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Binary Search Trees
Part 02

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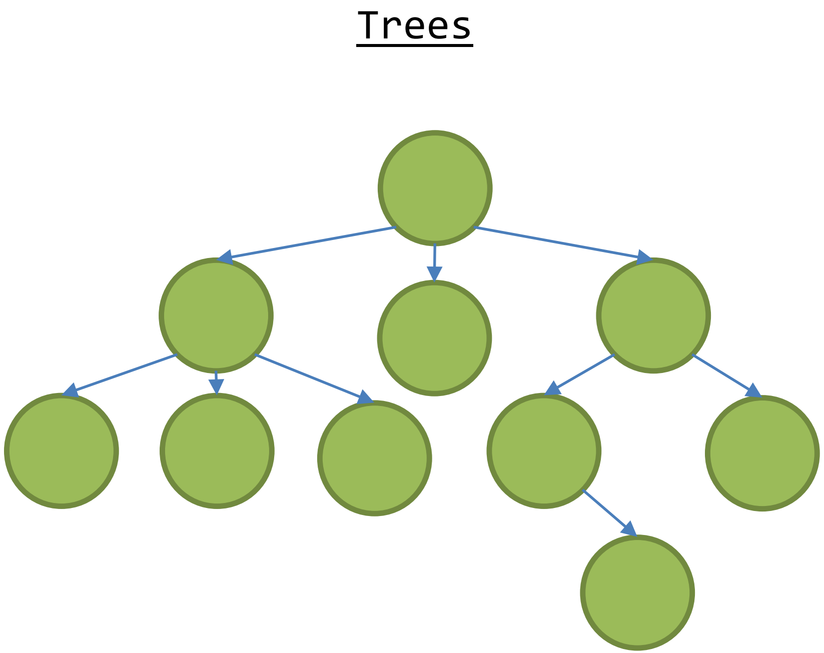
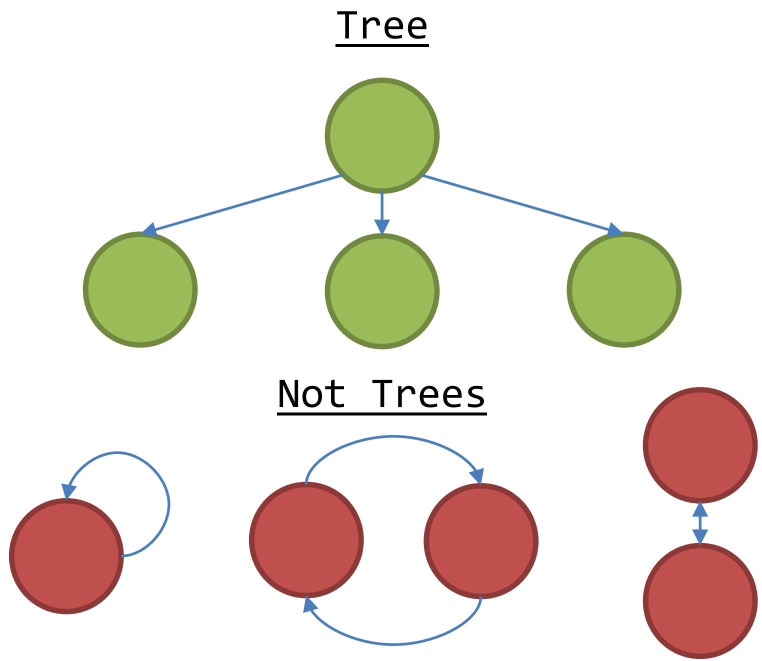
Trees

- Definition: A data structure that can be defined recursively as a collection of nodes, where each node is a data structure consisting of a value, together with a list of references (edges) to nodes, with the constraints that no reference is duplicated, and none points to the root.



Trees

- Trees Have
  - Nodes
  - Edges
- Trees CANNOT
  - Contain Self-Referencing Edges
  - Have Cycles
  - Be Disjointed

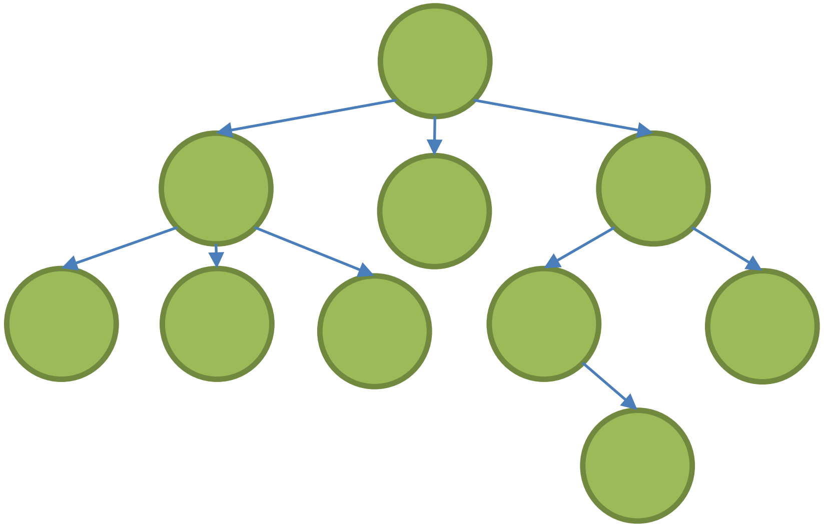


Trees

Common Terms

- **Root** – The top node in a tree.
- **Child** – A node's reference which is at a lower level
- **Parent** – The converse notion of *child*.
- **Siblings** – Nodes with the same parent.
- **Leaf** – a node with no children.
- **Degree** – number of sub trees of a node.
- **Edge** – connection between one node to another.
- **Path** – a sequence of nodes and edges connecting a node with a descendant.
- **Level** – The level of a node is defined by 1 + the number of connections between the node and the root.
- **Height of tree** –The height of a tree is the number of edges on the longest downward path between the root and a leaf.
- **Height of node** –The height of a node is the number of edges on the longest downward path between that node and a leaf.
- **Depth** –The depth of a node is the number of edges from the node to the tree's root node.

