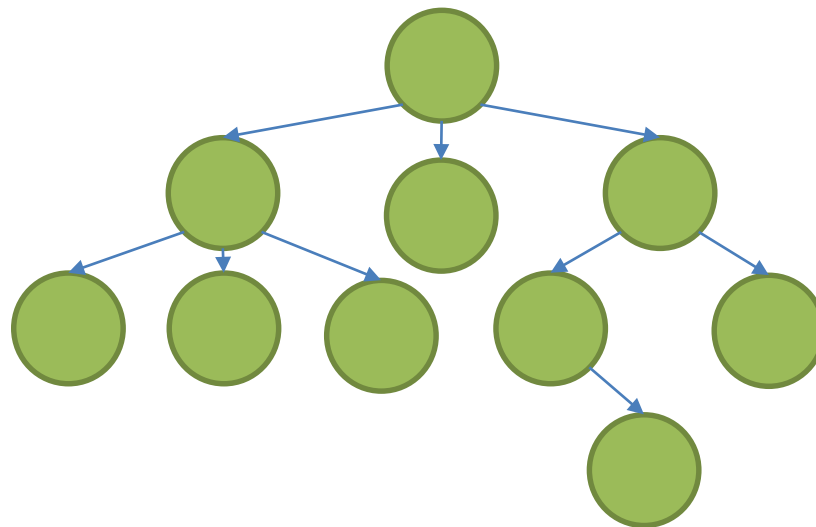


Binary Search Trees

Trees

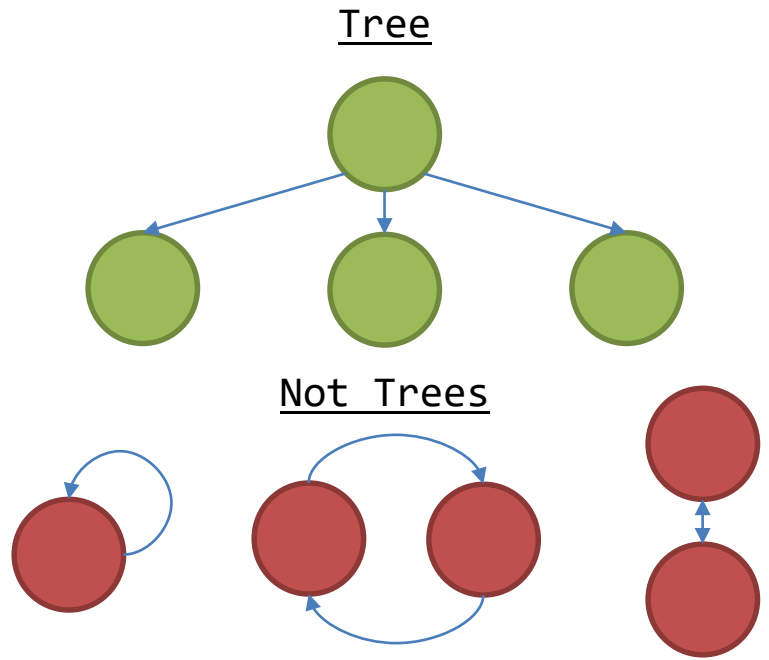
- Definition: A data structure that can be defined recursively as a collection of nodes, where each node is a data structure consisting of a value, together with a list of references (edges) to nodes, with the constraints that no reference is duplicated, and none points to the root.

Trees



Trees

- Trees Have
 - Nodes
 - Edges
- Trees CANNOT
 - Contain Self-Referencing Edges
 - Have Cycles
 - Be Disjointed

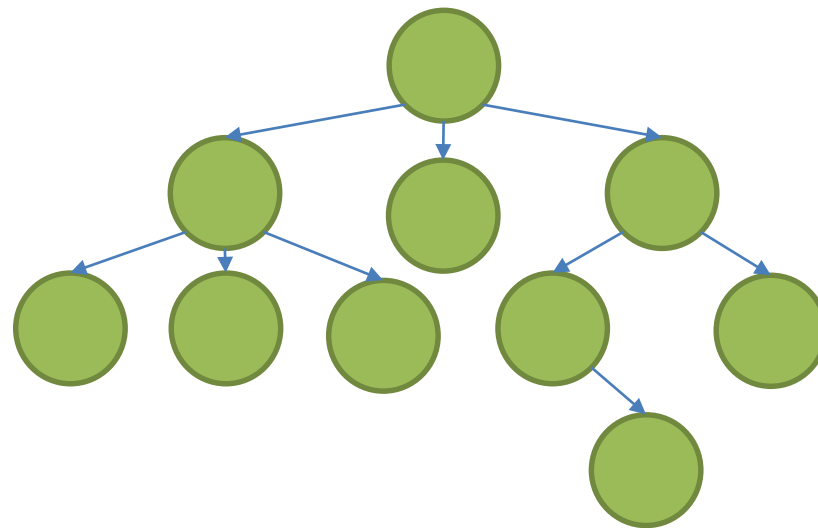


Trees

Common Terms

- **Root** – The top node in a tree.
- **Child** – A node's reference which is at a lower level
- **Parent** – The converse notion of *child*.
- **Siblings** – Nodes with the same parent.
- **Leaf** – a node with no children.
- **Degree** – number of sub trees of a node.
- **Edge** – connection between one node to another.
- **Path** – a sequence of nodes and edges connecting a node with a descendant.
- **Level** – The level of a node is defined by 1 + the number of connections between the node and the root.
- **Height of tree** –The height of a tree is the number of edges on the longest downward path between the root and a leaf.
- **Height of node** –The height of a node is the number of edges on the longest downward path between that node and a leaf.
- **Depth** –The depth of a node is the number of edges from the node to the tree's root node.

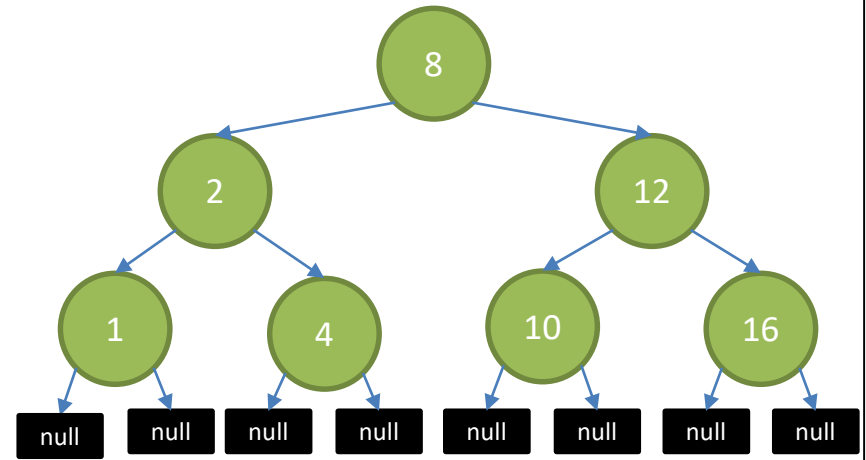
Trees



Binary Search Trees

- Tree Structure
- Node's data must be comparable
- Node's have at most two children
 - Left Child
 - Right Child
- Left child's value must be LESS THAN the parent's value
- Right child's value must be GREATER THAN the parent's value
- No Duplicate Values
- Assume Leaves are NULL references

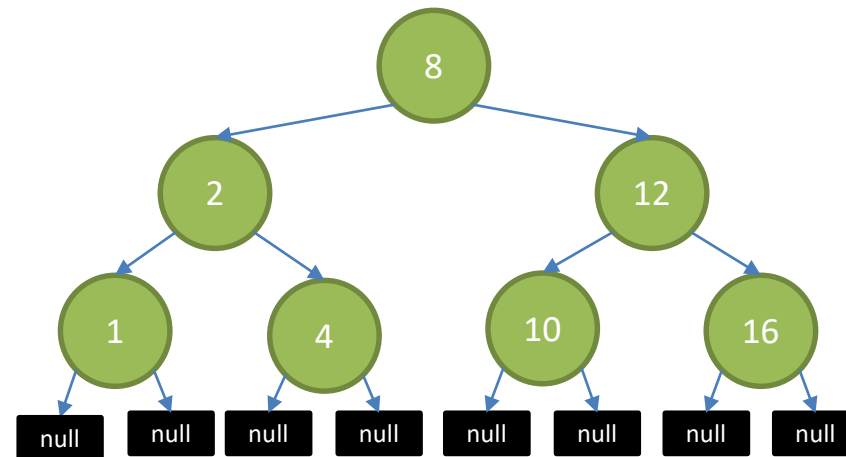
Binary Search Tree



Binary Search Trees

- Add
 - Construct a new Node
 - Start from the Root Node
 - Recursively go left and right until a leaf (NULL element) is found
 - Replace the leaf's reference with the newly created node's reference

