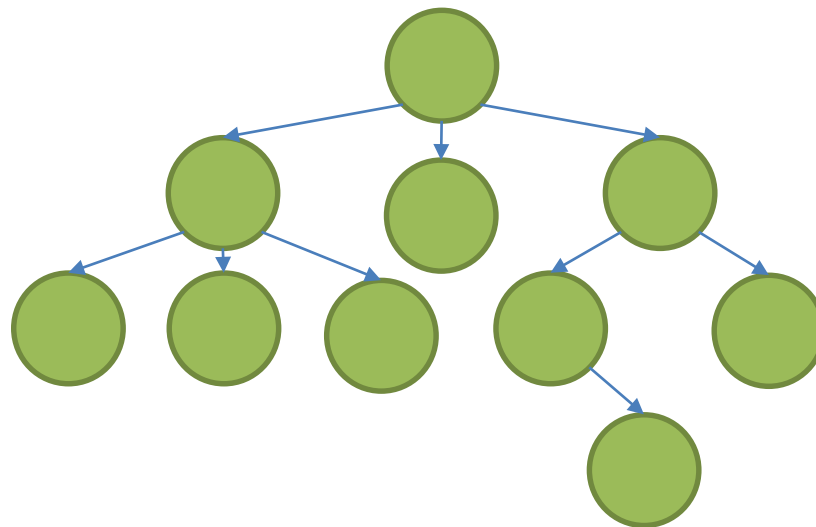


Heaps

Trees

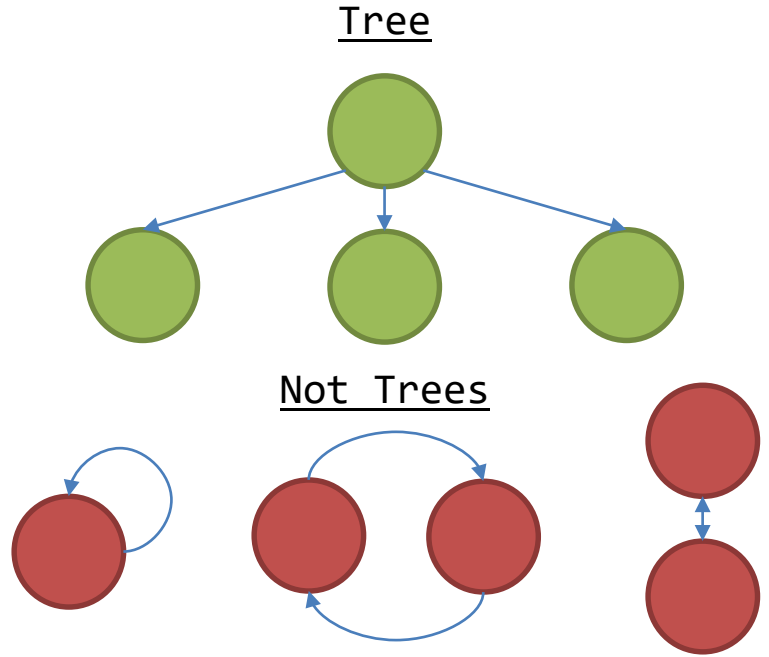
- Definition: A data structure that can be defined recursively as a collection of nodes, where each node is a data structure consisting of a value, together with a list of references (edges) to nodes, with the constraints that no reference is duplicated, and none points to the root.

Trees



Trees

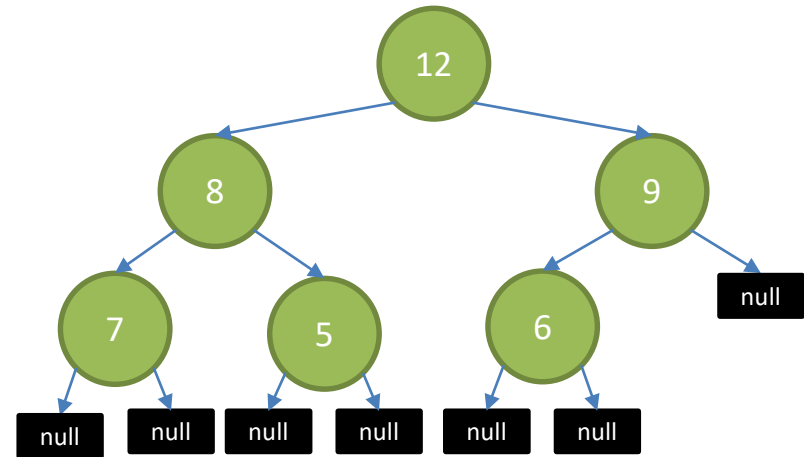
- Trees Have
 - Nodes
 - Edges
- Trees CANNOT
 - Contain Self-Referencing Edges
 - Have Cycles
 - Be Disjointed



Heaps

- Binary Tree Structure
- Node's data must be comparable
- Node's have at most two children
 - Left Child
 - Right Child
- Max Heap: Children must be less than or equal to the parent
- Min Heap: Children must be greater than or equal to the parent
- Assume Leaves are NULL references

Max Heap



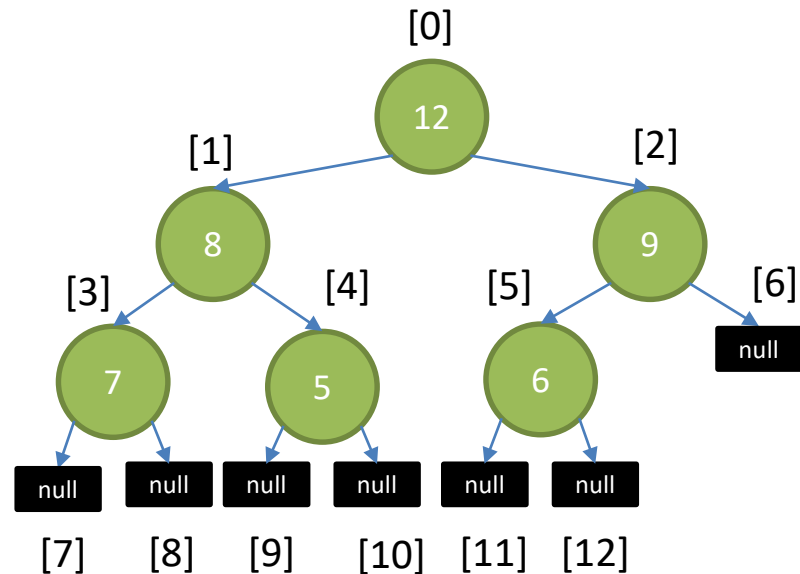
Heaps

- Array Heap
- Assume Root is at Index 0
- Left Child Index = Parent Index * 2 + 1
- Right Child Index = Parent Index * 2 + 2
- Parent Index = (Child Index-1)/2