Trees



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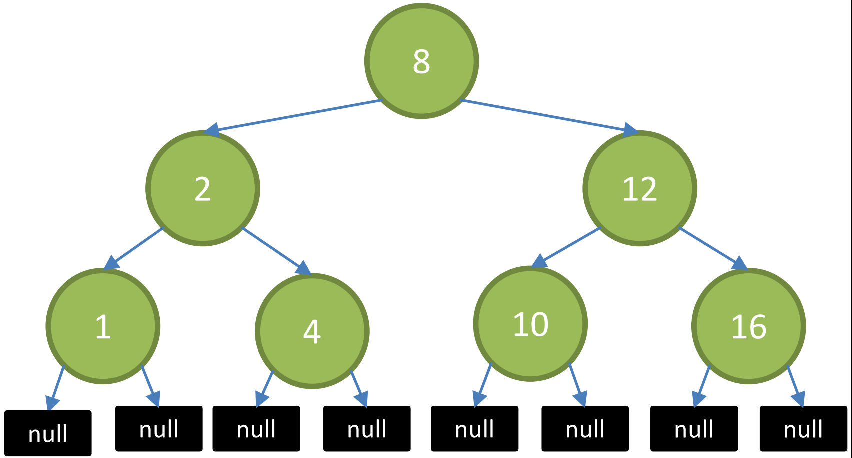
Binary Search Trees

- Tree Structure
- Node's data must be comparable
- Node's have at most two children
  - Left Child
  - Right Child
- Left child's value must be LESS THAN the parent's value
- Right child's value must be GREATER THAN the parent's value
- No Duplicate Values
- Assume Leaves are NULL references

Binary Search Tree



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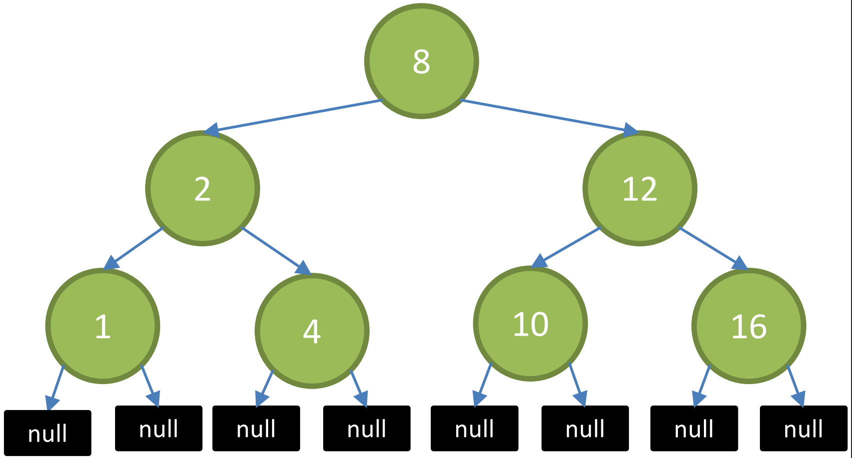
Binary Search Trees

- Search
  - Start from the Root
  - If it is a leaf then return false
  - If the target value matches the Node's data then return true
  - If the target value is less than the Node's data then recursively GO LEFT
  - If the target value is greater than the Node's data then recursively GO RIGHT

Binary Search Tree



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Binary Search Trees

- Remove
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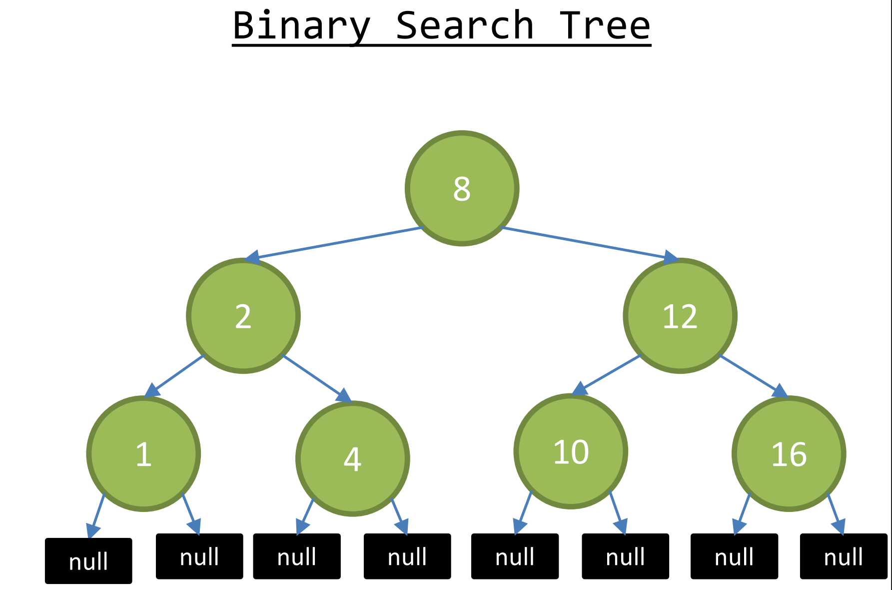

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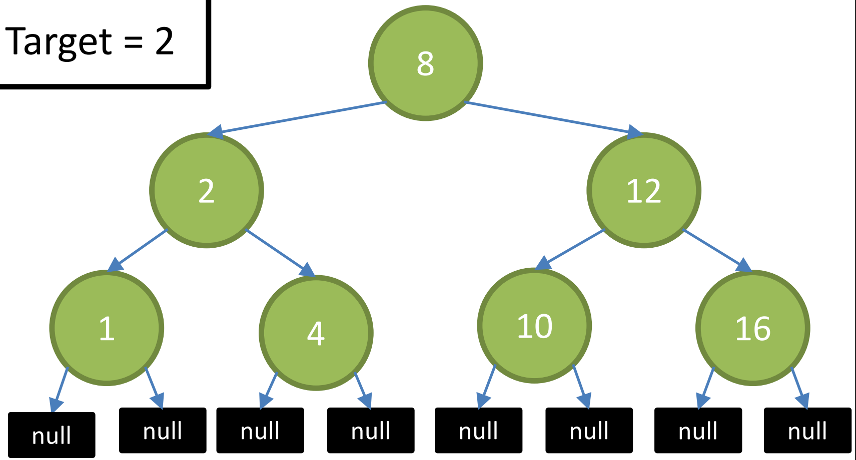
Remove 2 Example