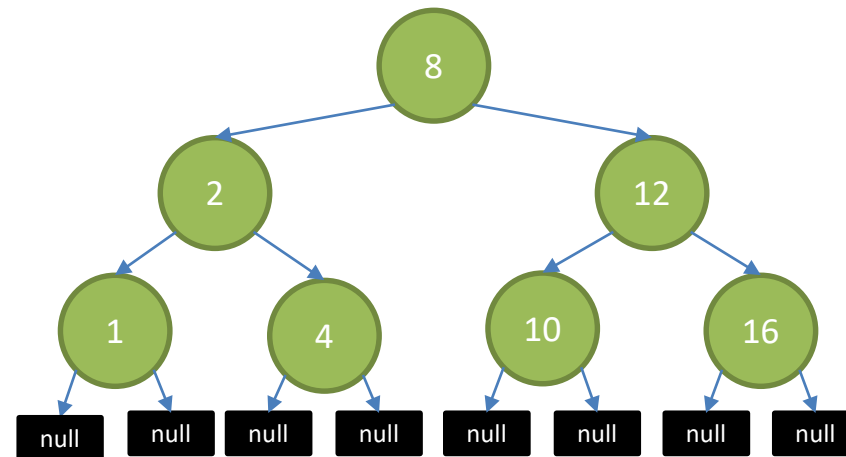
Binary Search Tree



Binary Search Trees

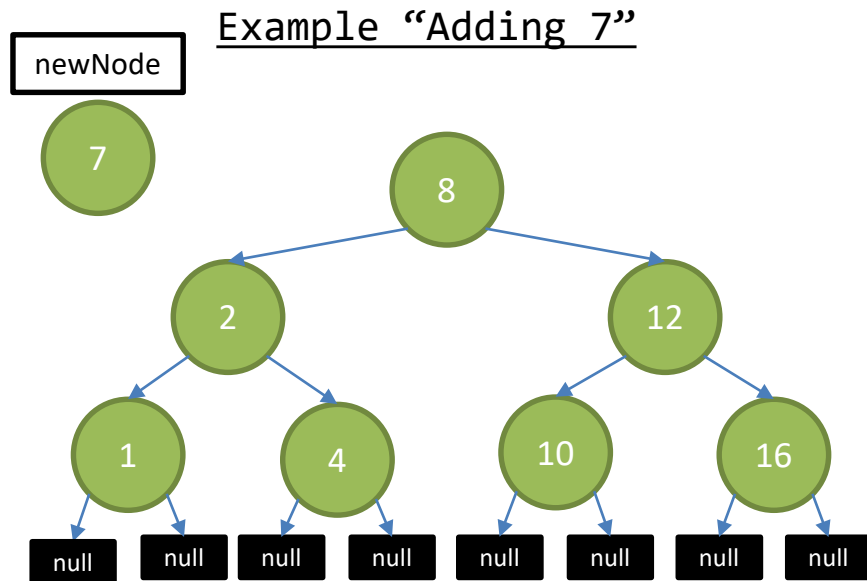
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Example “Adding 7”



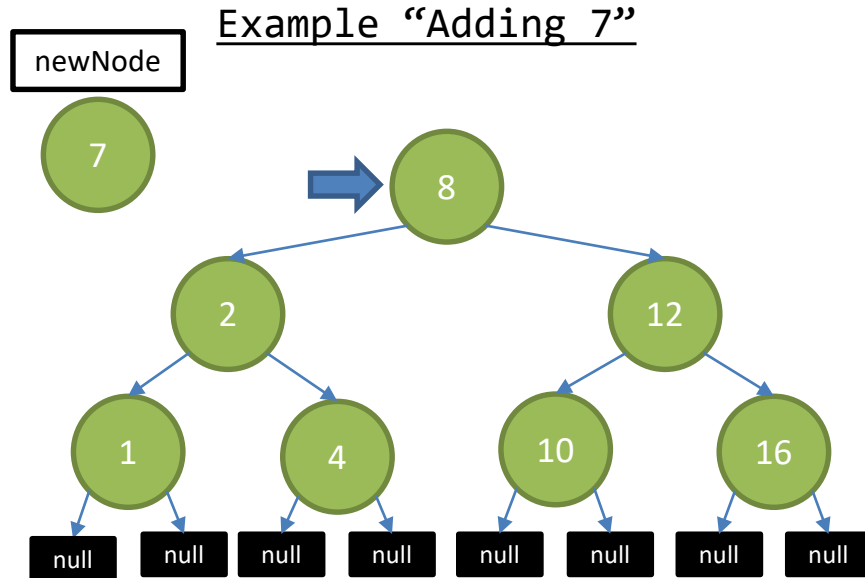
Binary Search Trees

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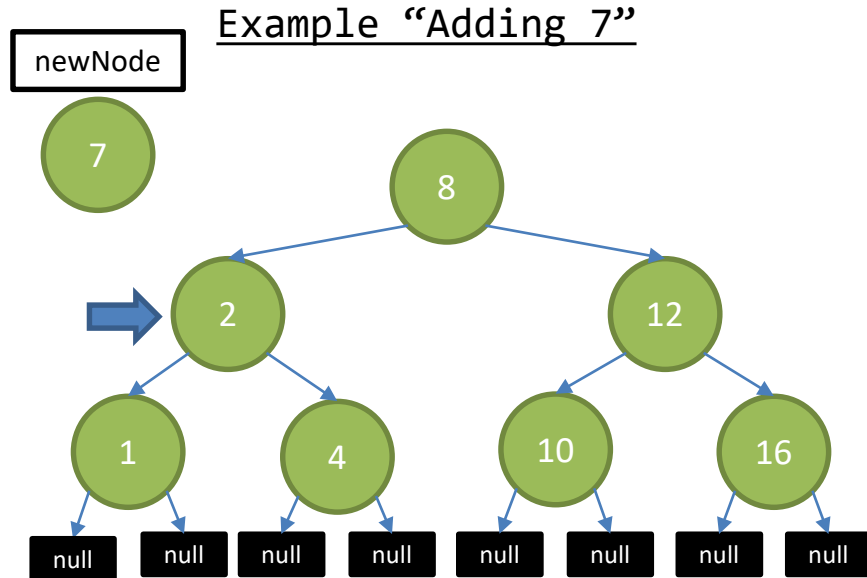
Binary Search Trees

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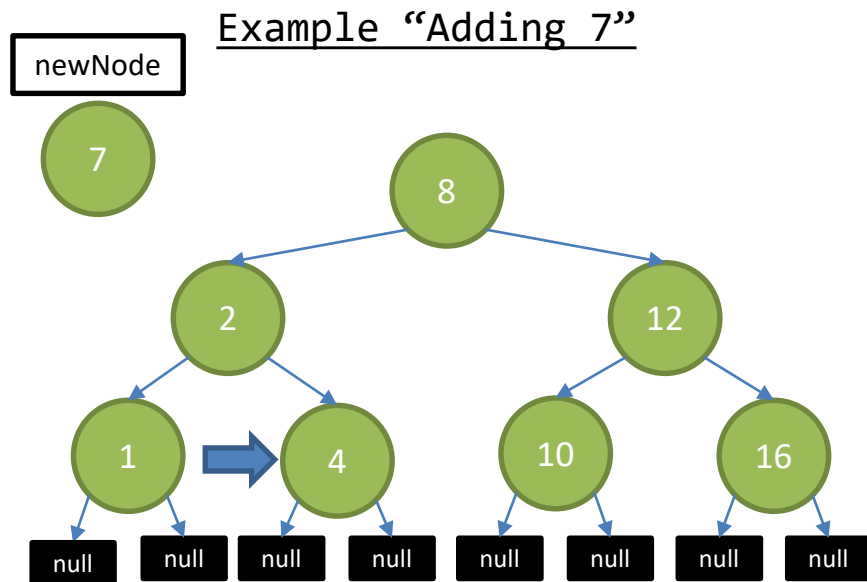
Binary Search Trees