Array Max Heap

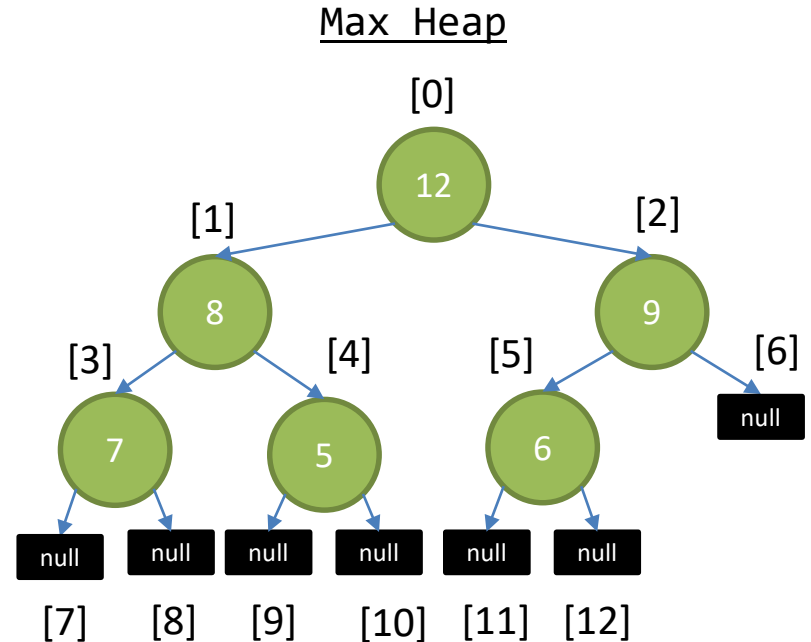
0	1	2	3	4	5	6	7	8	9	10	11	12
12	8	9	7	5	6	-	-	-	-	-	-	-

Max Heap



Heaps

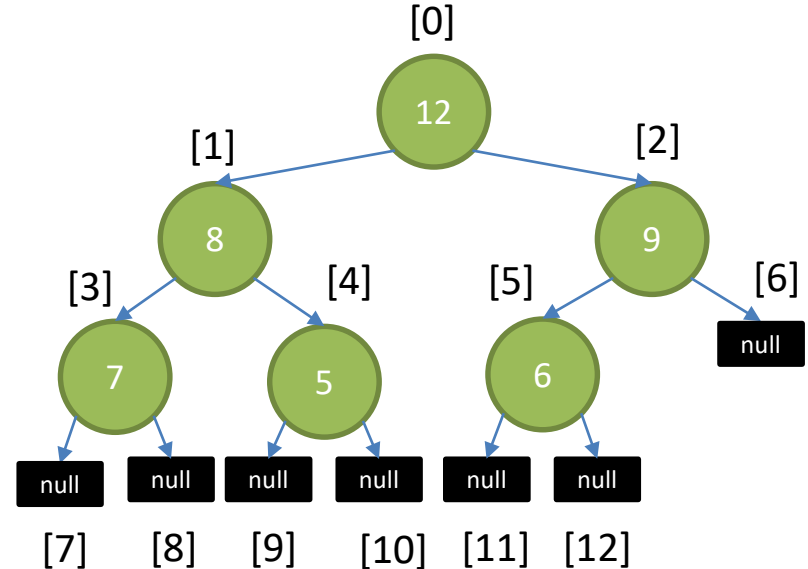
- Add
 - Replace the first leaf in breadth order with the new data
 - From that node “bubble up” the data if necessary
- Bubble Up
 - If the child’s data is larger than the parent then swap that information
 - Continue swapping child data with parent data until the parent is larger than the child or we reach the root index



Heaps

- Add
 - Replace the first leaf in breadth order with the new data
 - From that node “bubble up” the data if necessary
- Bubble Up
 - If the child’s data is larger than the parent then swap that information
 - Continue swapping child data with parent data until the parent is larger than the child or we reach the root index

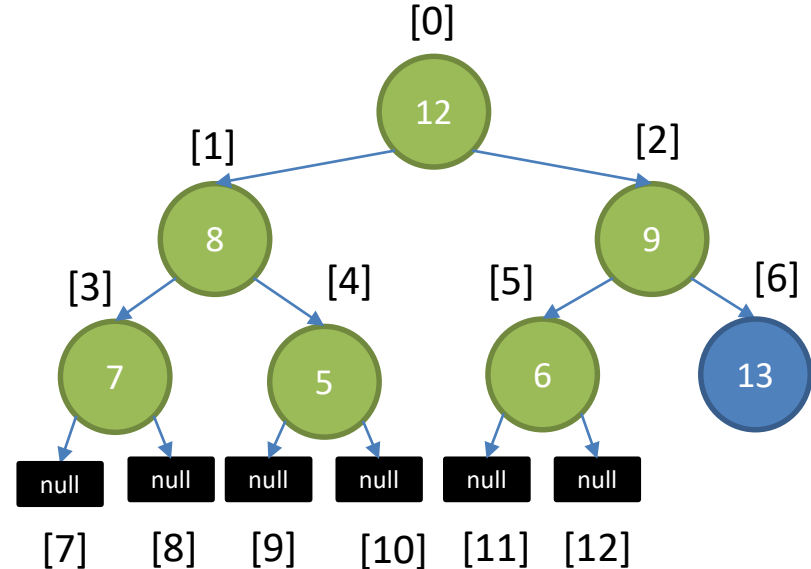
Example Adding “13”



Heaps

- Add
 - Replace the first leaf in breadth order with the new data
 - From that node “bubble up” the data if necessary
- Bubble Up
 - If the child’s data is larger than the parent then swap that information
 - Continue swapping child data with parent data until the parent is larger than the child or we reach the root index

