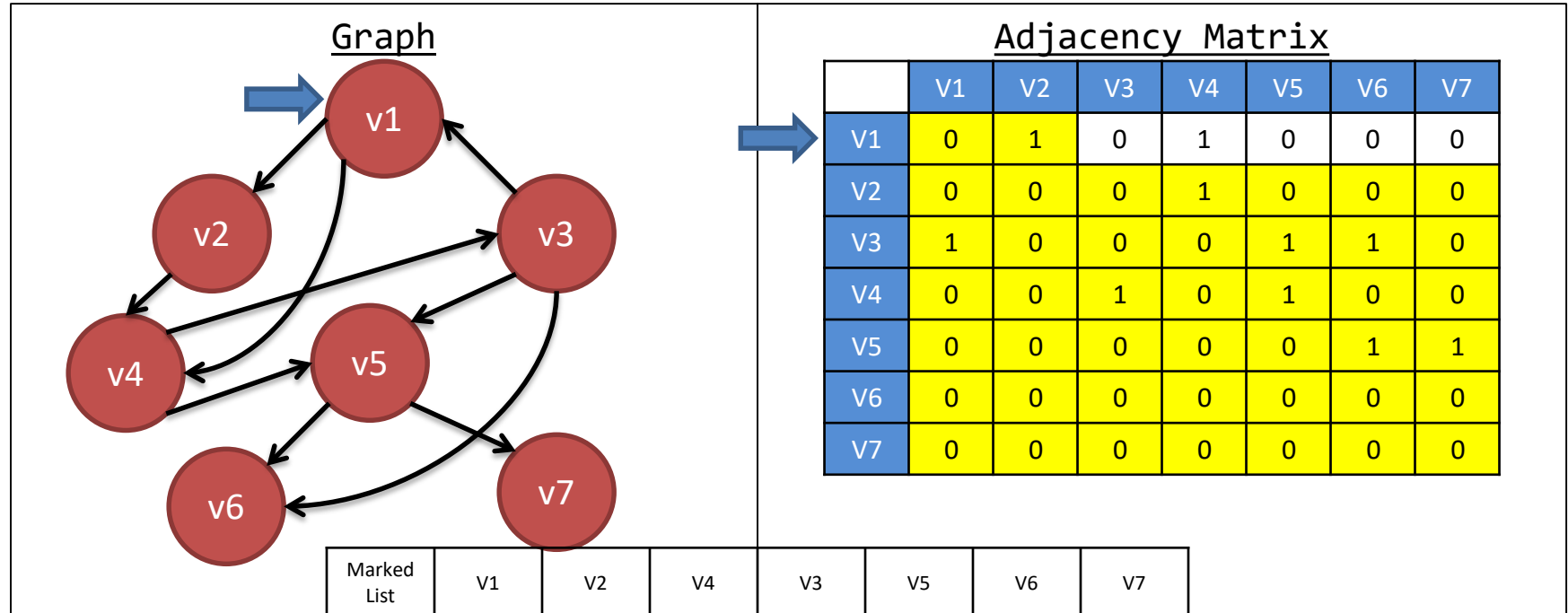
DFS Example



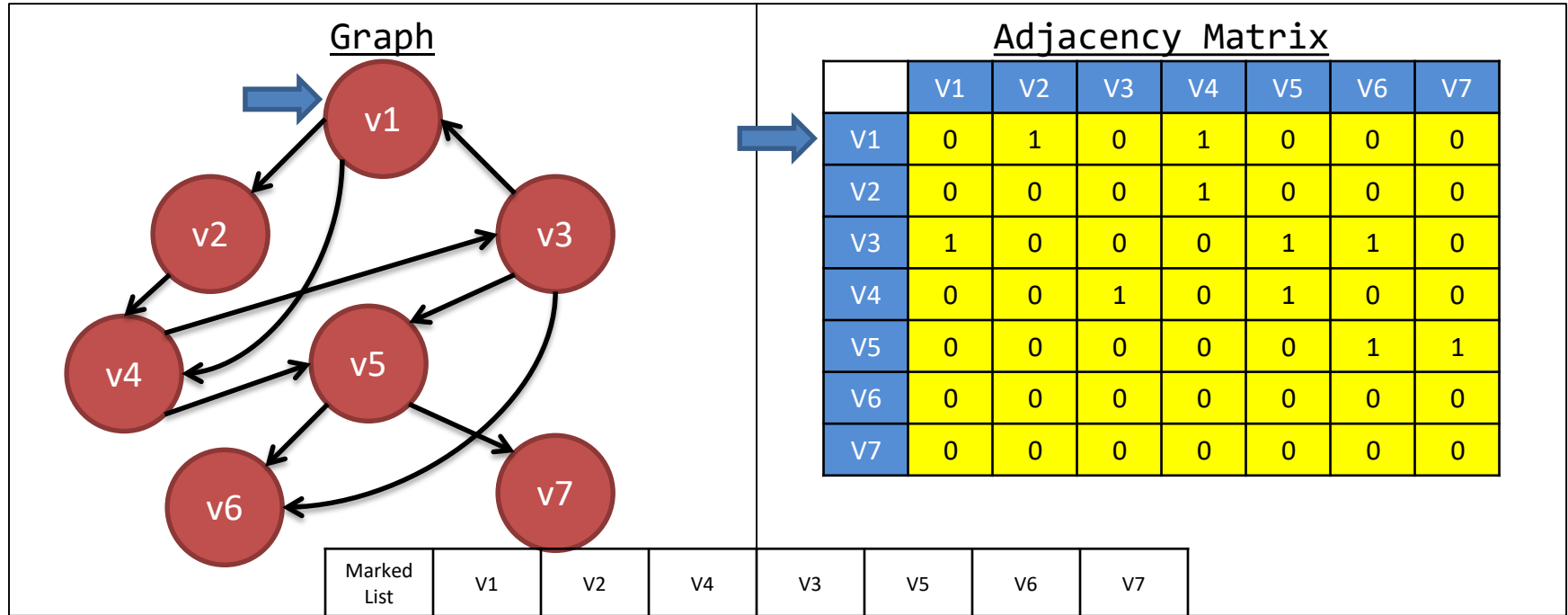
DFS Example



DFS Example

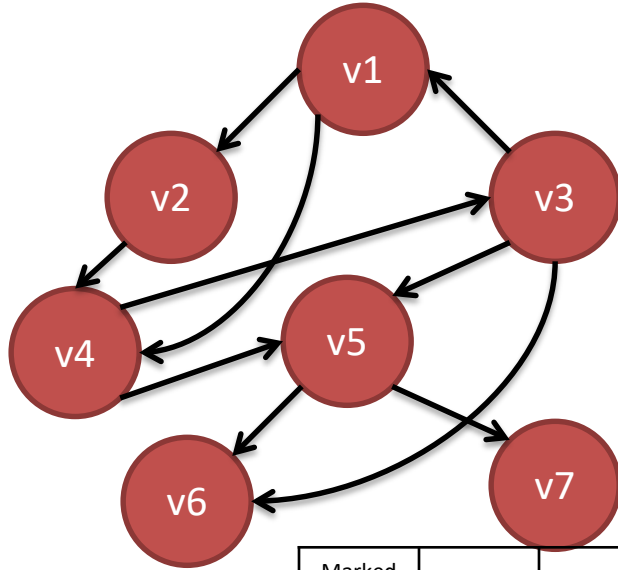


DFS Example



DFS Example

Graph



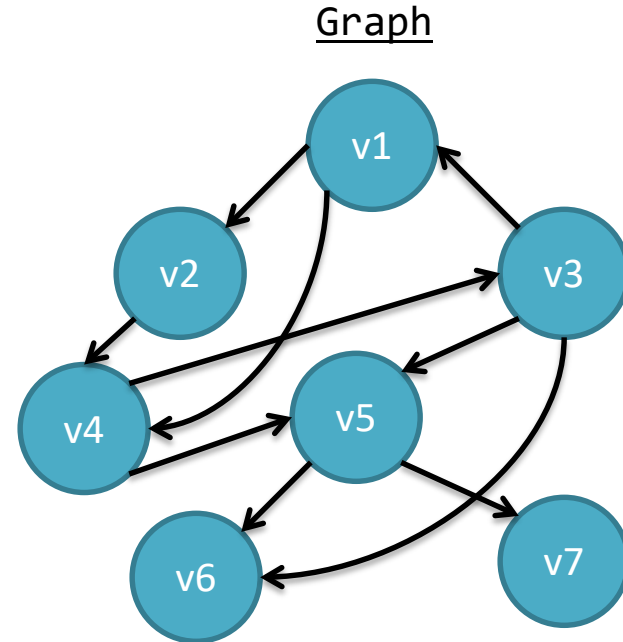
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3	v5	v6	v7

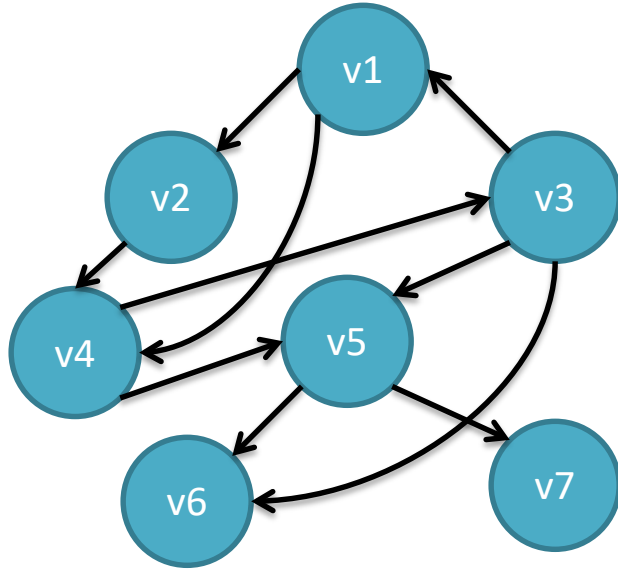
BFS Algorithm

- Assume that each vertex has a unique identifier and we have a List of “marked vertices” and a Queue of vertices
1. Start from an arbitrary vertex
 2. Add the vertex’s identifier to the marked vertices list and the vertex queue
 3. Dequeue a vertex
 4. Enqueue all outgoing edges from that vertex as long as the vertex is not already in the queue or marked list
 5. Follow an outgoing edge
 6. Repeat Step 3 until the Queue is empty