Target = 2



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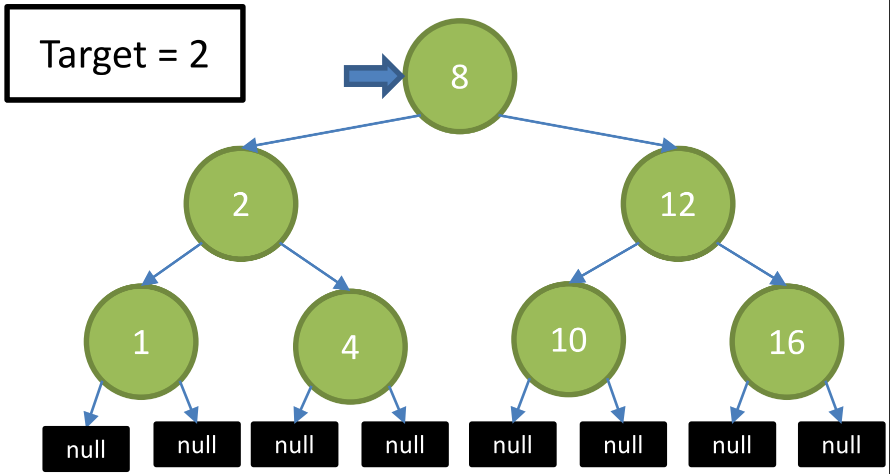

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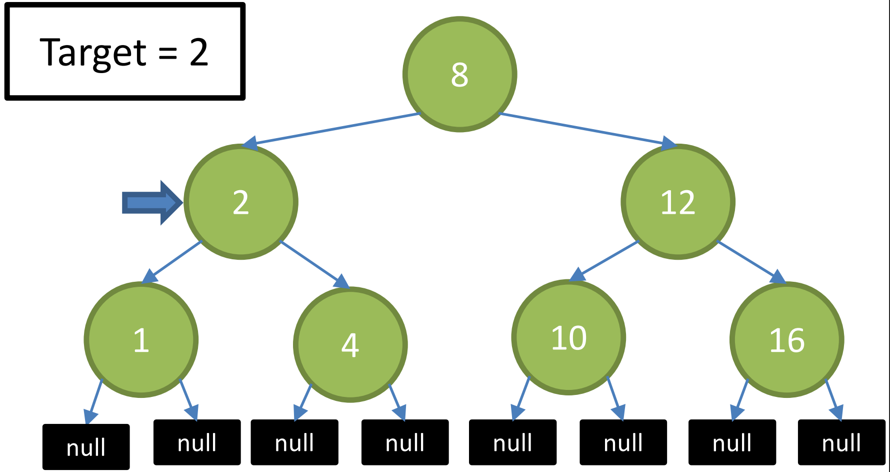

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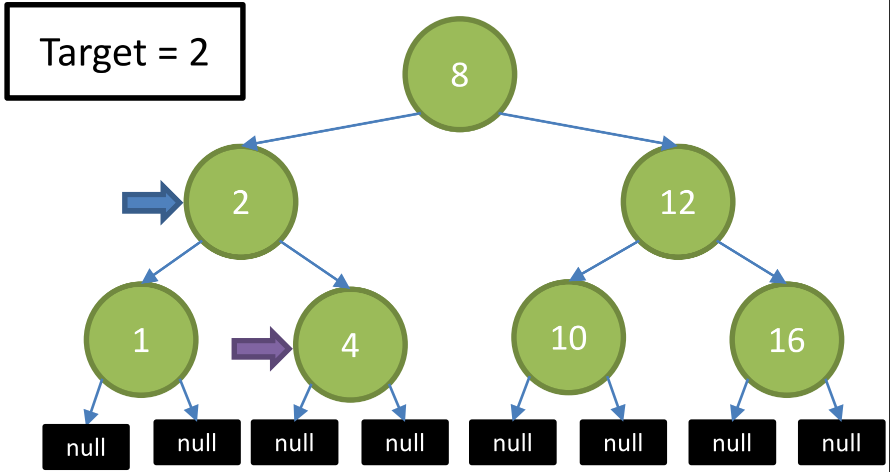

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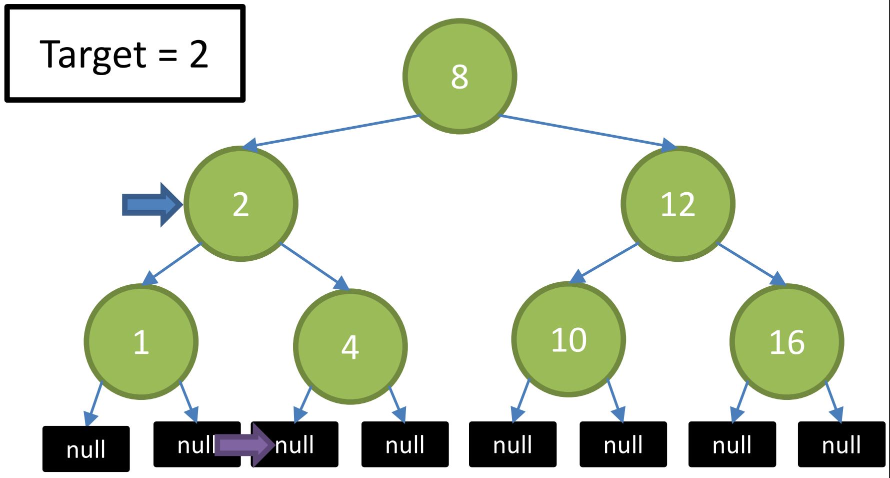