- Add
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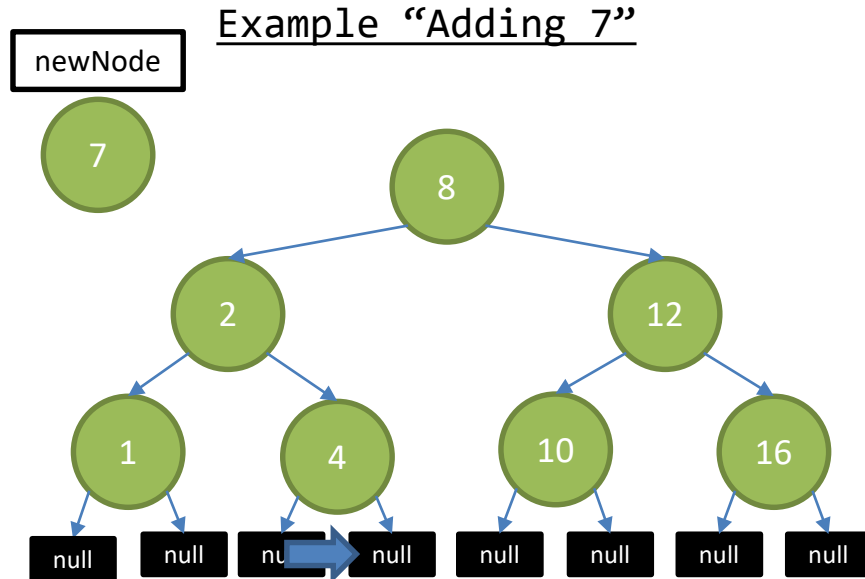
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Binary Search Trees

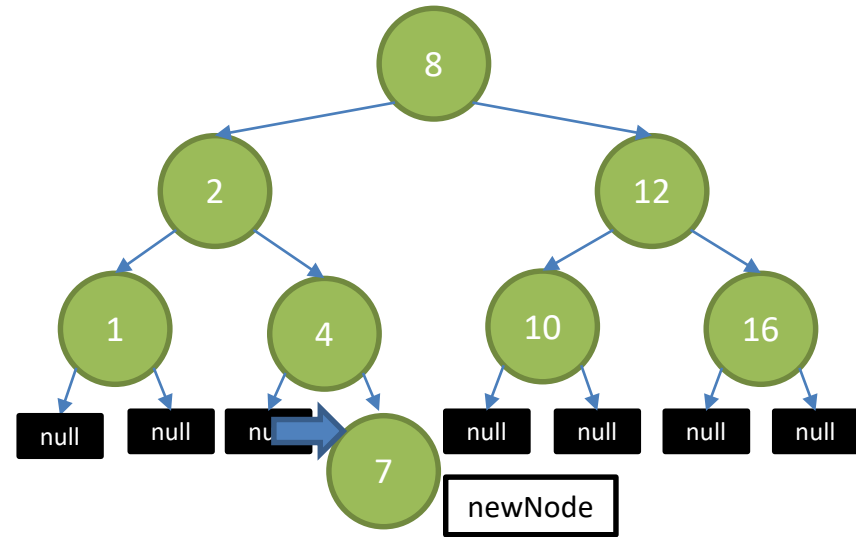
- Add
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Binary Search Trees

- Add
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 - Start from the Root Node
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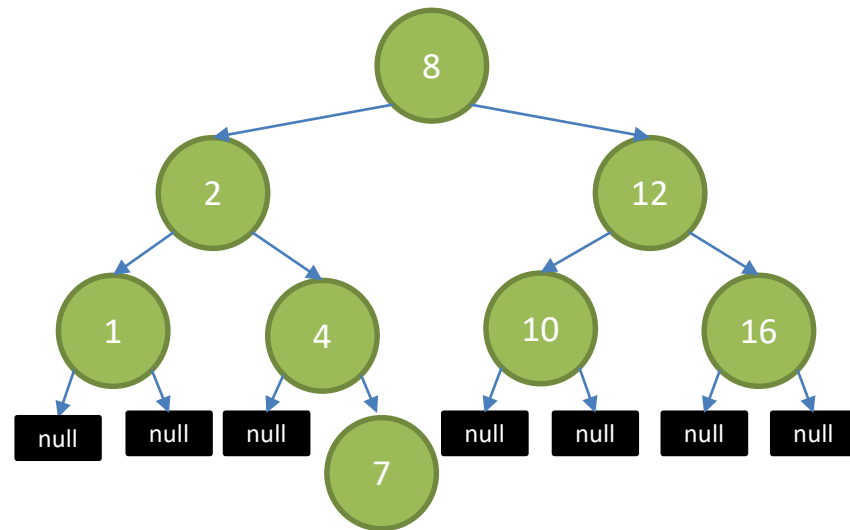
Example "Adding 7"



Binary Search Trees

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 - Start from the Root Node
 - Recursively go left and right until a leaf (NULL element) is found
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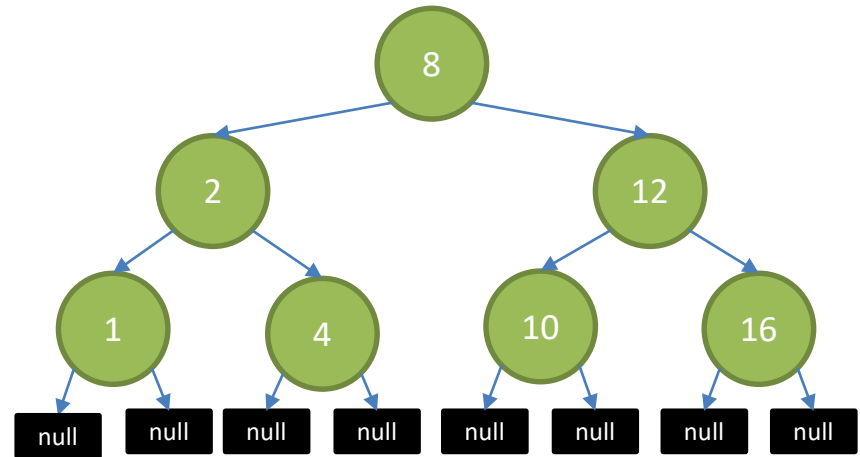
Example "Adding 7"



Binary Search Trees

- Traversals
 - Provide a way to traverse (“walk”) through the data structure
 - Not straight forward like linear structures
- 2 Versions
 - Depth-first (Stack Based)
 - Breadth-first (Queue Based)
- Depth-First
 - Travel deeper until a leaf is reached, then backtrack and do the same for a sibling node
- Breadth-First
 - Travel wider then move to the next level

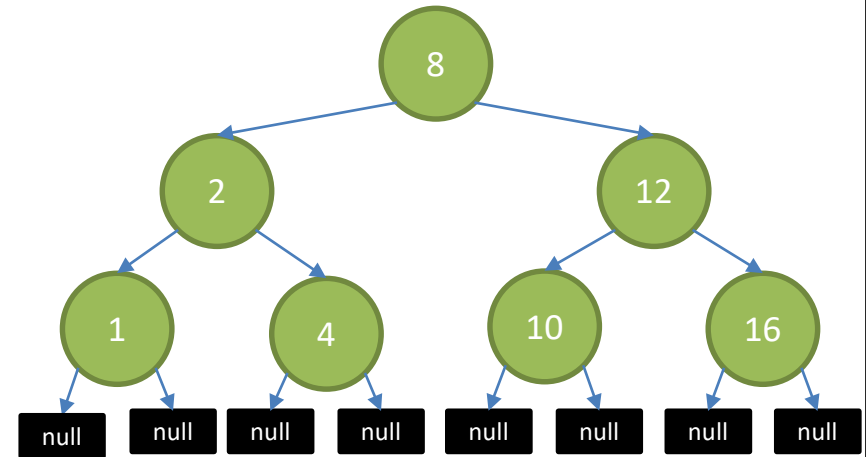
Binary Search Tree



Binary Search Trees

- Depth-First Traversal Operators
 - LEFT: Traversal recursively down the left subtree
 - RIGHT: Traverse recursively down the right subtree
 - PROCESS: Process the node such as printing a value or return true if they are equal
- Depth-First Traversals Types
 - Pre-Order
 - In-Order
 - Post-Order

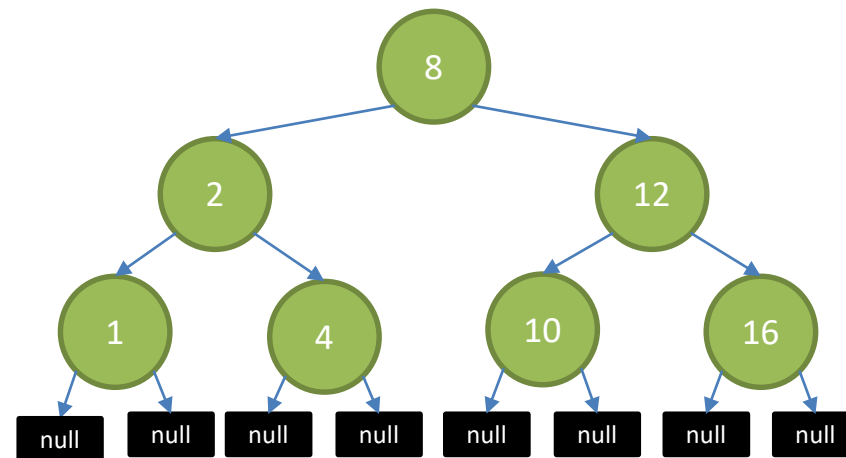
Binary Search Tree



Binary Search Trees

- Pre-Order
 - PROCESS, LEFT, RIGHT
 - “Pass by the Left”
- In-Order
 - LEFT, PROCESS, RIGHT
 - “Pass Underneath”
- Post-Order
 - LEFT, RIGHT, PROCESS
 - “Pass by the Right”