BFS Example

Graph

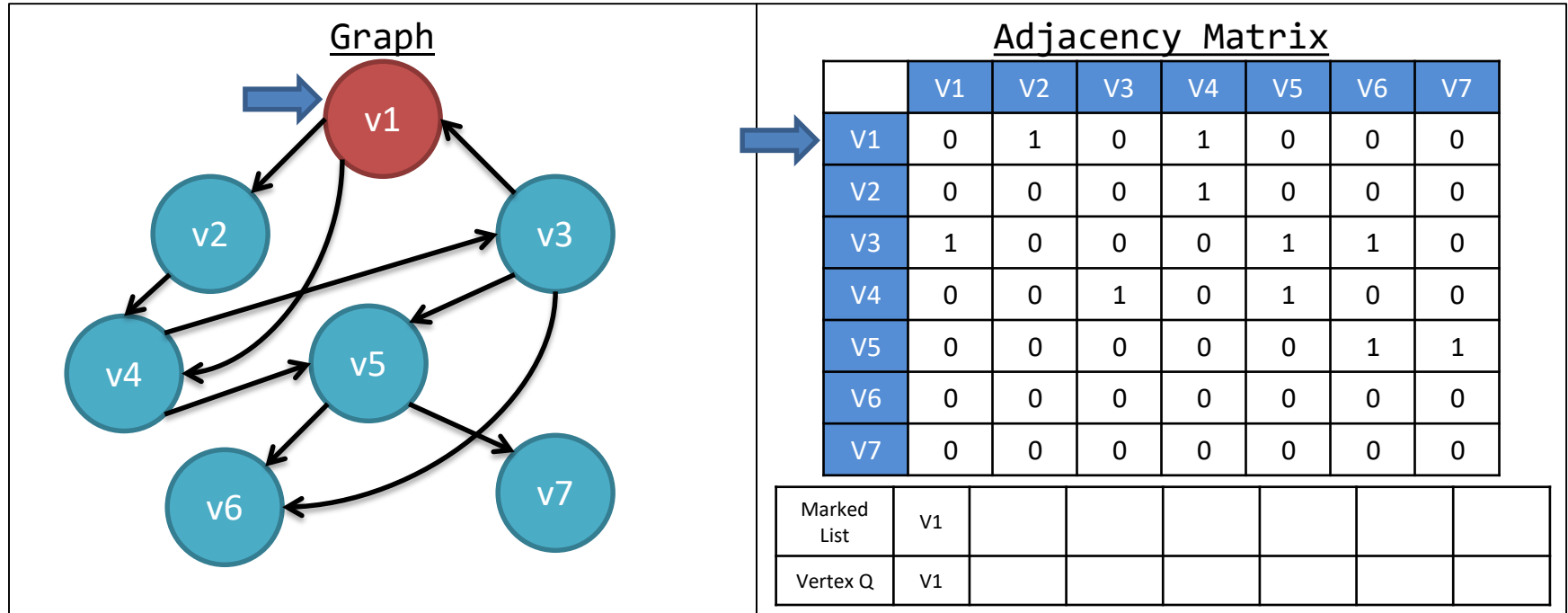


Adjacency Matrix

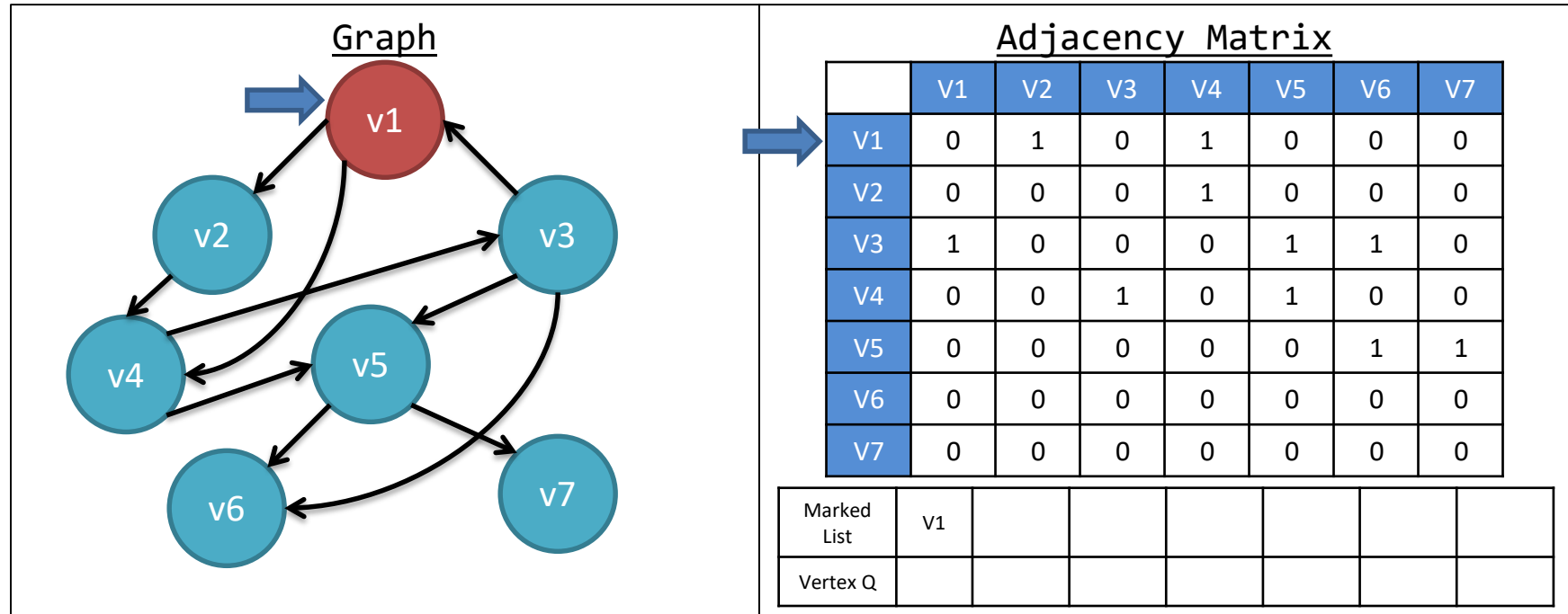
	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List							
Vertex Q							