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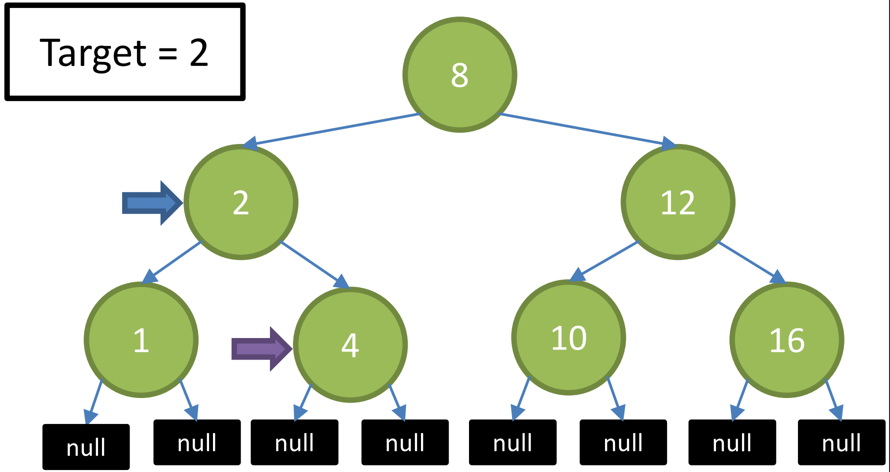

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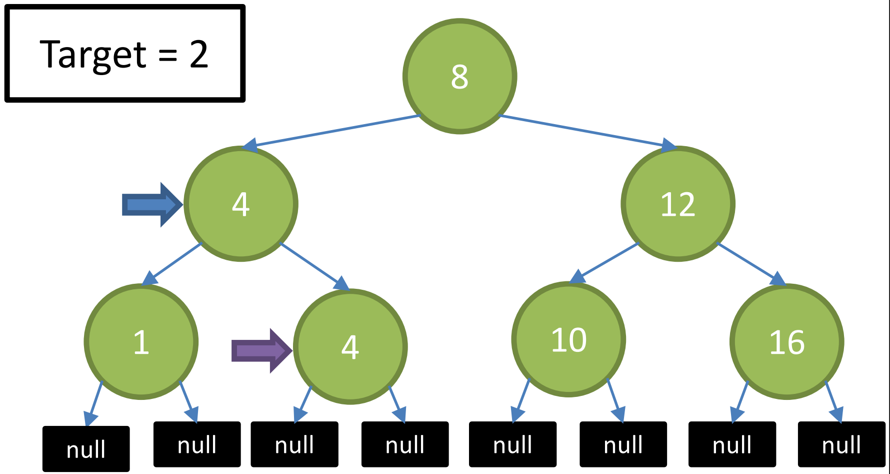

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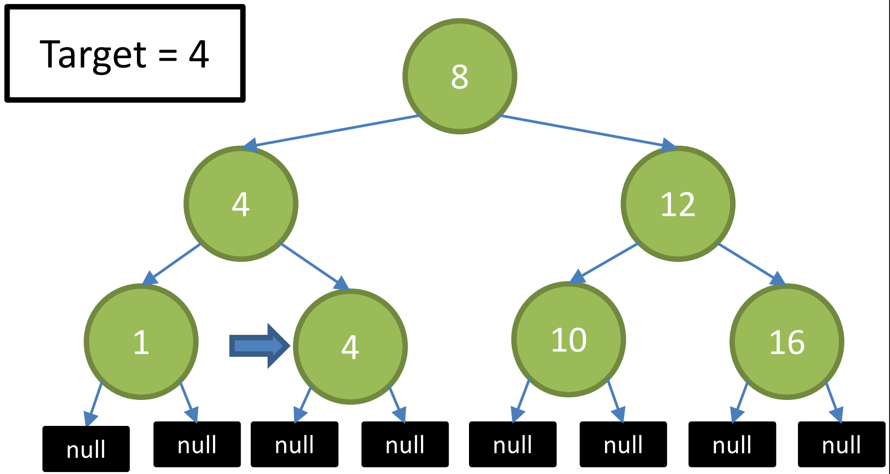

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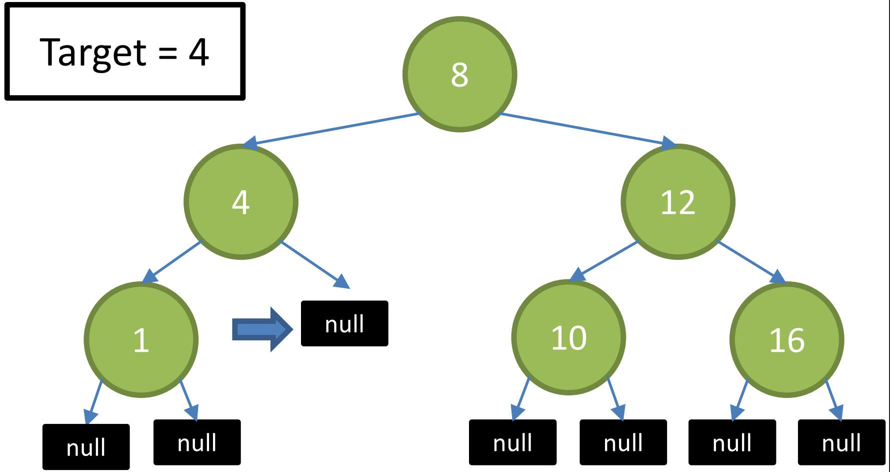