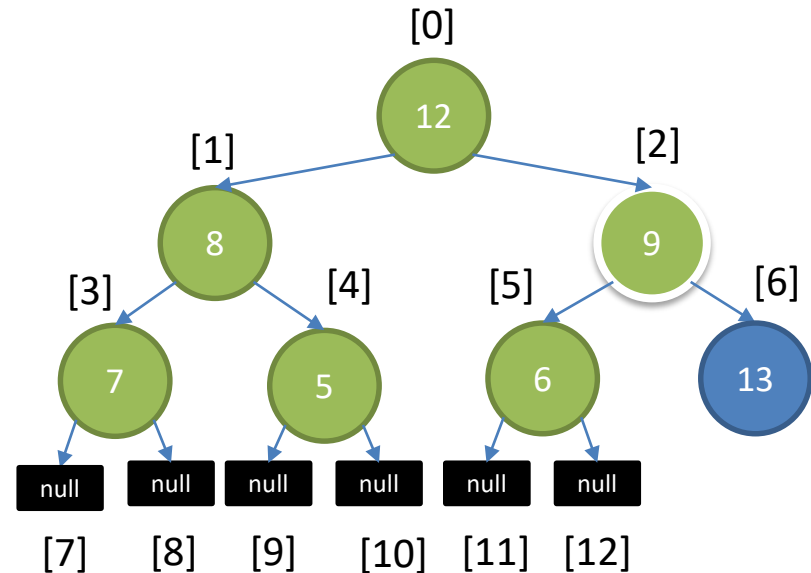
Example Adding “13”



Heaps

- Add
 - Replace the first leaf in breadth order with the new data
 - From that node “bubble up” the data if necessary
- Bubble Up
 - If the child’s data is larger than the parent then swap that information
 - Continue swapping child data with parent data until the parent is larger than the child or we reach the root index

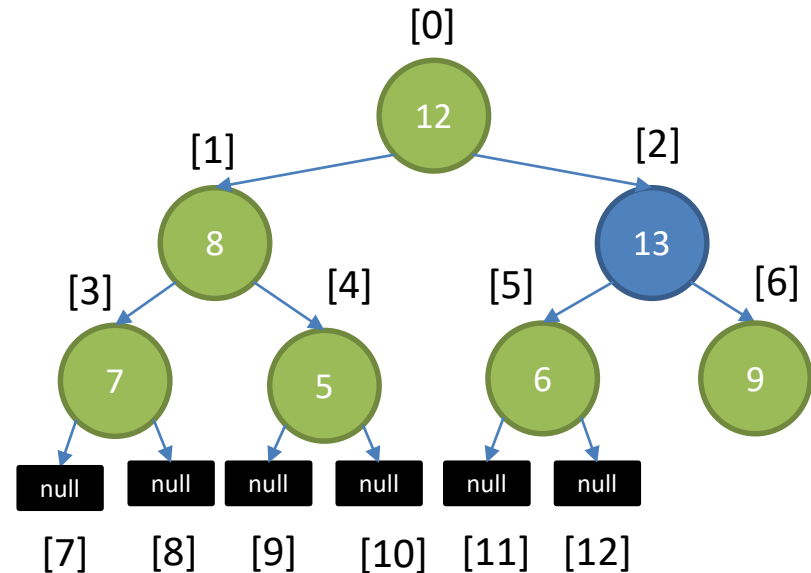
Example Adding “13”



Heaps

- Add
 - Replace the first leaf in breadth order with the new data
 - From that node “bubble up” the data if necessary
- Bubble Up
 - If the child’s data is larger than the parent then swap that information
 - Continue swapping child data with parent data until the parent is larger than the child or we reach the root index

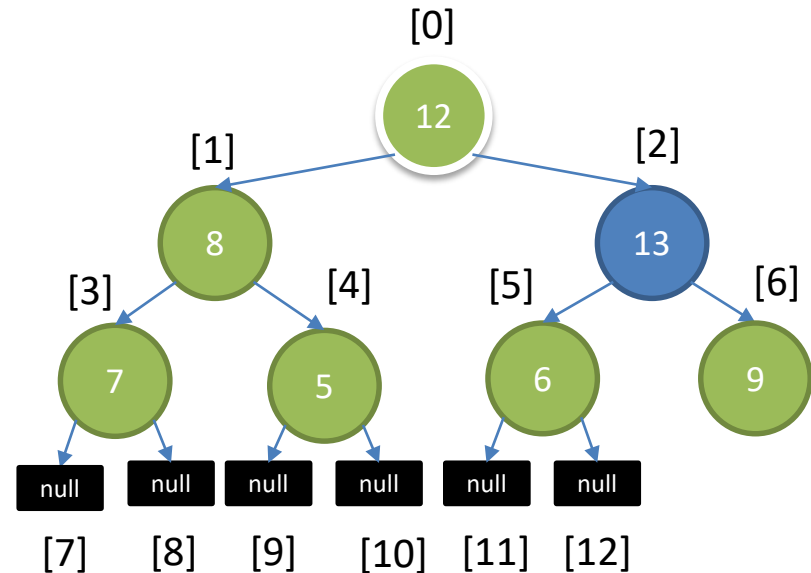
Example Adding “13”



Heaps

- Add
 - Replace the first leaf in breadth order with the new data
 - From that node “bubble up” the data if necessary
- Bubble Up
 - If the child’s data is larger than the parent then swap that information
 - Continue swapping child data with parent data until the parent is larger than the child or we reach the root index

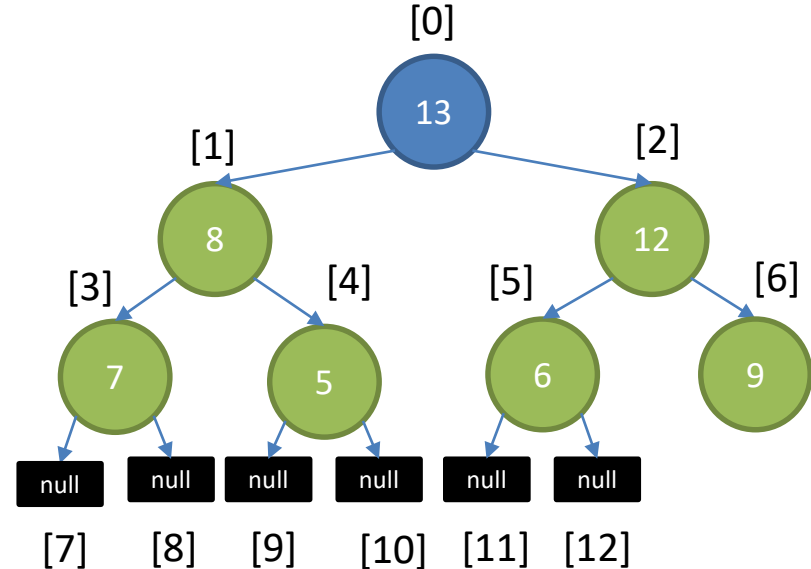
Example Adding “13”