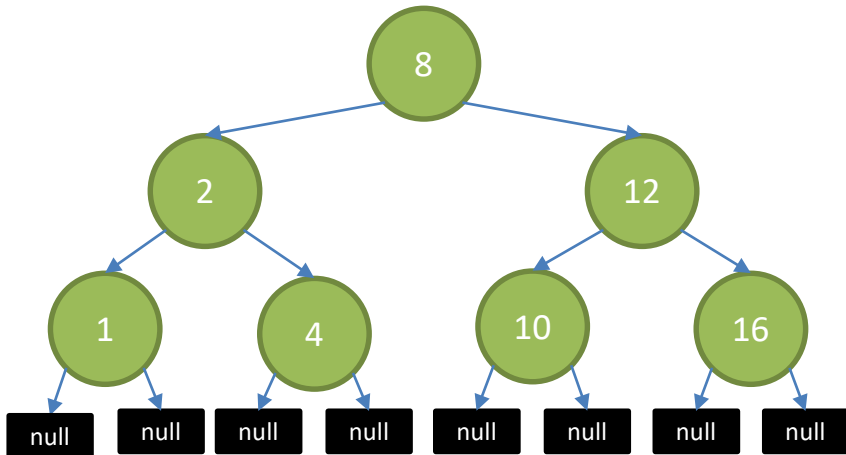
Binary Search Tree



Binary Search Trees

Stack Example

Binary Search Tree



Main Method

Call Stack

Binary Search Trees

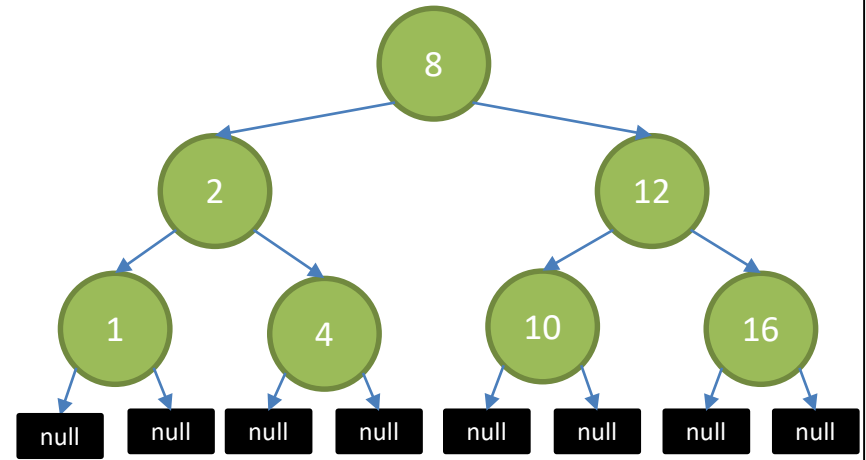
Stack Example

```
public void printPreorder()  
{  
    printPreorder(root);  
}
```

Main Method

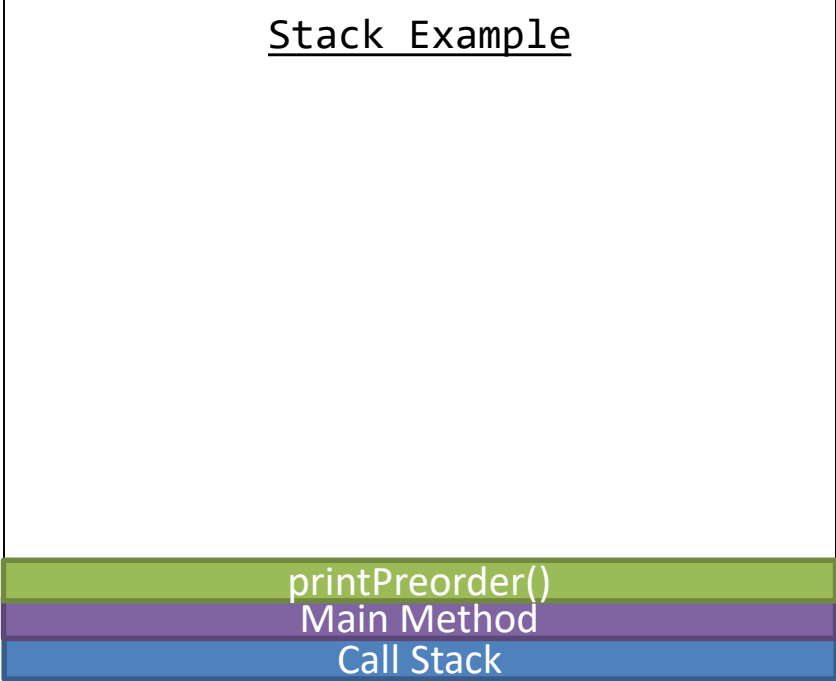
Call Stack

Binary Search Tree

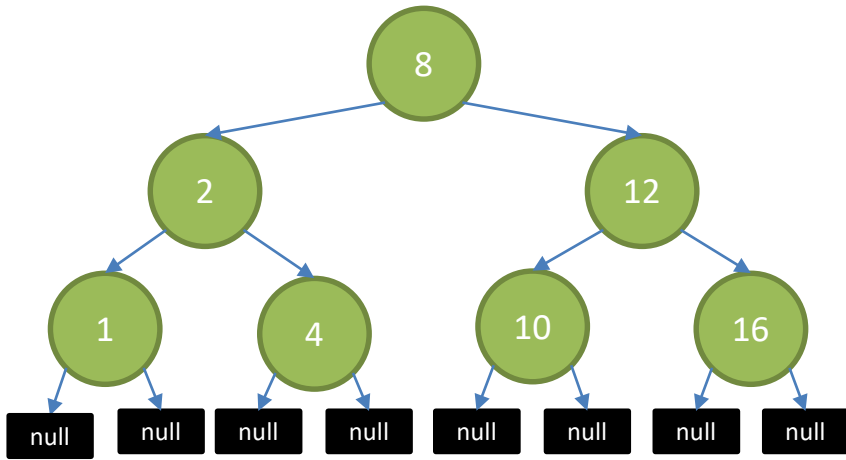


Binary Search Trees

Stack Example



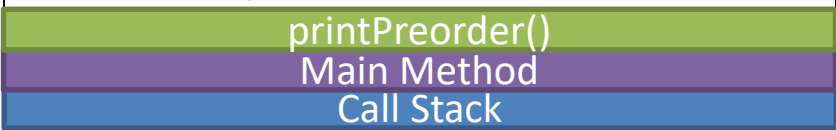
Binary Search Tree



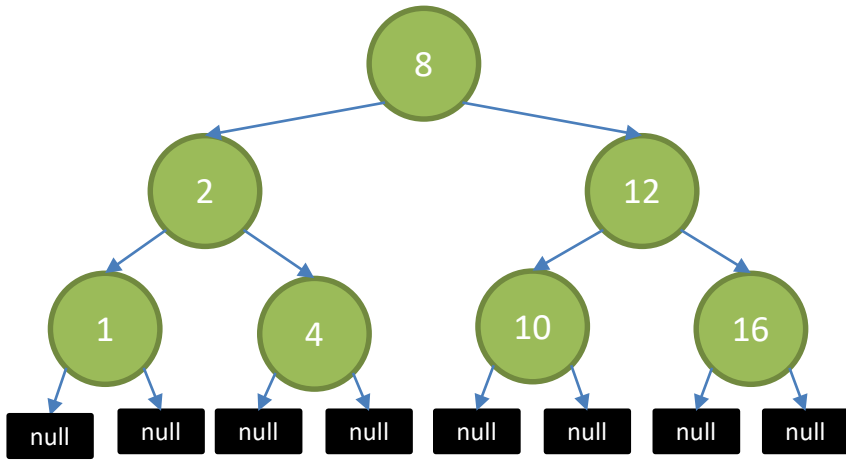
Binary Search Trees

Stack Example

```
private void printPreorder(Node aNode)
{
    if(aNode == null)
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    System.out.println(aNode.data);
    printPreorder(aNode.leftChild);
    printPreorder(aNode.rightChild);
}
```