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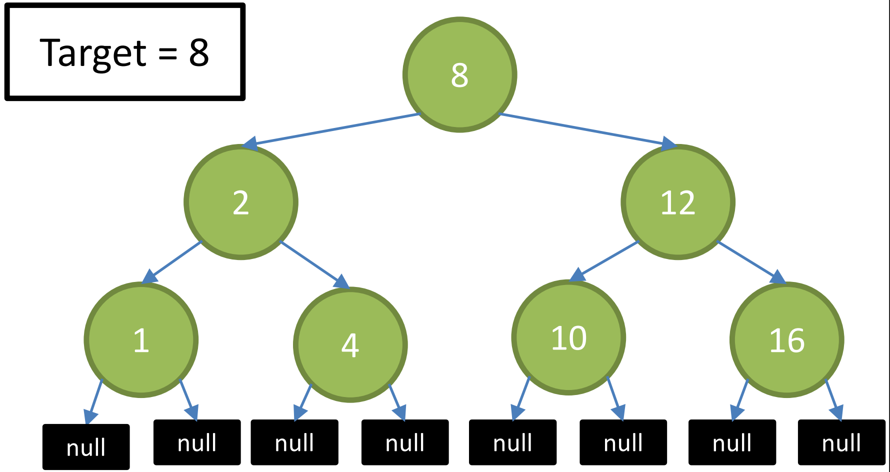

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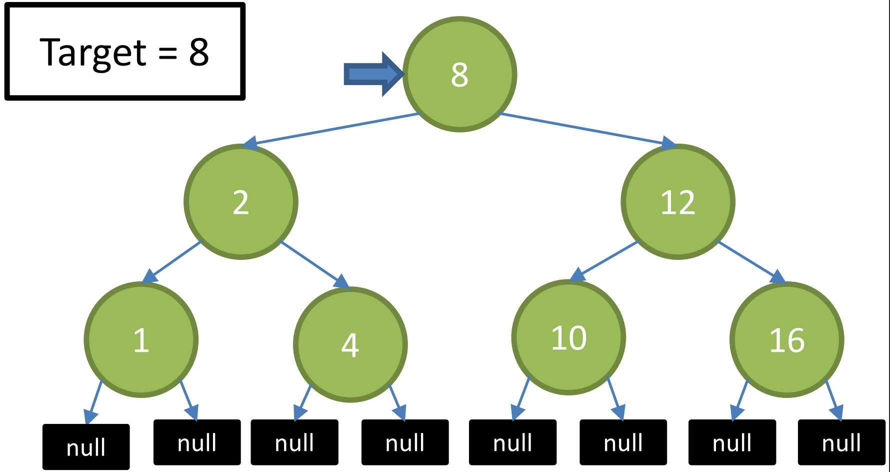

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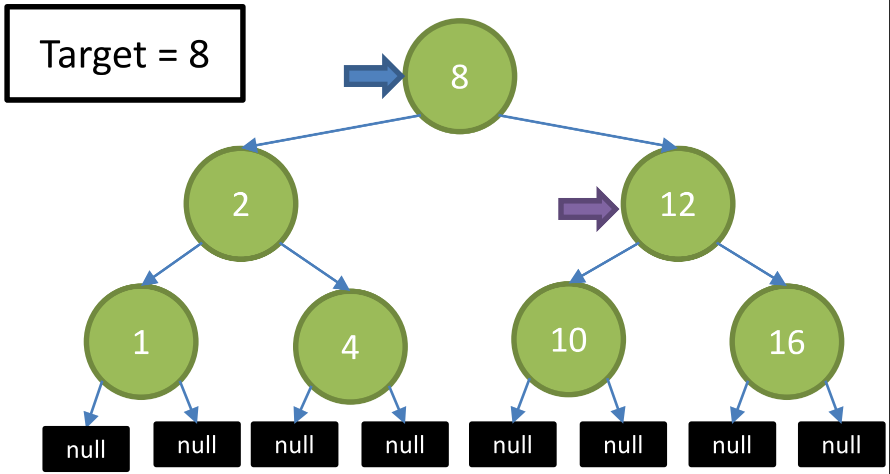

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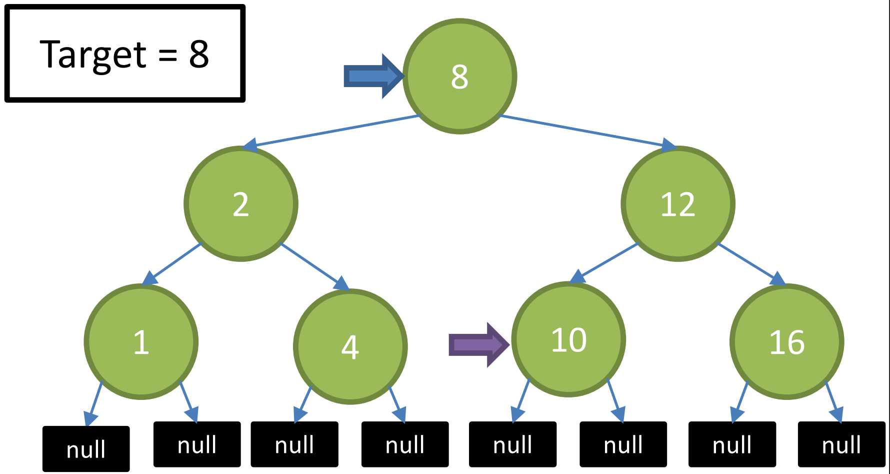

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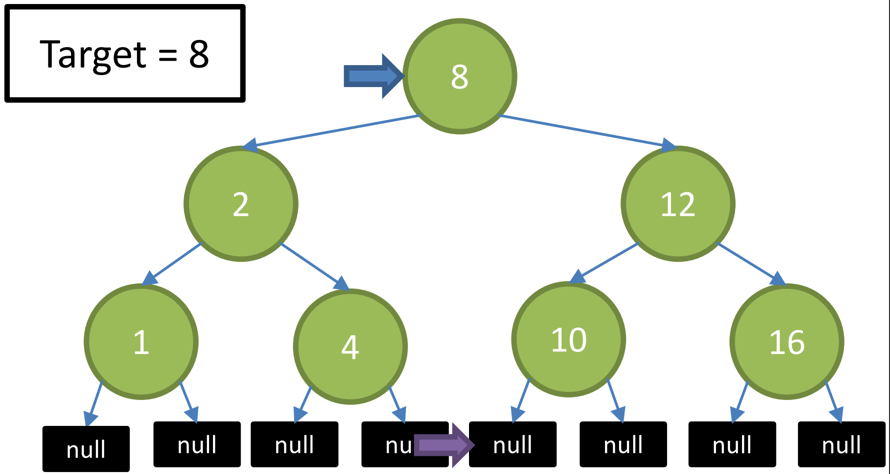