Heaps

- Add
 - Replace the first leaf in breadth order with the new data
 - From that node “bubble up” the data if necessary
- Bubble Up
 - If the child’s data is larger than the parent then swap that information
 - Continue swapping child data with parent data until the parent is larger than the child or we reach the root index

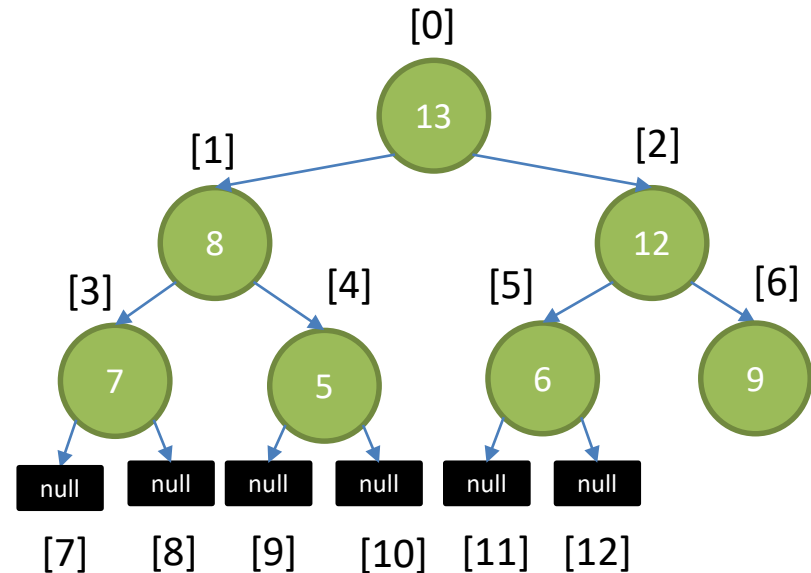
Example Adding “13”



Heaps

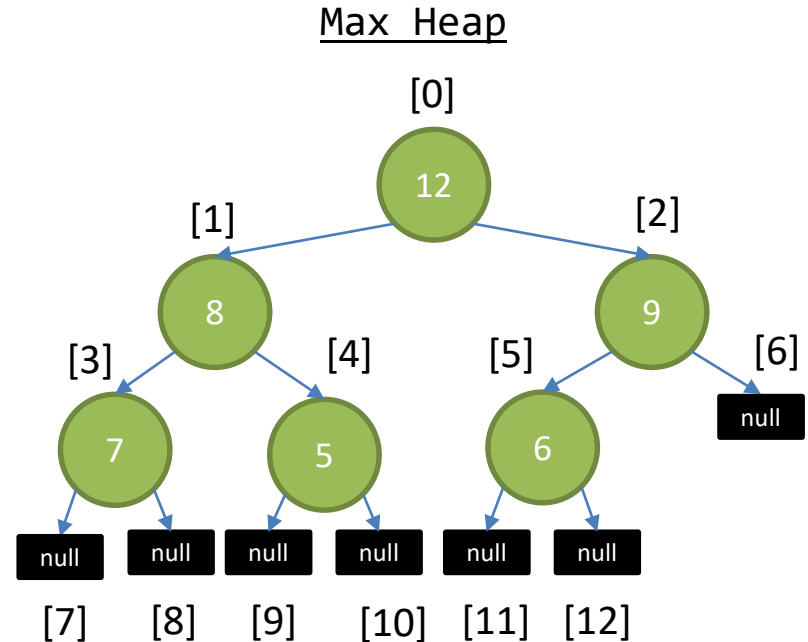
- Add
 - Replace the first leaf in breadth order with the new data
 - From that node “bubble up” the data if necessary
- Bubble Up
 - If the child’s data is larger than the parent then swap that information
 - Continue swapping child data with parent data until the parent is larger than the child or we reach the root index

Example Adding “13”



Heaps

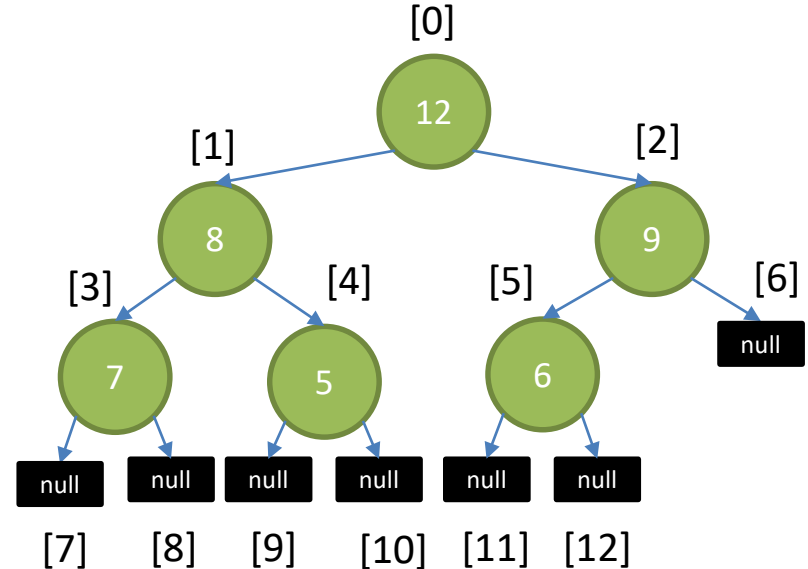
- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds