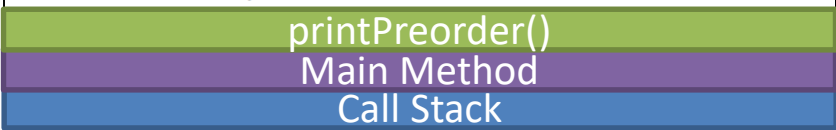
Binary Search Tree



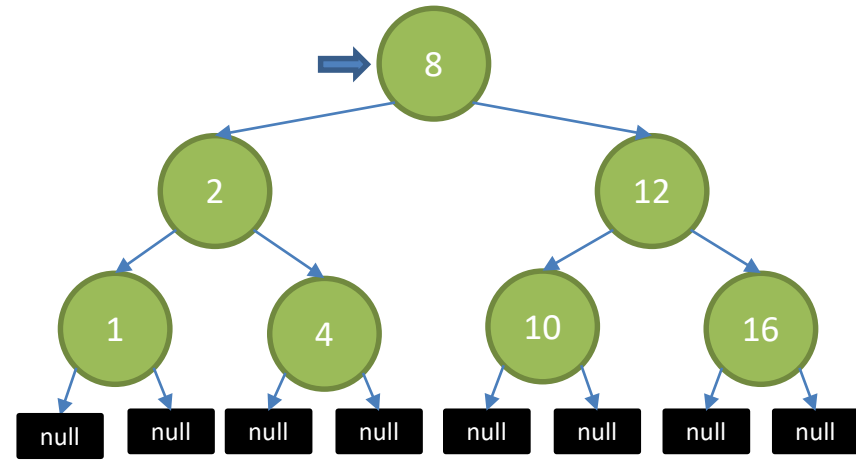
Binary Search Trees

Stack Example

```
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    if(aNode == null)
        return;
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}
```



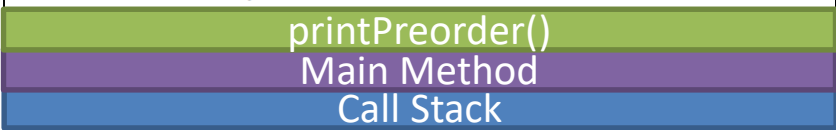
Binary Search Tree



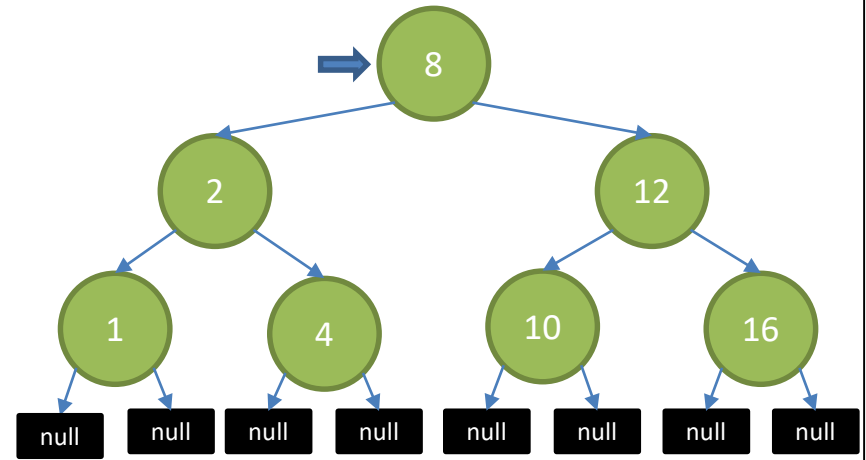
Binary Search Trees

Stack Example

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Binary Search Tree



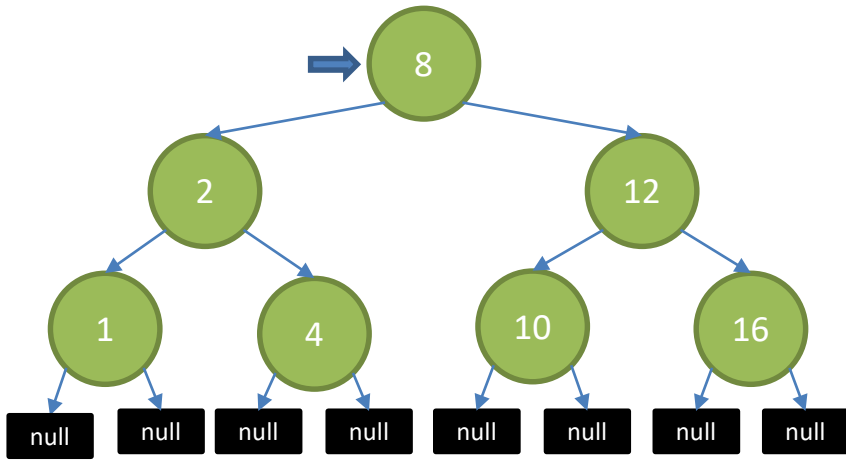
Binary Search Trees

Stack Example

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Binary Search Tree

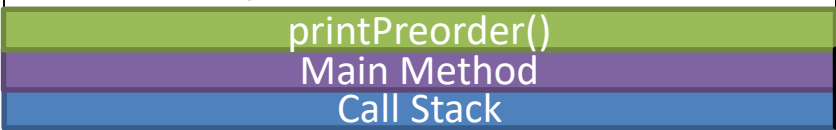


Console: 8

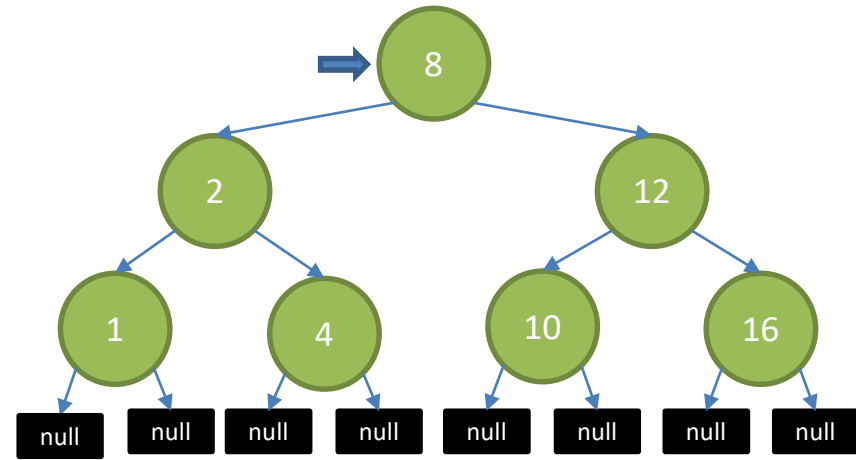
Binary Search Trees

Stack Example

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private void printPreorder(Node aNode)
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```



Binary Search Tree



Console: 8

Binary Search Trees

Stack Example

```

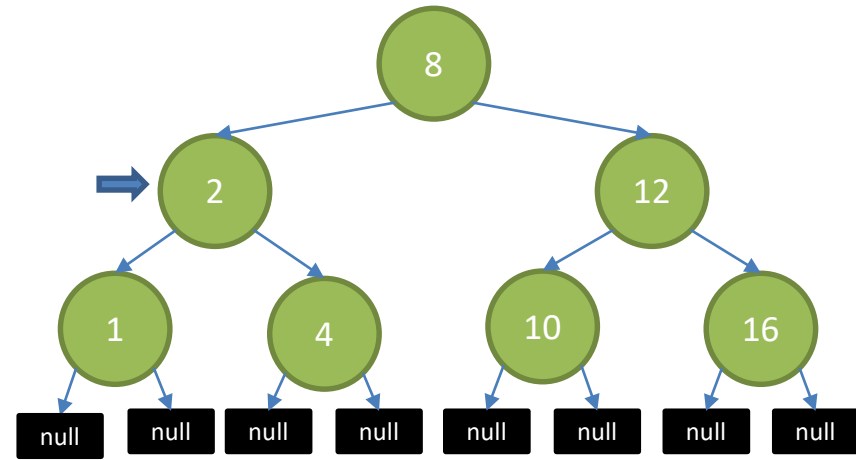
➔ private void printPreorder(Node aNode)
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private void printPreorder(Node aNode)
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}