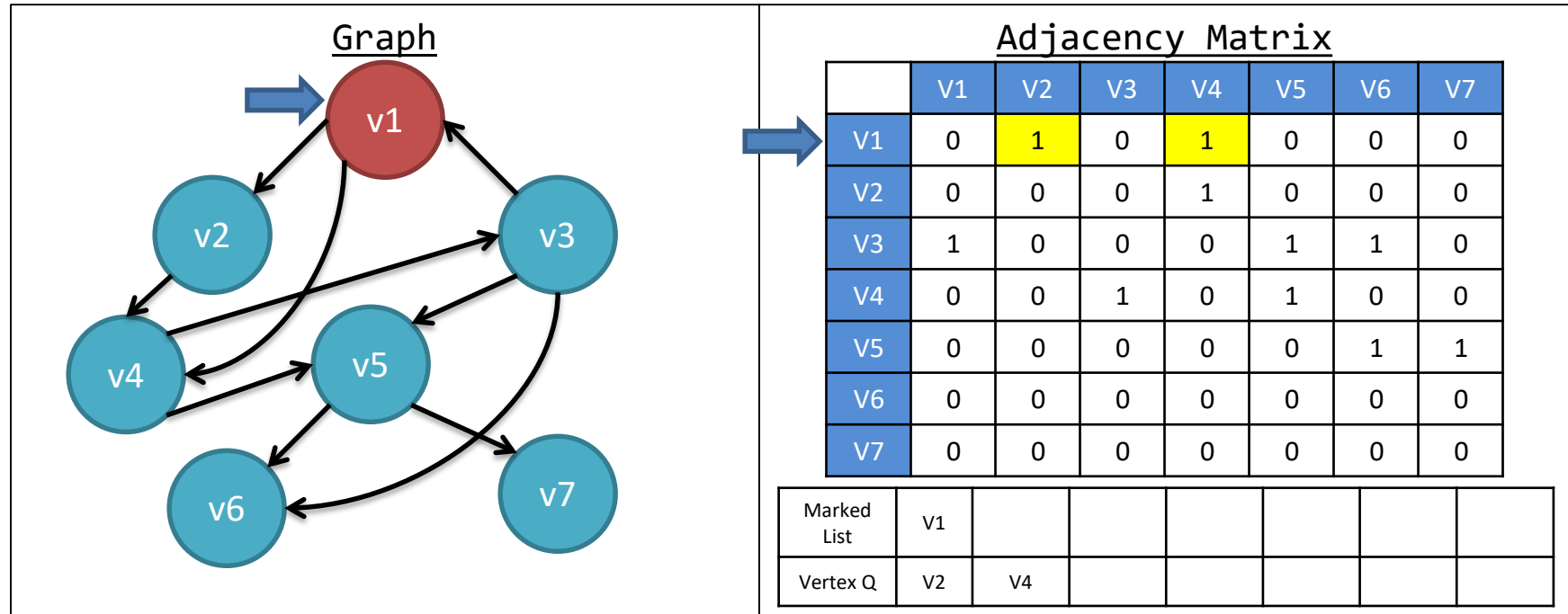
BFS Example



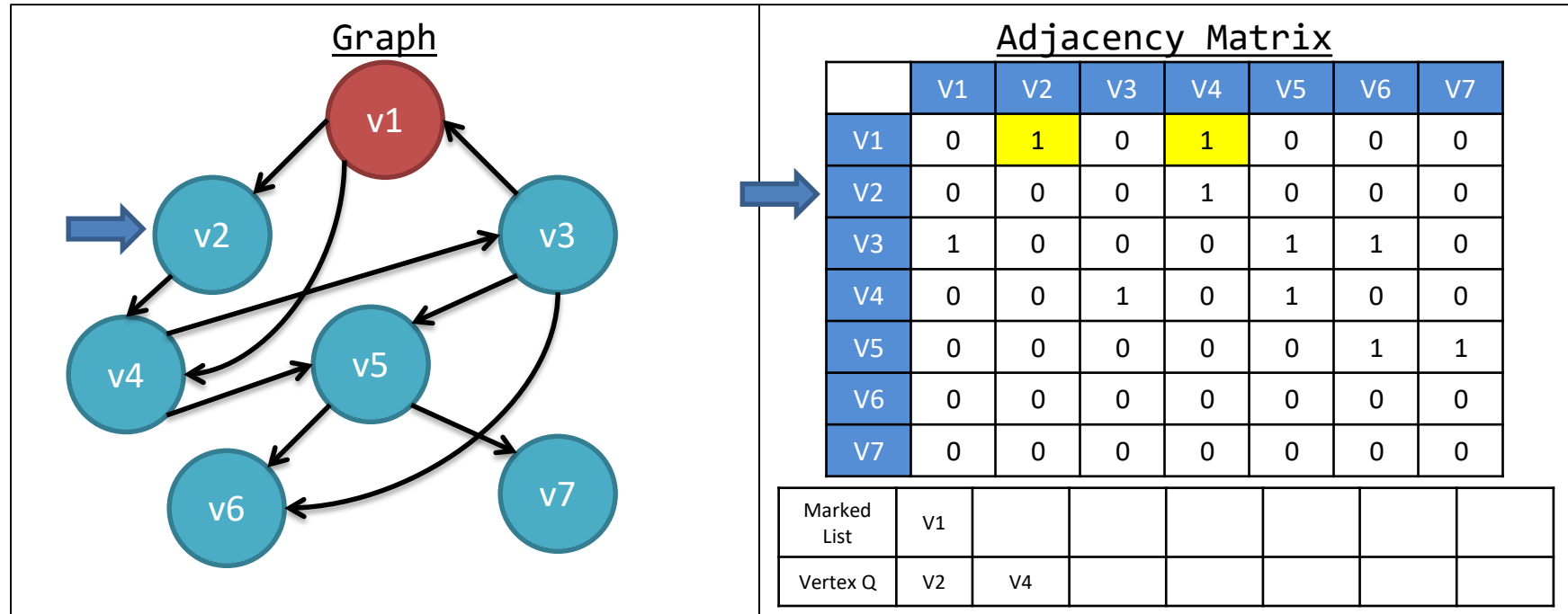
BFS Example



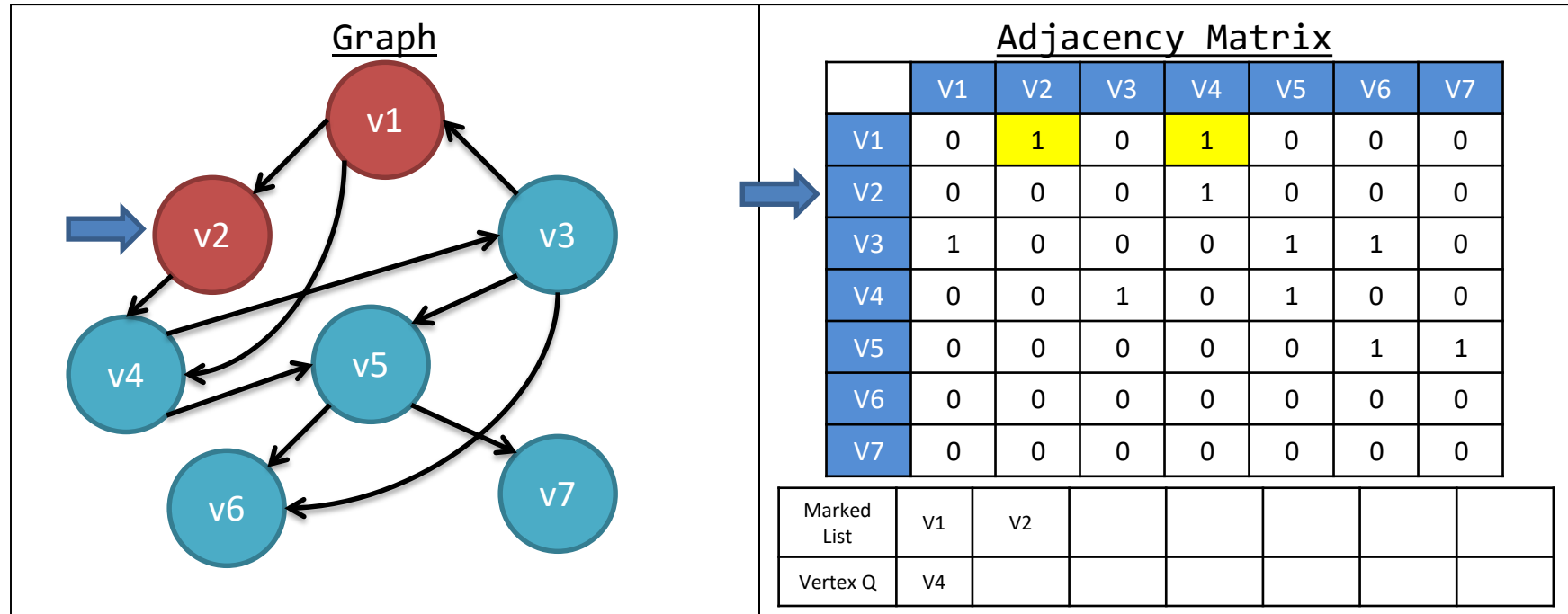
BFS Example



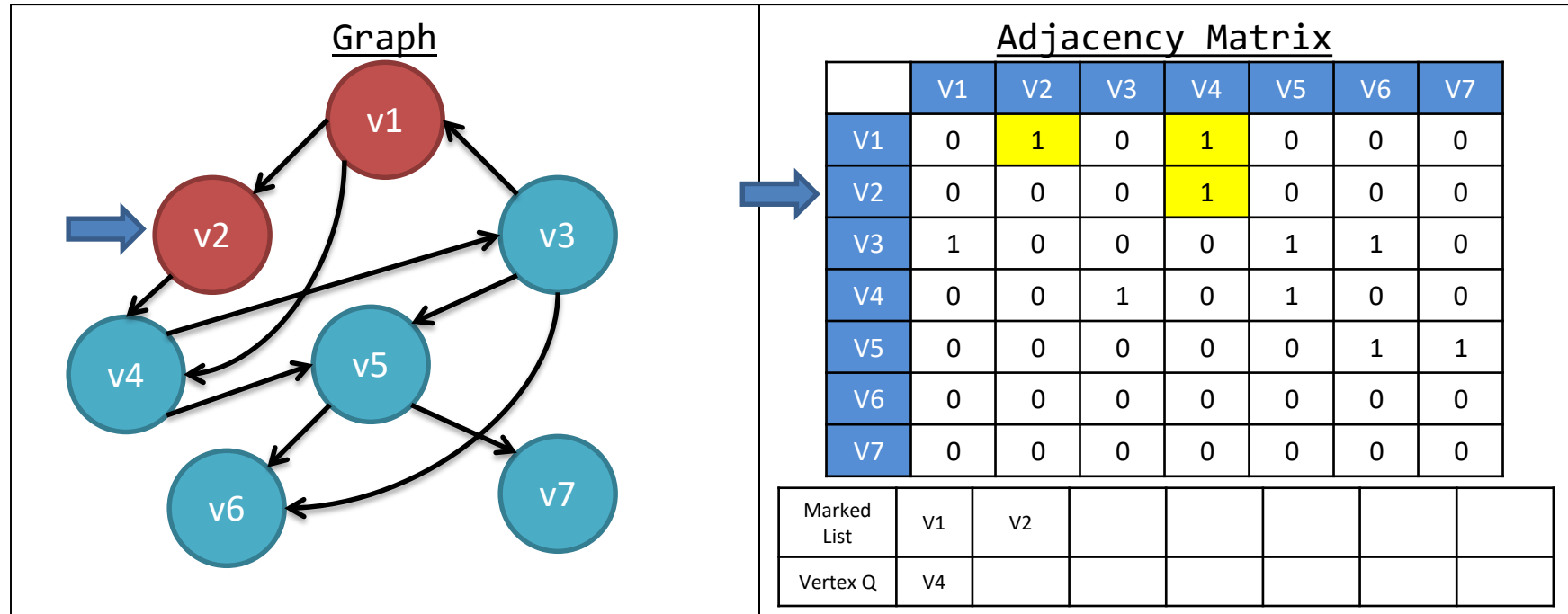