Heaps

- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds

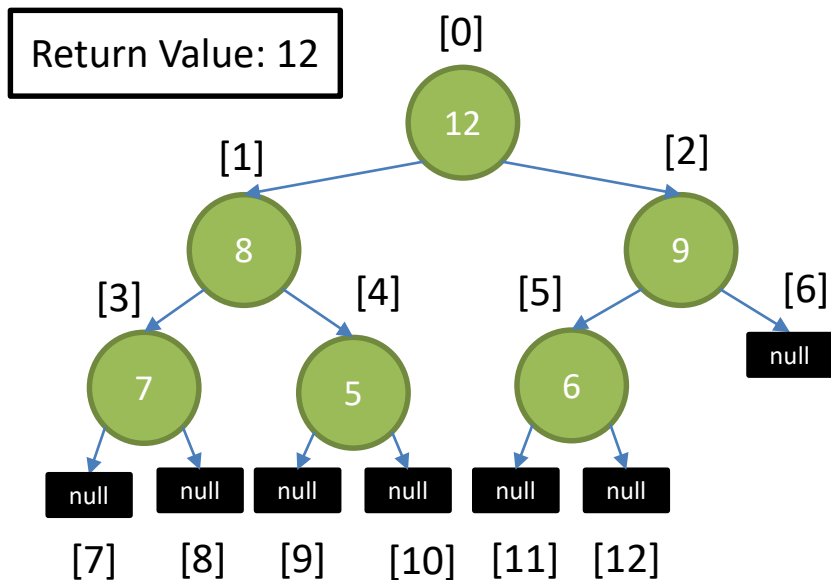
Example Remove



Heaps

- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds

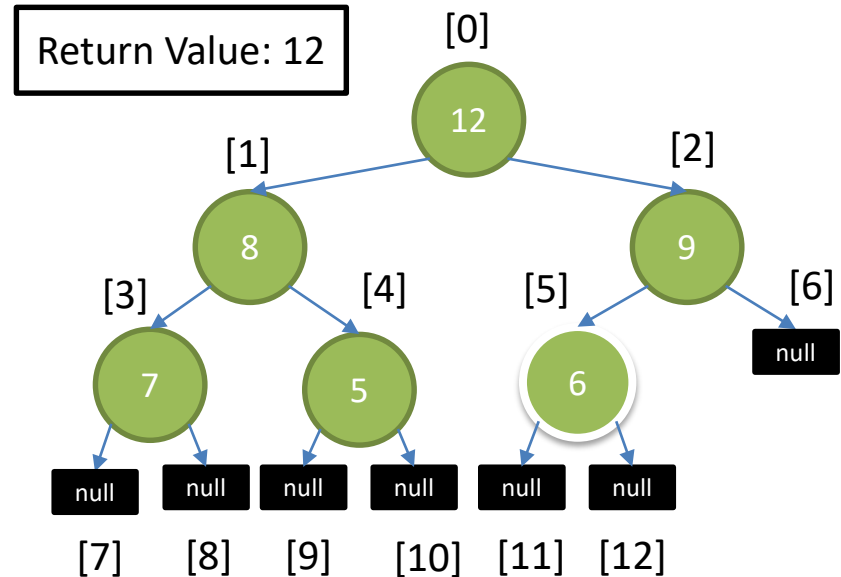
Example Remove



Heaps

- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds

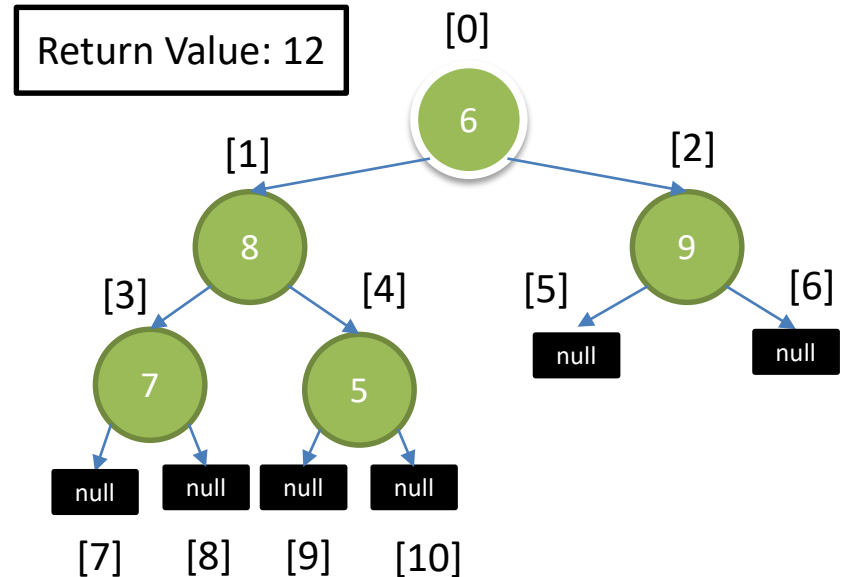
Example Remove



Heaps

- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds

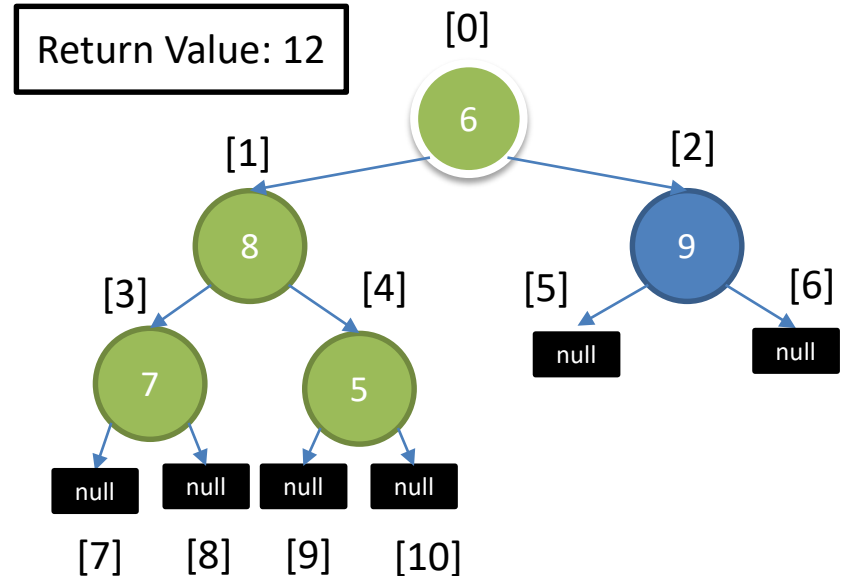
Example Remove



Heaps

- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds

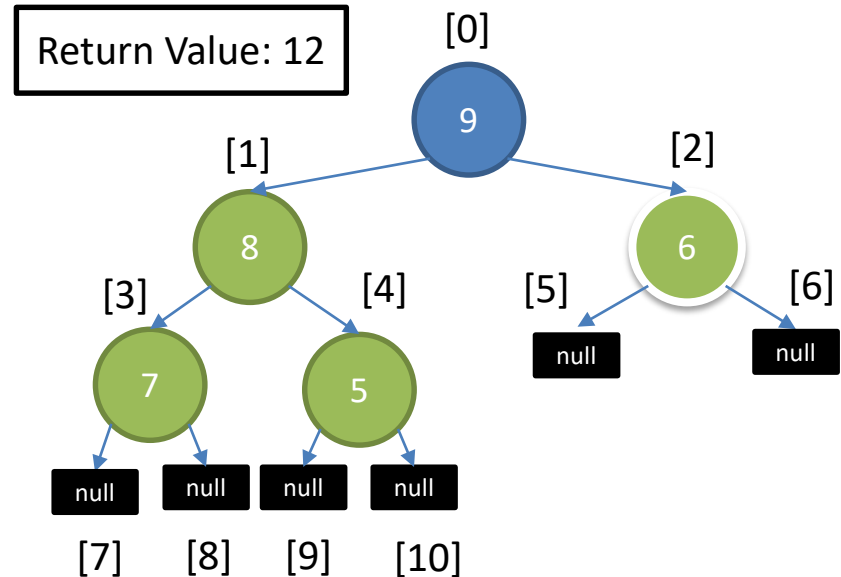