```



Binary Search Tree



Console: 8

Binary Search Trees

Stack Example

```

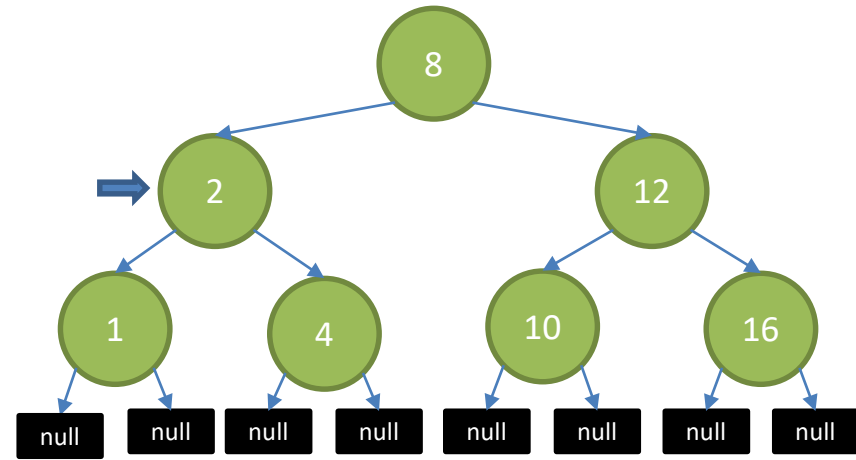
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```



Binary Search Tree



Console: 8

Binary Search Trees

Stack Example

```

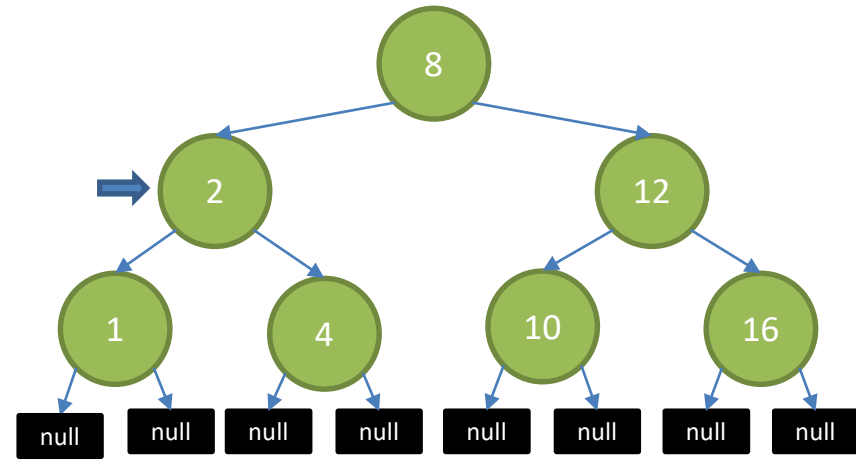
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}

```



Binary Search Tree



Console: 8 2

Binary Search Trees

Stack Example

```

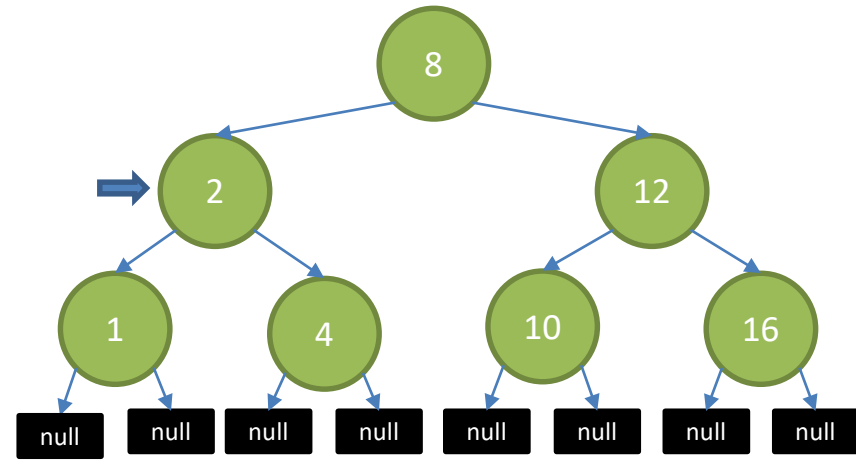
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```



Binary Search Tree



Console: 8 2

Binary Search Trees

Stack Example

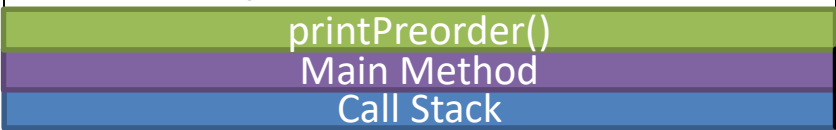
```

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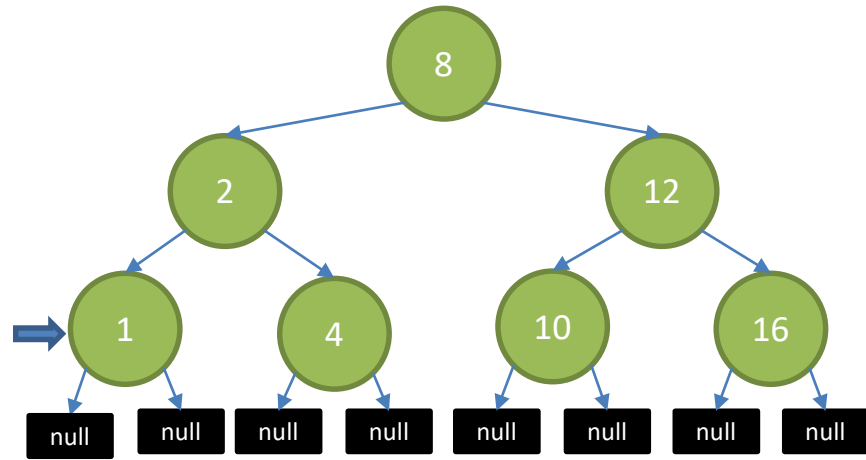
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```



Binary Search Tree



Console: 8 2

Binary Search Trees

Stack Example

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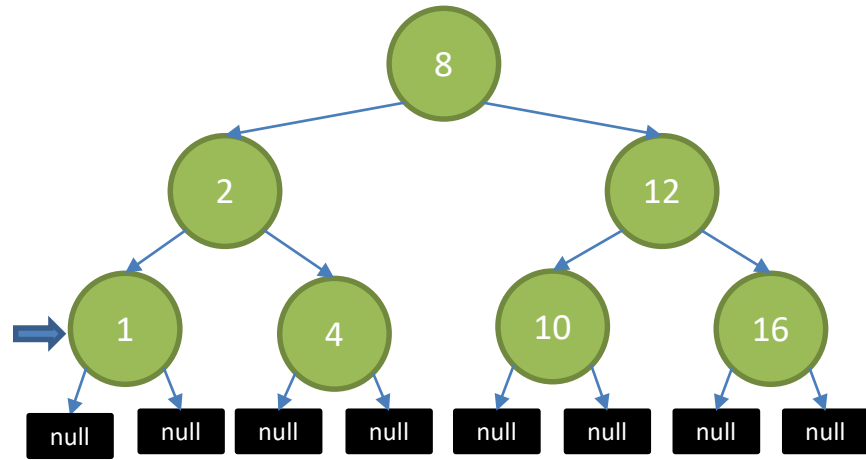
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```



Binary Search Tree



Console: 8 2

Binary Search Trees

Stack Example

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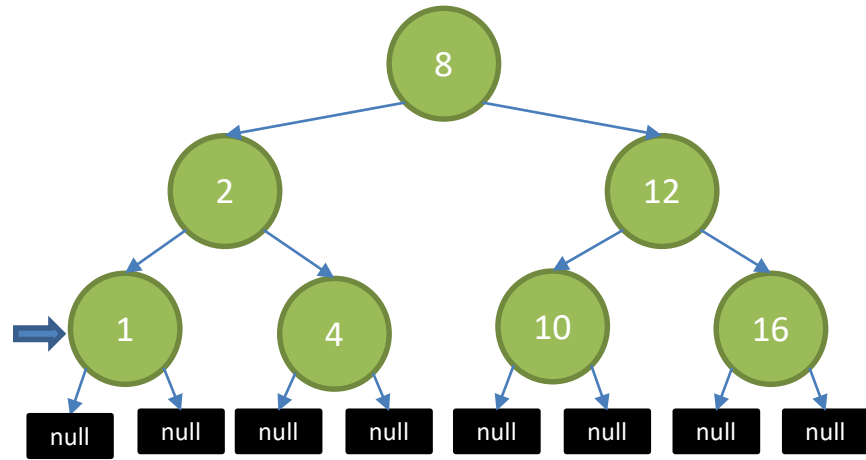
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```



Binary Search Tree



Console: 8 2 1

Binary Search Trees

Stack Example

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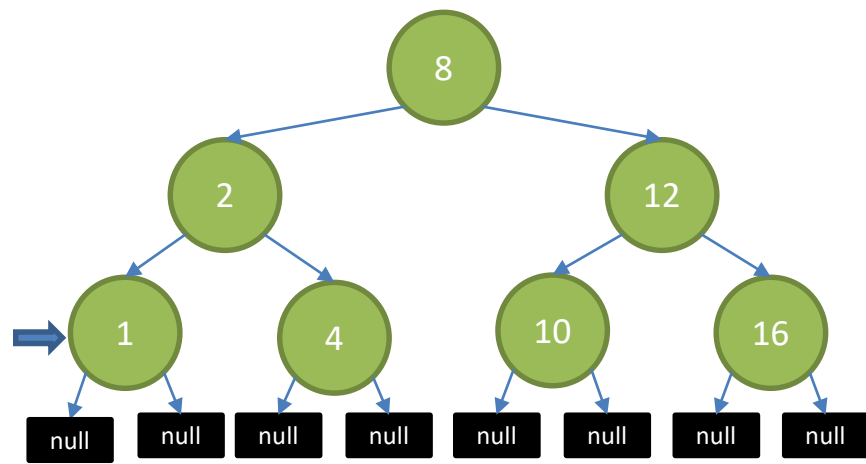
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```



Binary Search Tree



Console: 8 2 1

Binary Search Trees

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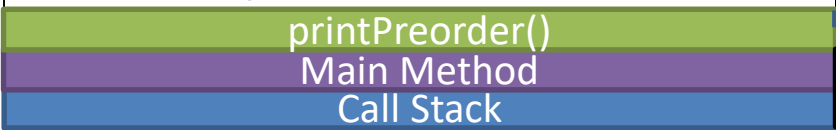
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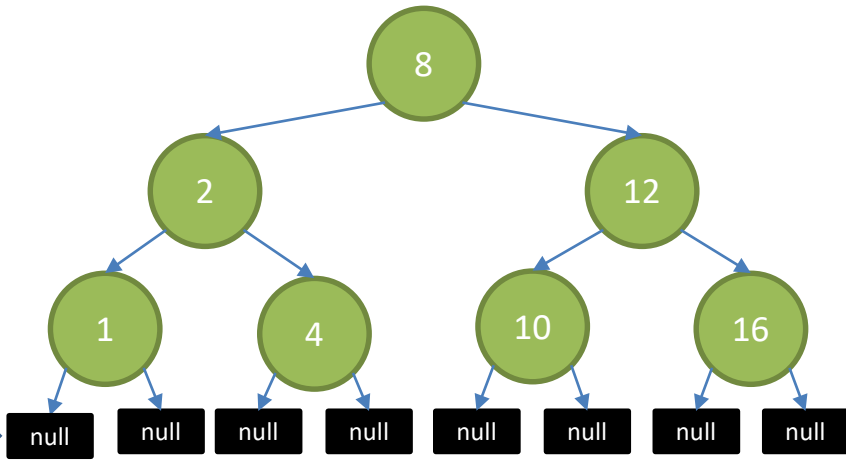
private void printPreorder(Node aNode)
{
    if(aNode == null)
        return;
    System.out.println(aNode.data);
    printPreorder(aNode.leftChild);
    printPreorder(aNode.rightChild);
}

➡