BFS Example



BFS Example

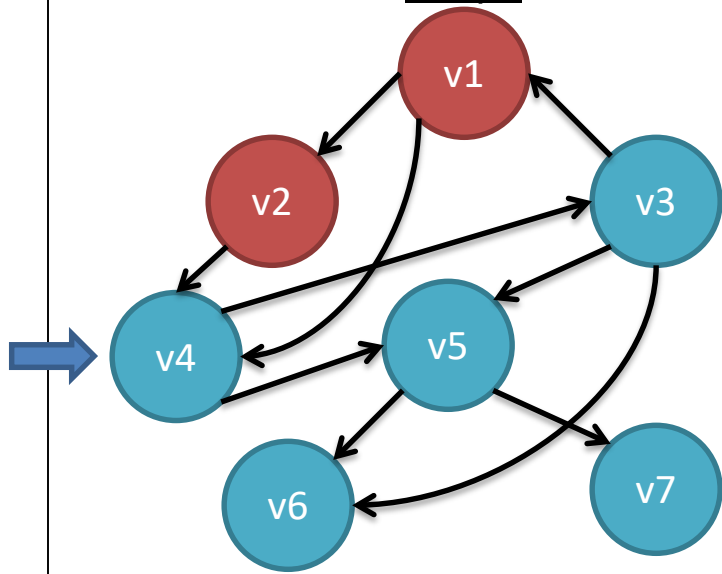


BFS Example



BFS Example

Graph

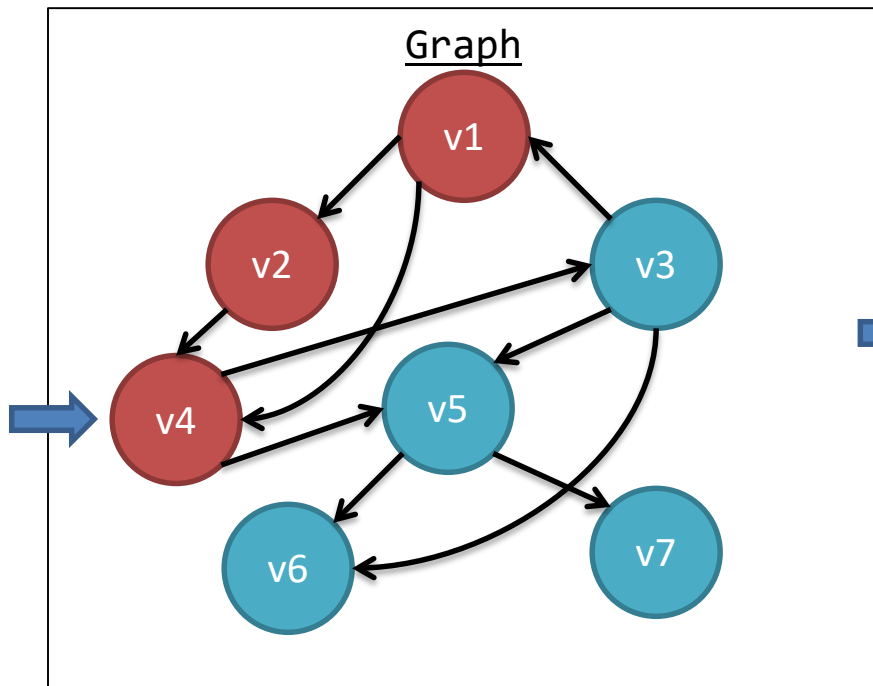


Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2					