Example Remove



Heaps

- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds

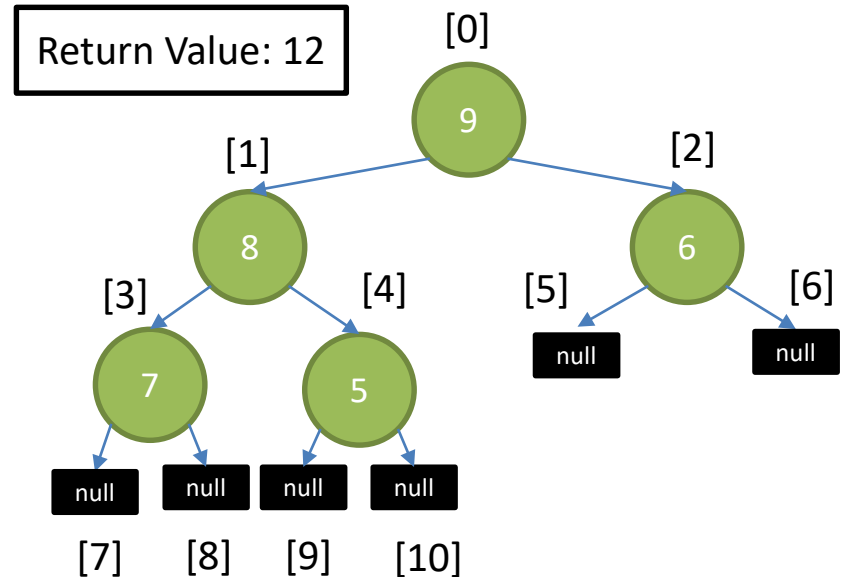
Example Remove



Heaps

- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds

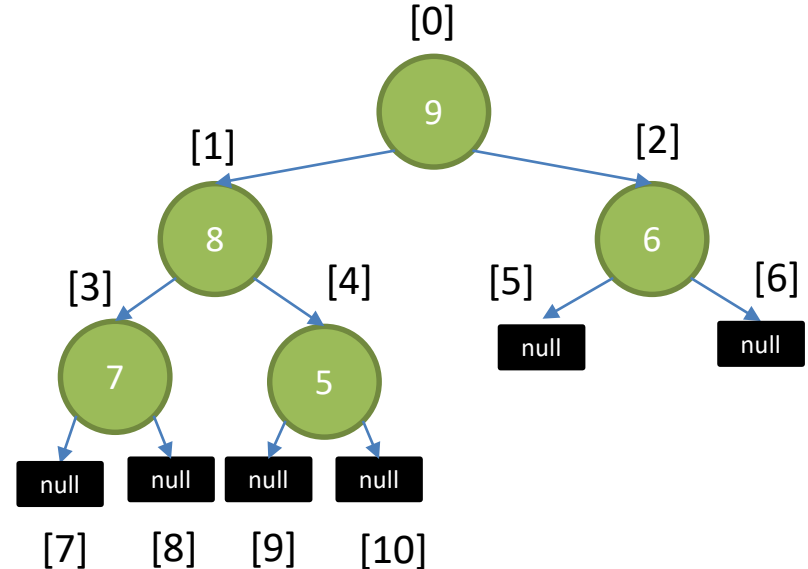
Example Remove



Heaps

- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds

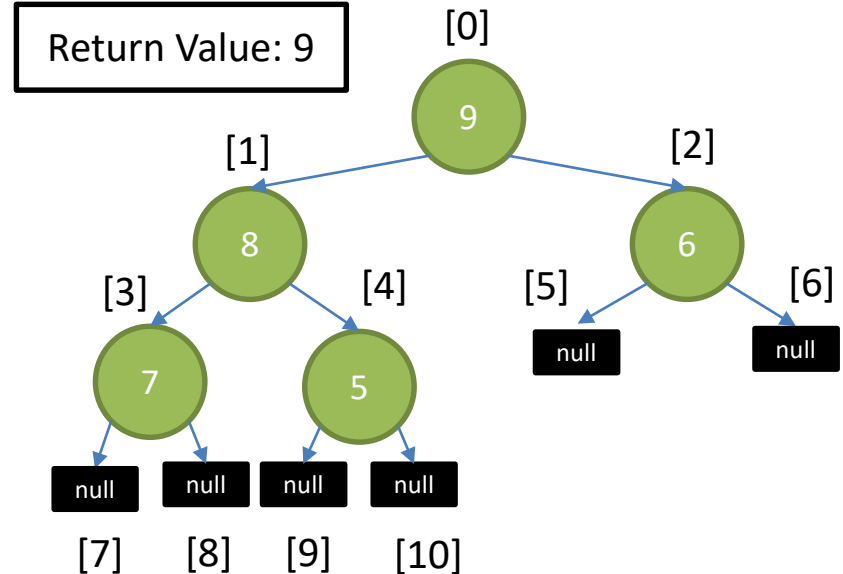
Example Remove



Heaps

- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds

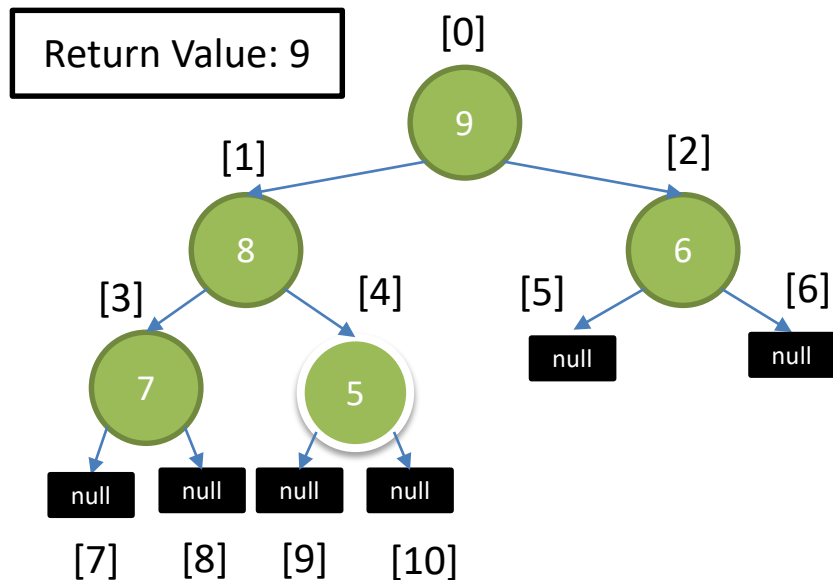
Example Remove Again



Heaps

- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds

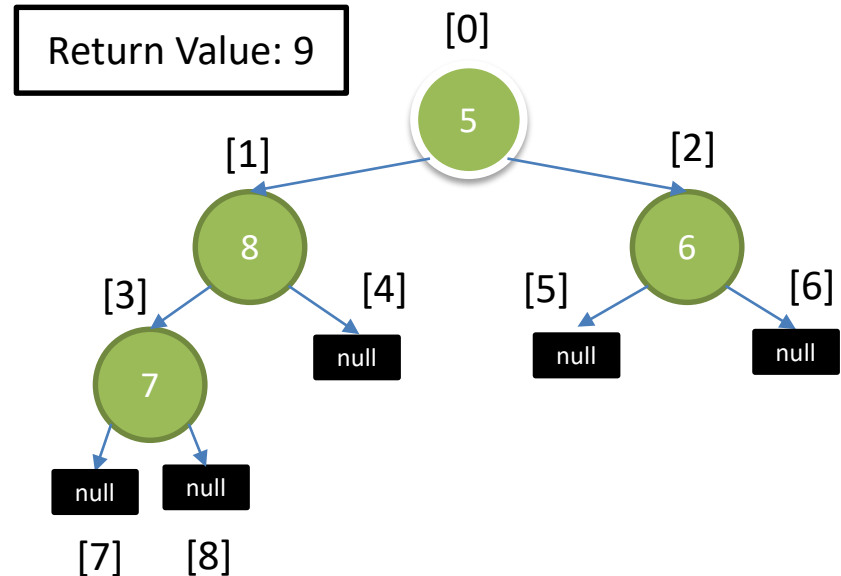