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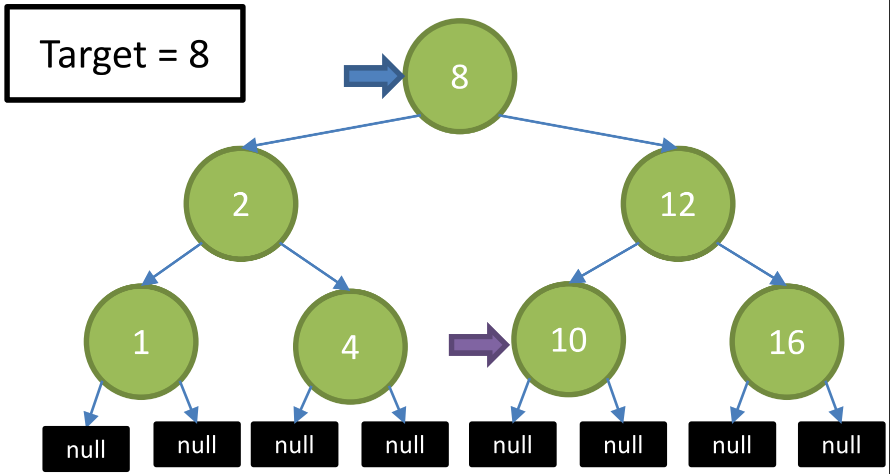

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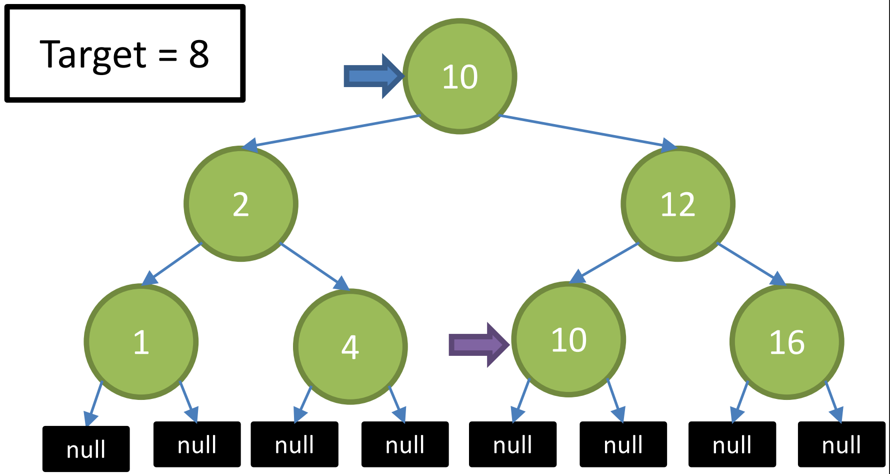

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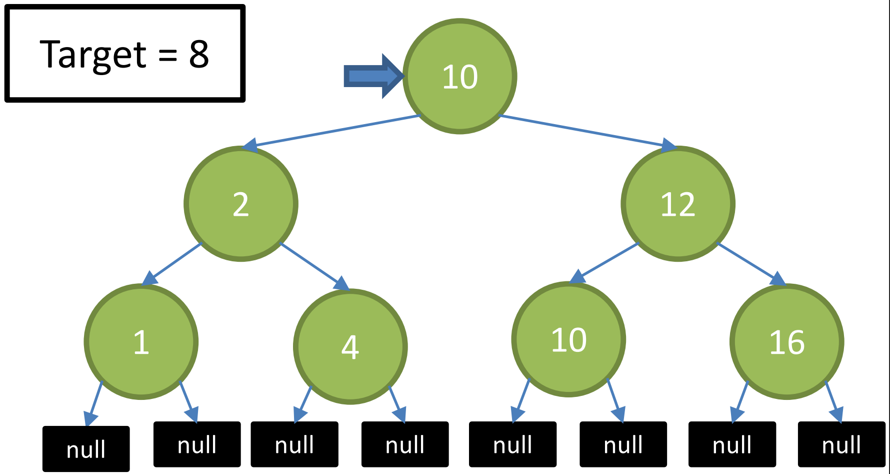

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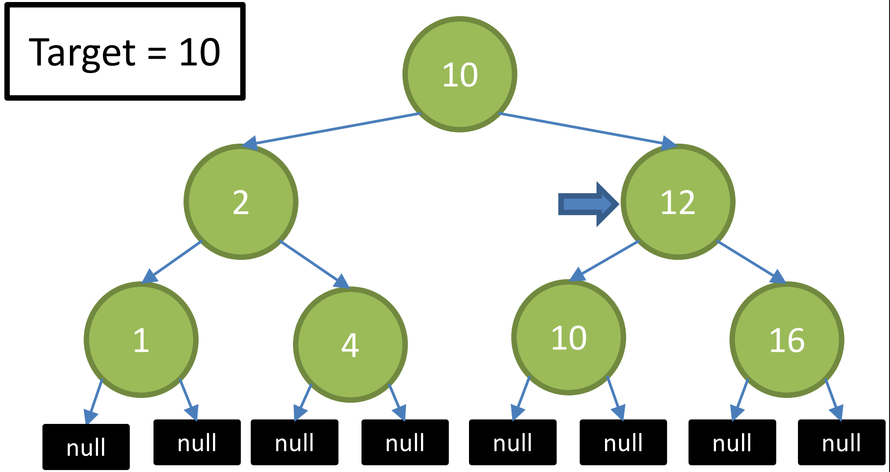