```



Binary Search Tree



Console: 8 2 1

Binary Search Trees

```

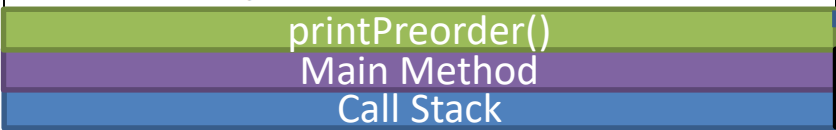
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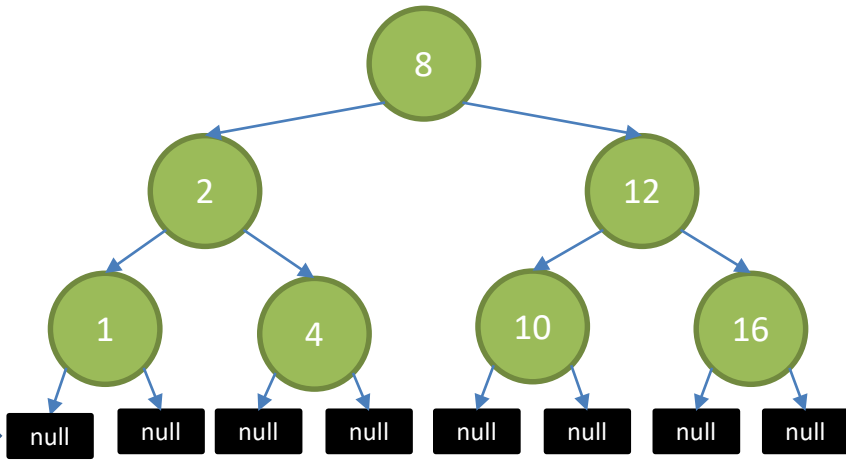
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```



Binary Search Tree



Console: 8 2 1

Binary Search Trees

```

private void printPreorder(Node aNode)
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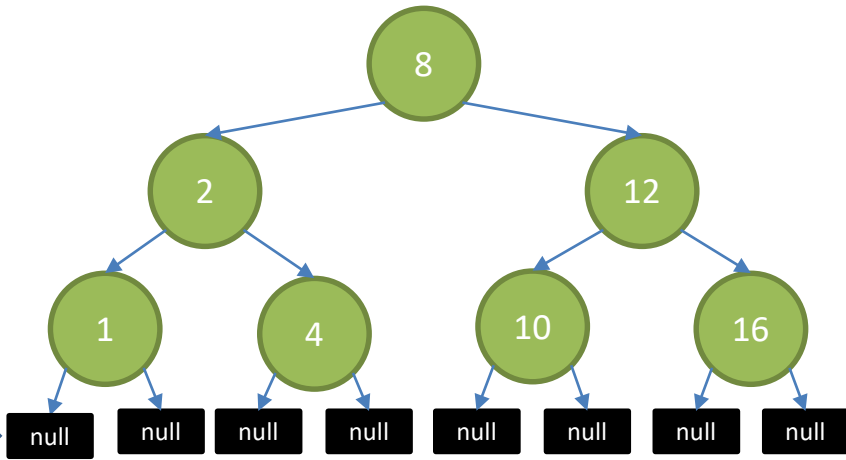
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```