Example Remove Again



Heaps

- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds

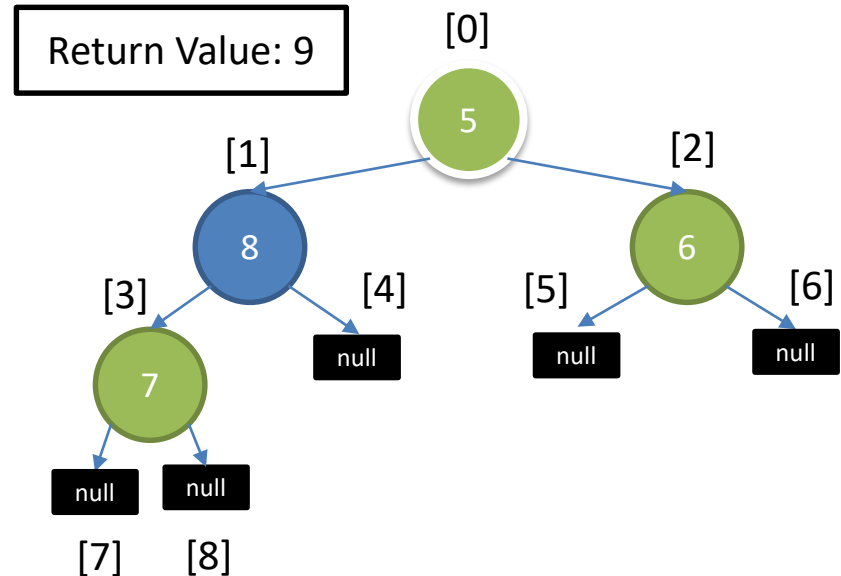
Example Remove Again



Heaps

- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds

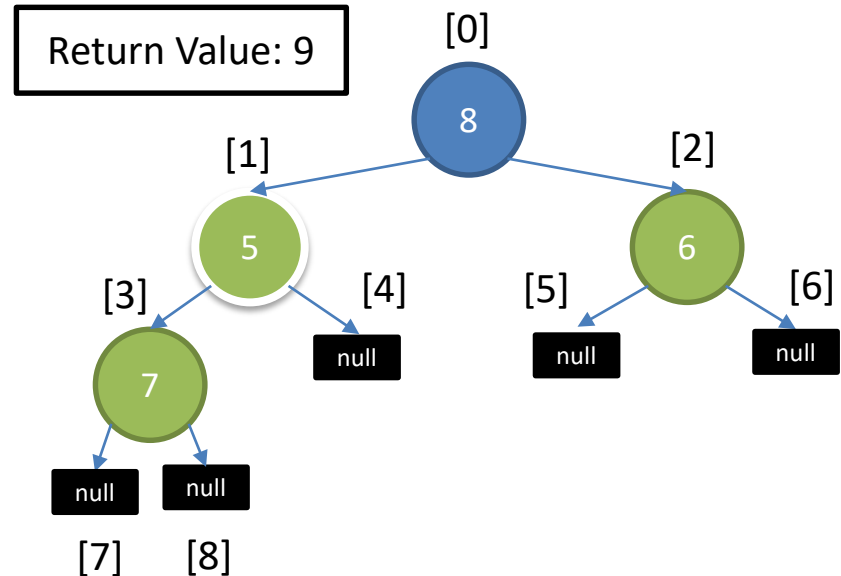
Example Remove Again



Heaps

- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds

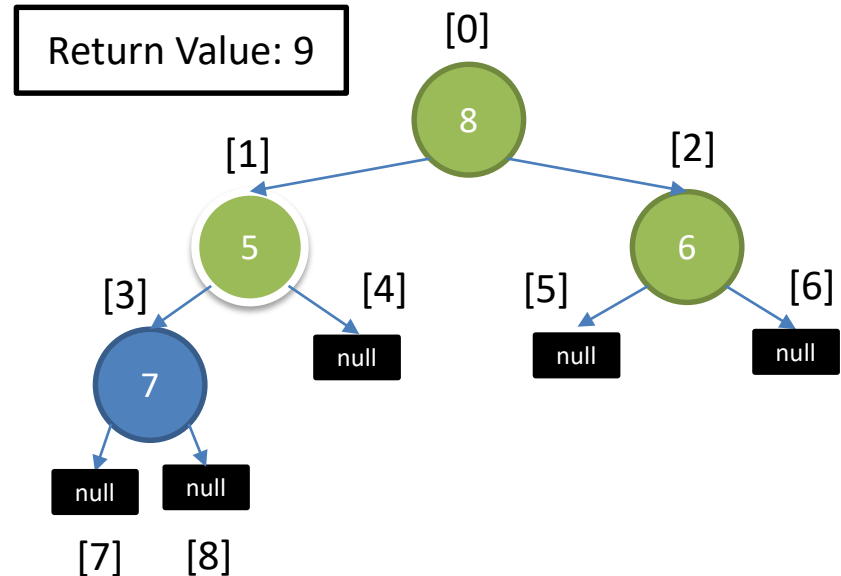
Example Remove Again



Heaps

- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds

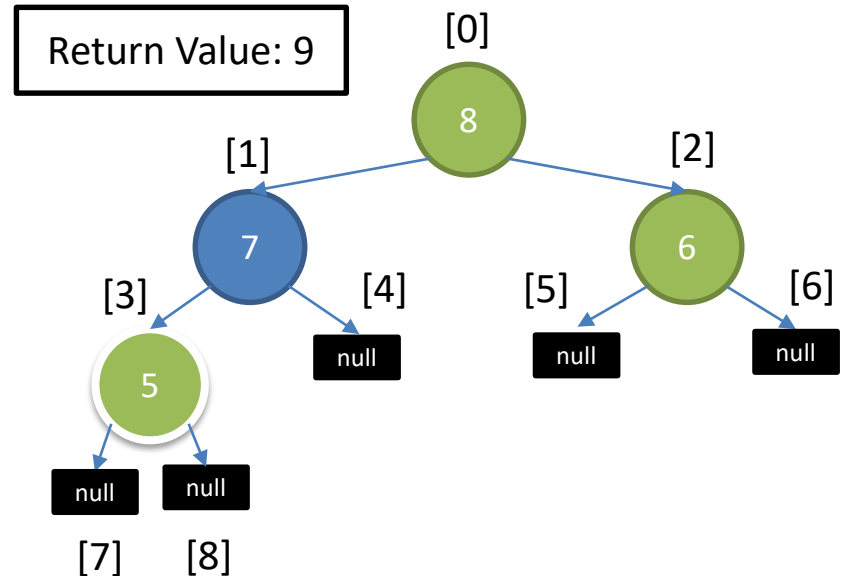
Example Remove Again



Heaps

- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds