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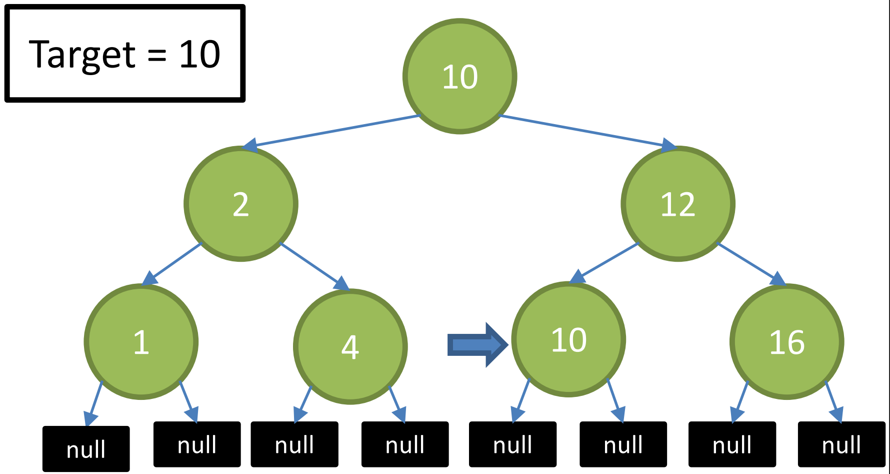

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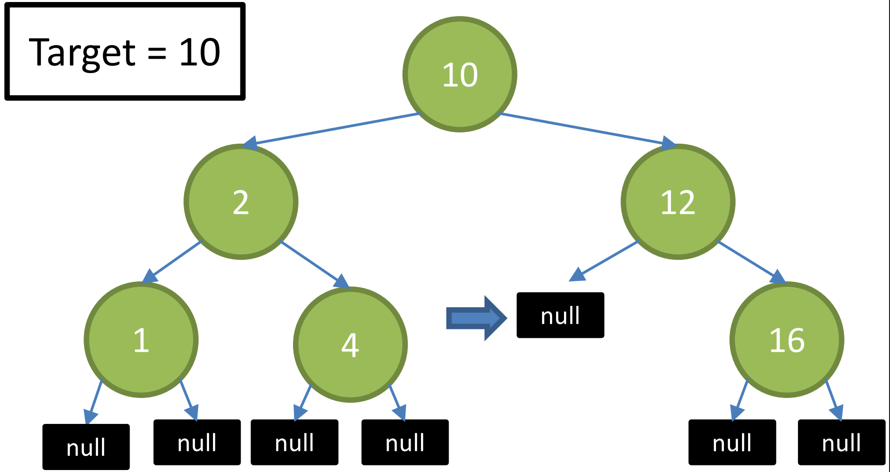

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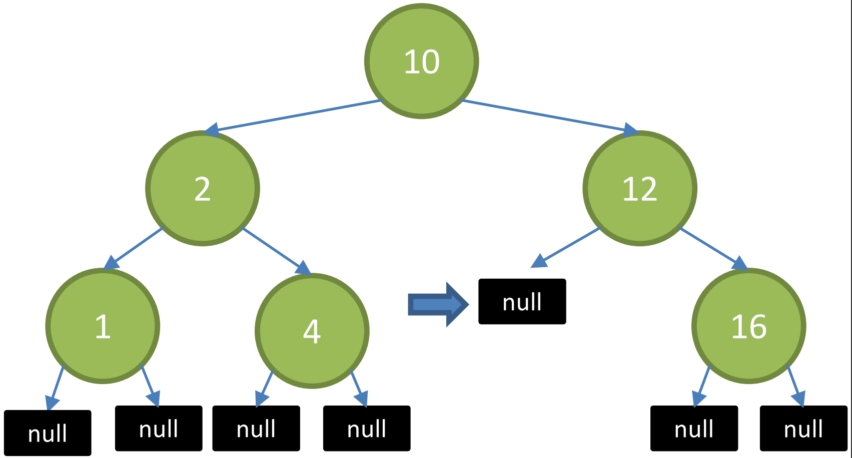

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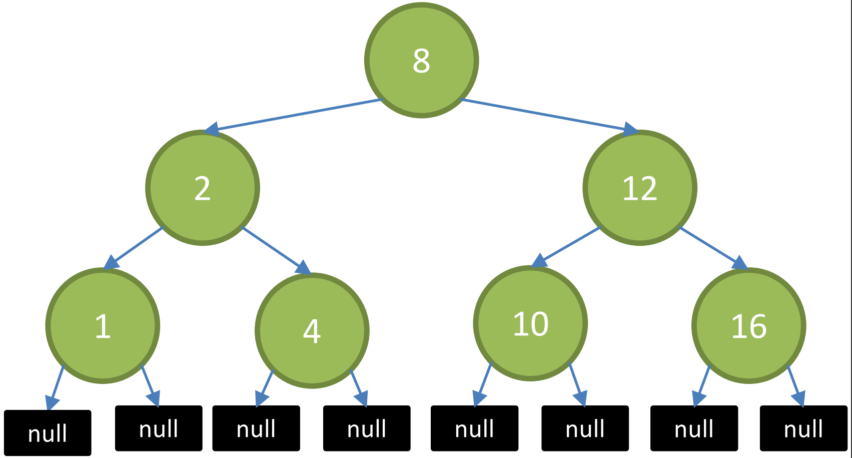
Binary Search Trees

- Complexity depends on the structure of the tree
- Balanced Trees
  - From the root to any leaf there are AT MOST lg(n) edges
- Unbalanced Trees
  - Have at least one path from root to a leaf that is more than lg(n) edges