Vertex Q	v4						

BFS Example



Adjacency Matrix

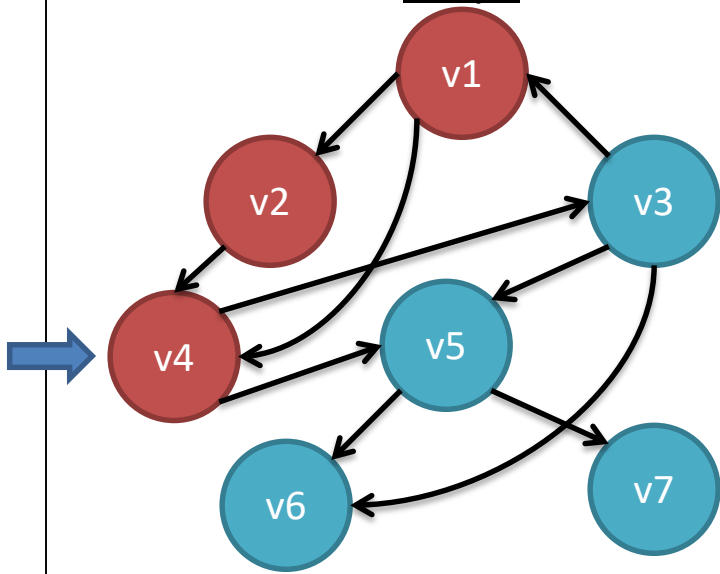
	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

A blue arrow points from the graph to the matrix.