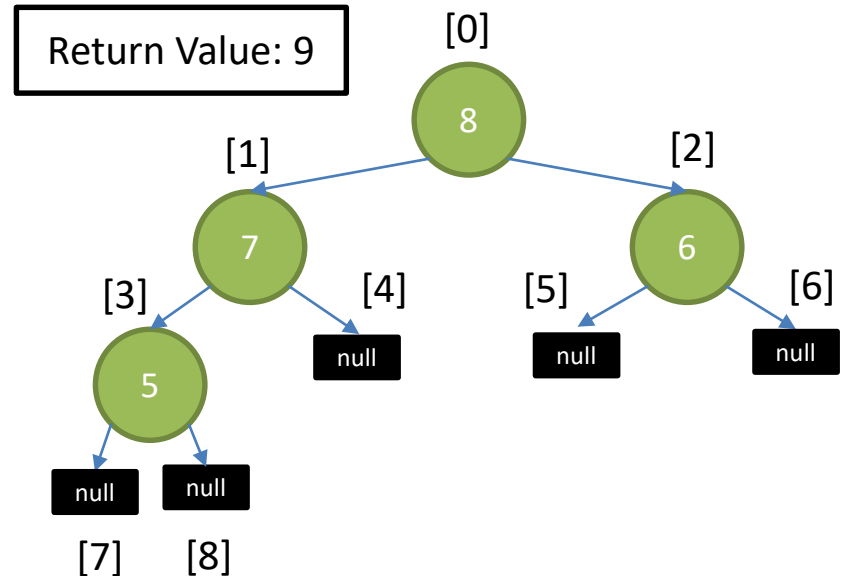
Example Remove Again



Heaps

- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds

Example Remove Again



Heaps

- Remove
 - Store the data at the Root
 - Replace the Root data with the Data in the last node in Breadth Order
 - Starting from the root, “Bubble Down” that information
 - Return the stored value, previously at the root
- Bubble Down
 - Pick the larger of the 2 children
 - If its value is larger than the parent, then swap those values
 - Continue this until the parent’s value is larger or we reach the tree’s bounds

Example Remove Again