Balanced Tree



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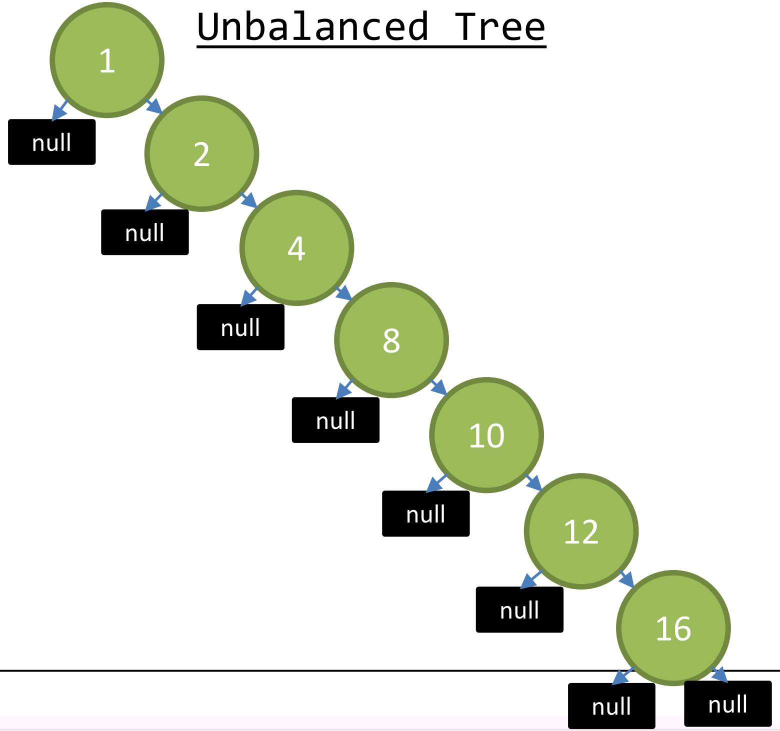

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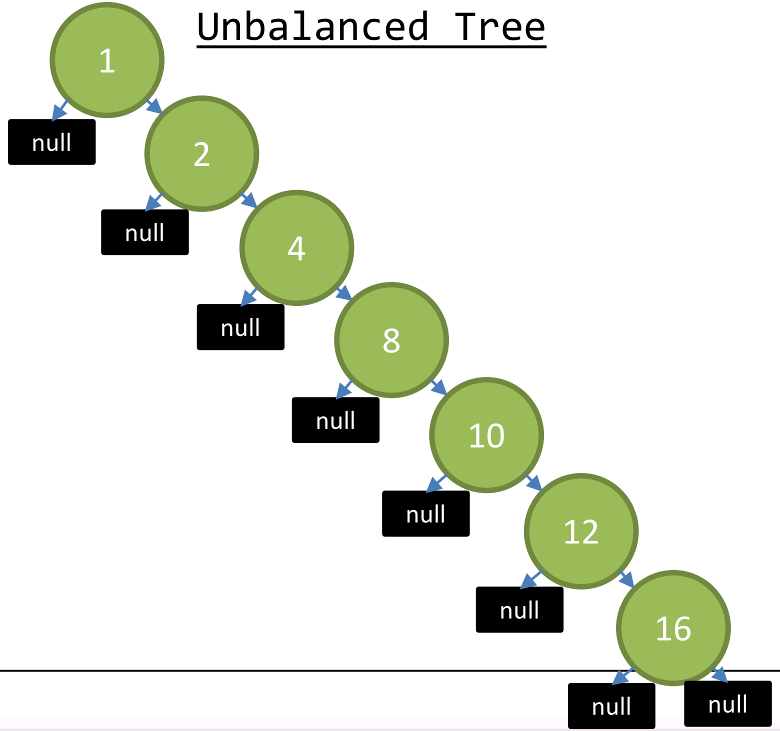
Binary Search Trees

- Unbalanced Tree
  - Add = O(n)
  - Search = O(n)
  - Remove = O(n)

Unbalanced Tree



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Binary Search Trees

- Balanced Tree
  - Add = O(lg(n))
  - Search = O(lg(n))
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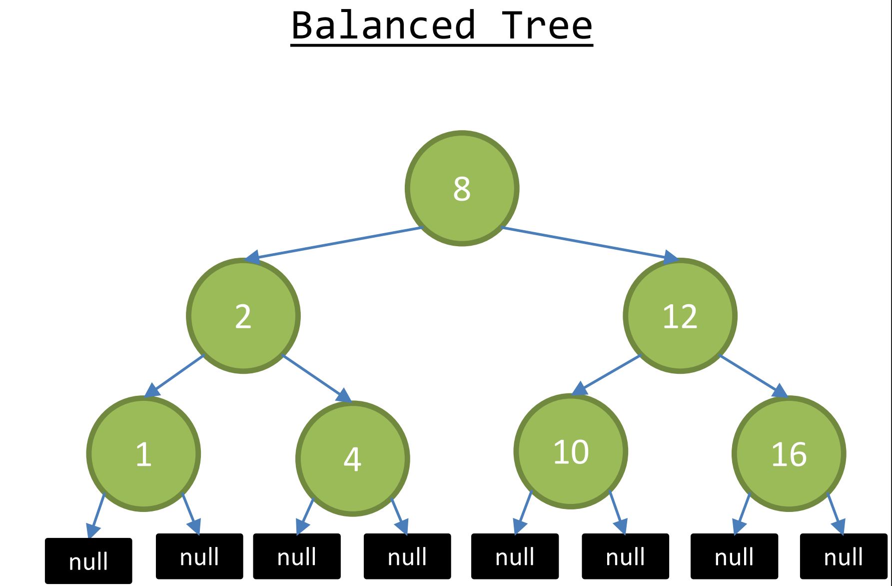

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Binary Search Trees

- Self-Balancing Trees
  - Change references until the tree is balanced
  - Based on criteria like Height or Node "Color"
- Rotations are used to Balance the Tree
  - Left Rotations
  - Right Rotations
- Popular Self-Balancing Trees
  - AVL
  - Red / Black Tree