Marked List	v1	v2	v4				
Vertex Q							

DFS Example

Graph

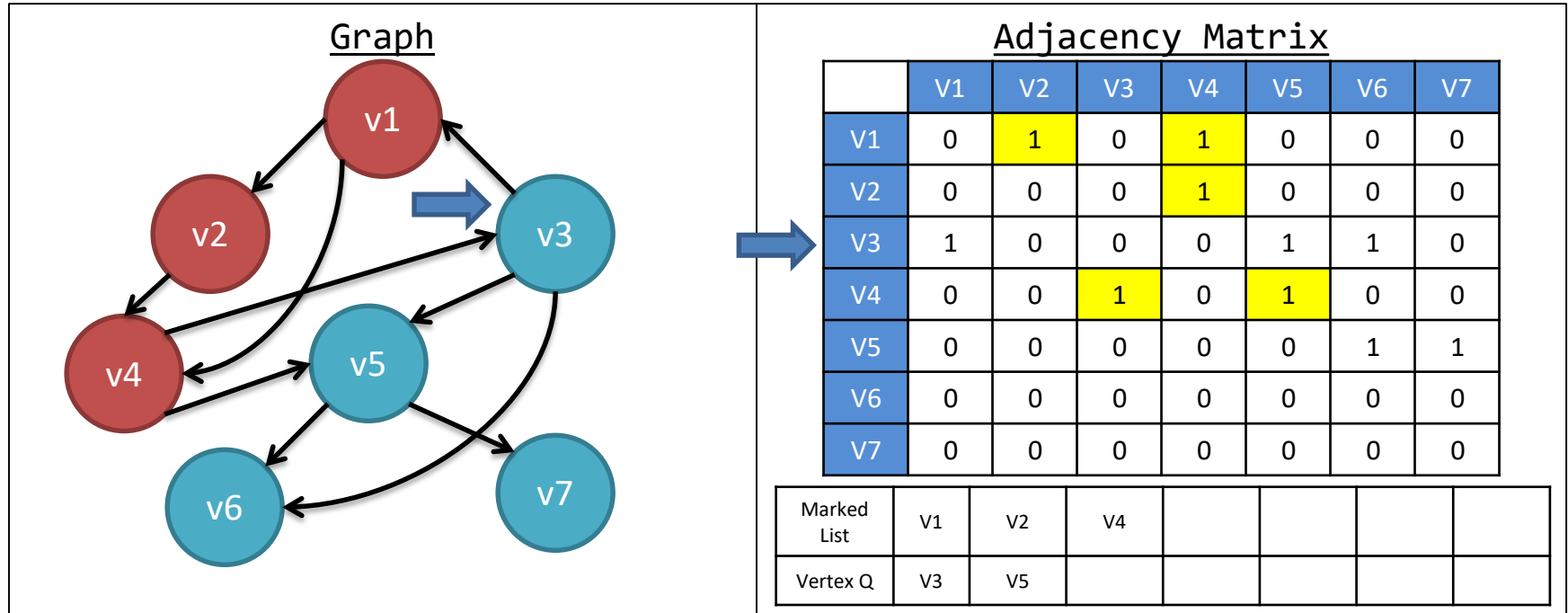


Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

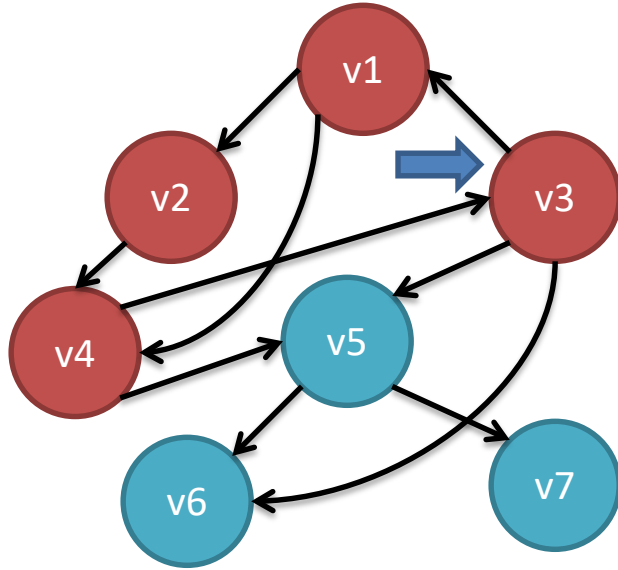
Marked List	v1	v2	v4				
Vertex Q	v3	v5					

BFS Example



BFS Example

Graph



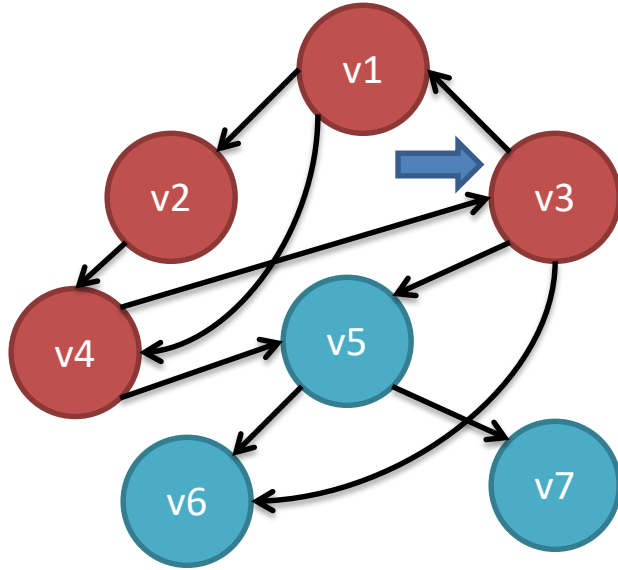
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3			
Vertex Q	v5						

BFS Example

Graph



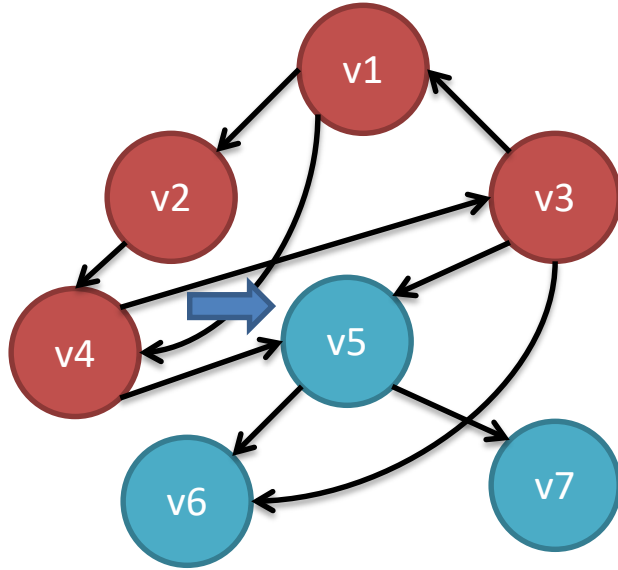
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3			
Vertex Q	v5	v6					

BFS Example

Graph



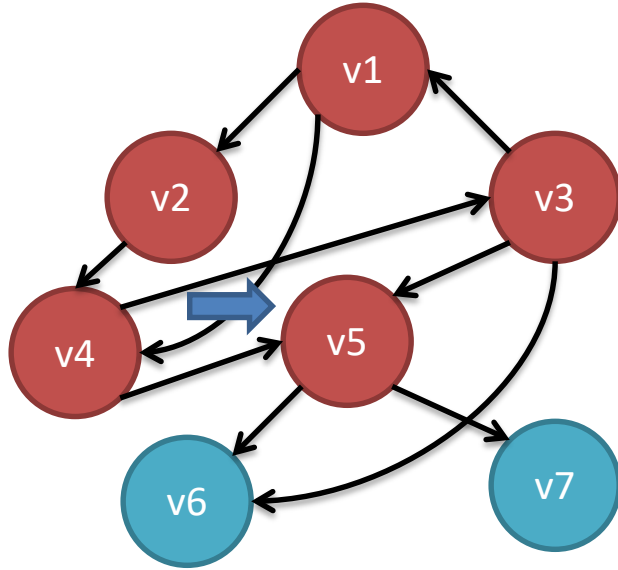
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3			
Vertex Q	v5	v6					

BFS Example

Graph



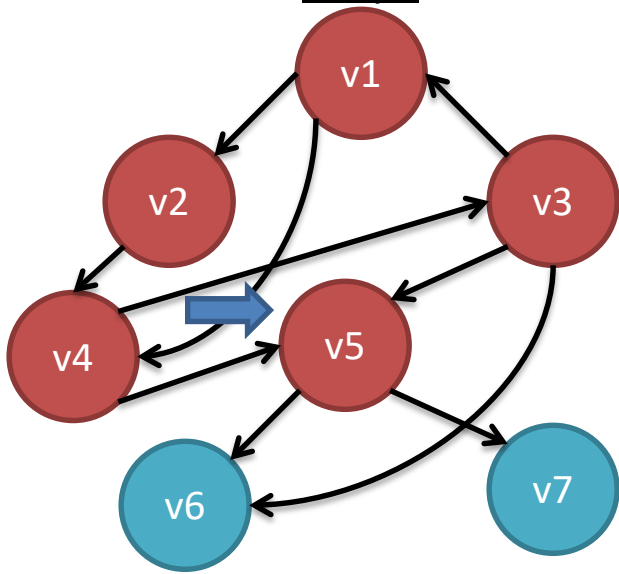
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3	v5		
Vertex Q	v6						

BFS Example

Graph



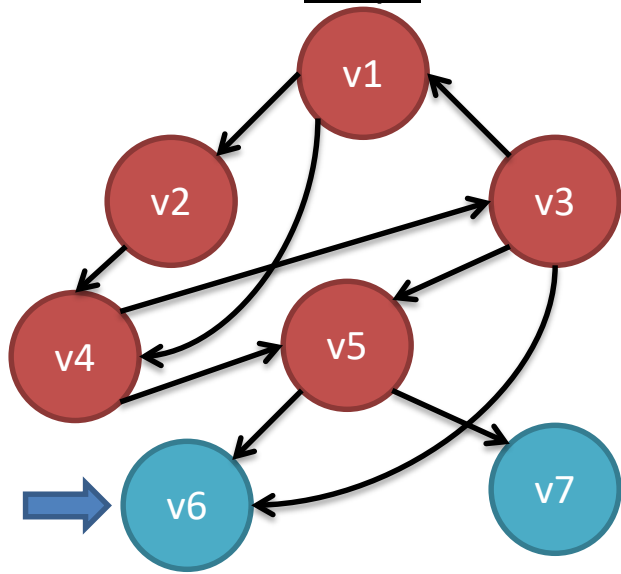
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3	v5		
Vertex Q	v6	v7					

BFS Example

Graph



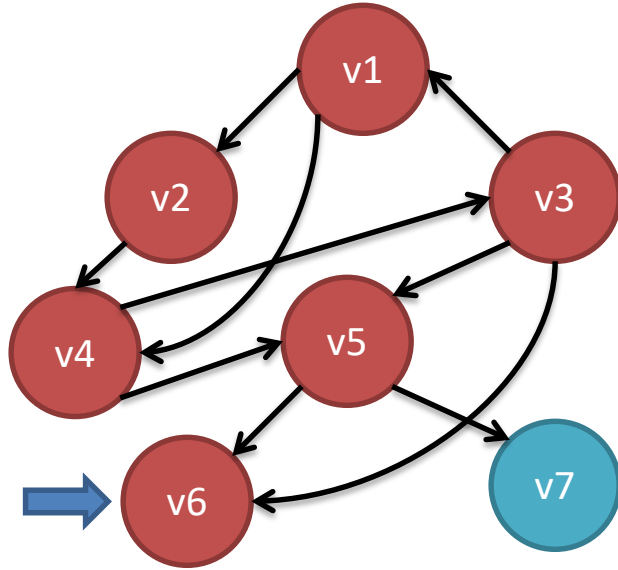
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3	v5		
Vertex Q	v6	v7					

BFS Example

Graph



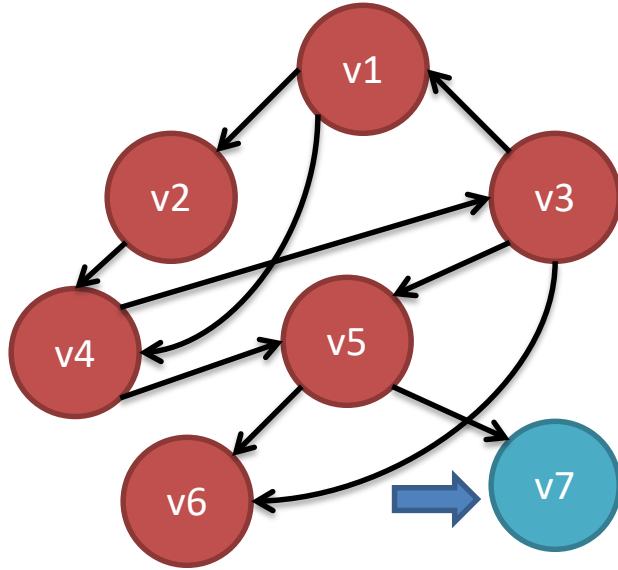
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3	v5	v6	
Vertex Q	v7						

BFS Example

Graph



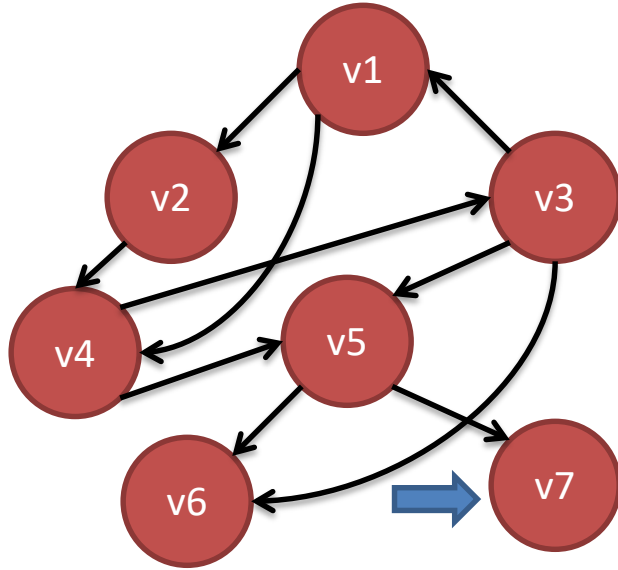
Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3	v5	v6	
Vertex Q	v7						

BFS Example

Graph



Adjacency Matrix

	V1	V2	V3	V4	V5	V6	V7
V1	0	1	0	1	0	0	0
V2	0	0	0	1	0	0	0
V3	1	0	0	0	1	1	0
V4	0	0	1	0	1	0	0
V5	0	0	0	0	0	1	1
V6	0	0	0	0	0	0	0
V7	0	0	0	0	0	0	0

Marked List	v1	v2	v4	v3	v5	v6	v7
Vertex Q							