Binary Search Tree



Console: 8 2 1

Binary Search Trees

Stack Example

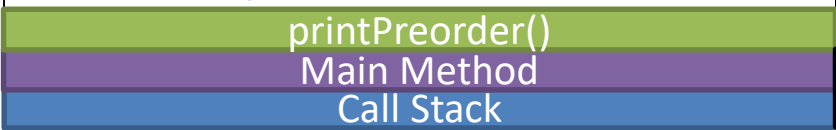
```

private void printPreorder(Node aNode)
{
    if(aNode == null)
        return;
    System.out.println(aNode.data);
    printPreorder(aNode.leftChild);
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}

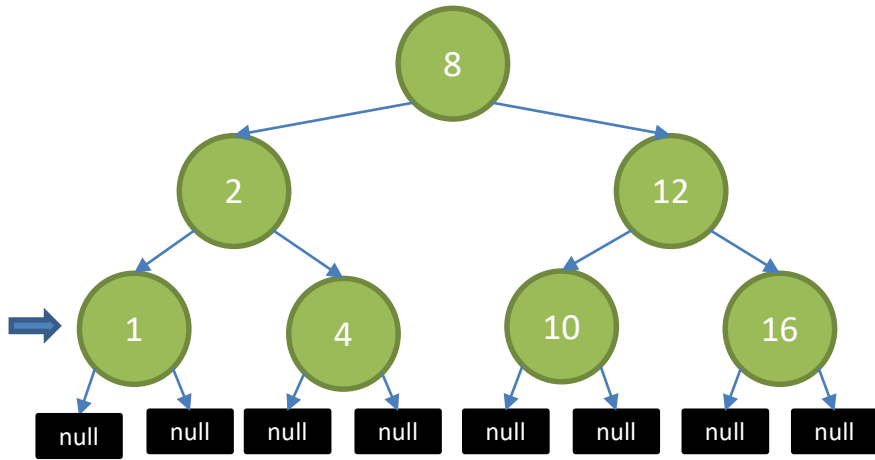
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Binary Search Tree



Console: 8 2 1

Binary Search Trees

Stack Example

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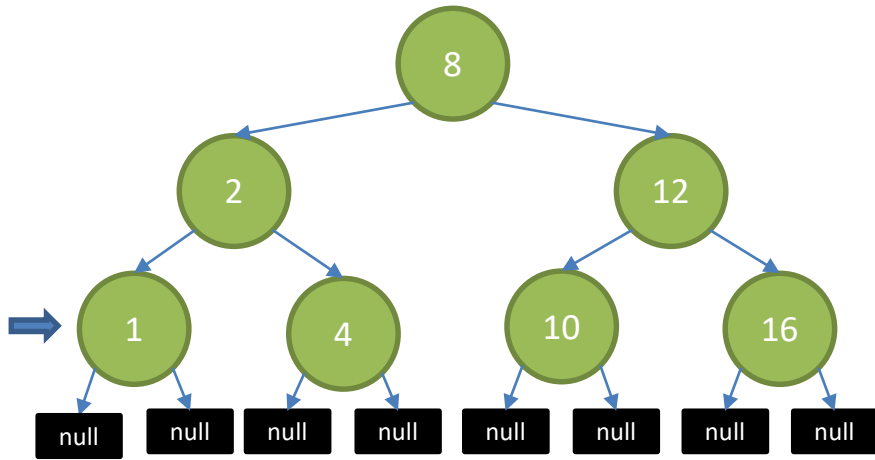
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Binary Search Tree



Console: 8 2 1

Binary Search Trees

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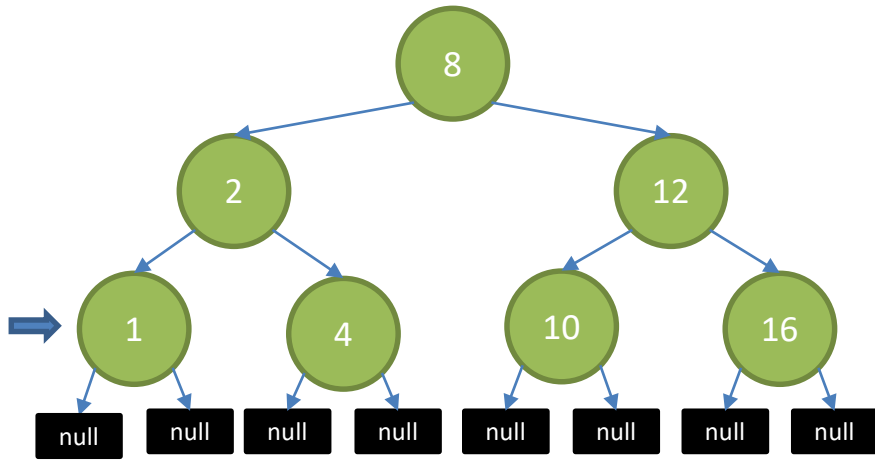
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Binary Search Tree



Console: 8 2 1

Binary Search Trees

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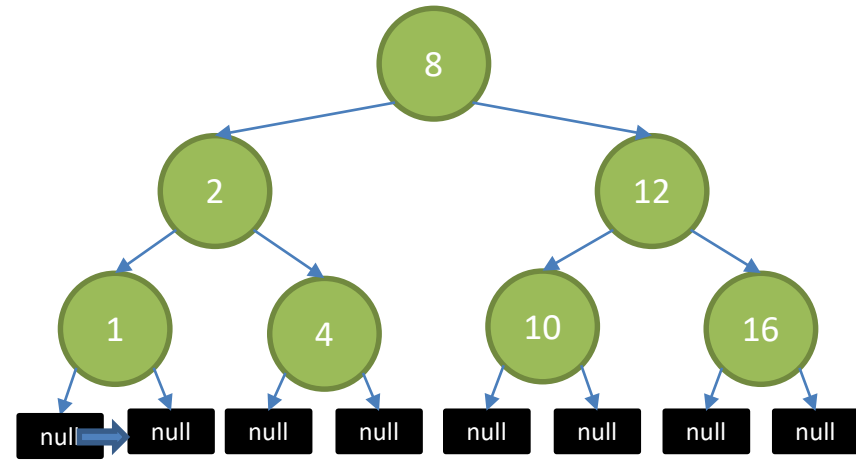
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➡

```



Binary Search Tree



Console: 8 2 1

Binary Search Trees

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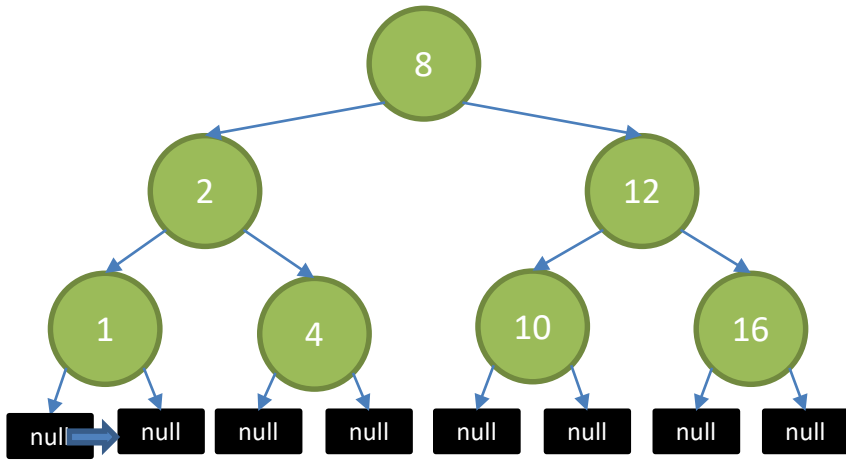
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Binary Search Tree



Console: 8 2 1

Binary Search Trees

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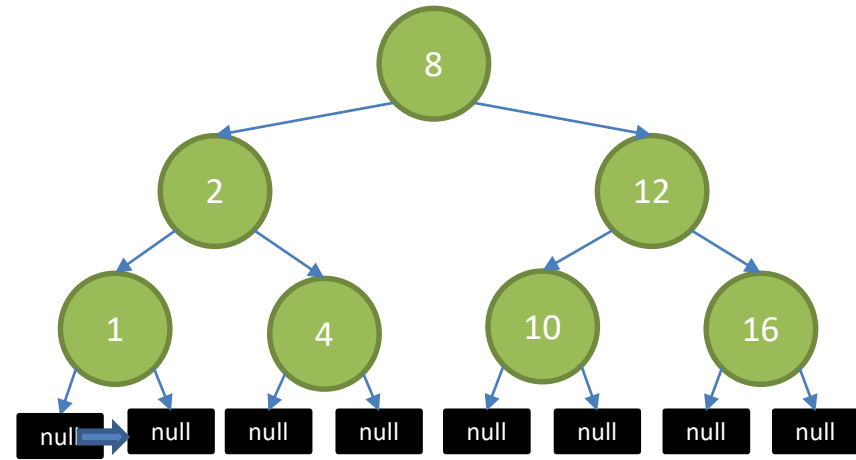
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Binary Search Tree



Console: 8 2 1

Binary Search Trees

Stack Example

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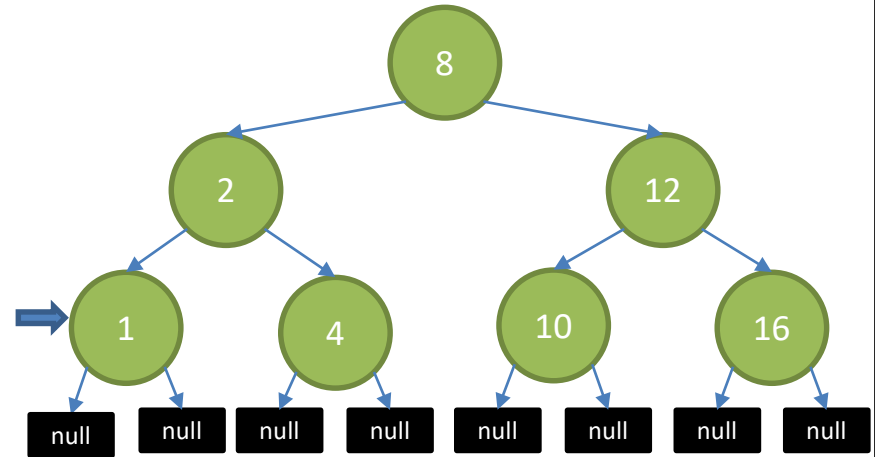
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}
```



printPreorder()
Main Method
Call Stack

Binary Search Tree



Console: 8 2 1

Binary Search Trees

Stack Example

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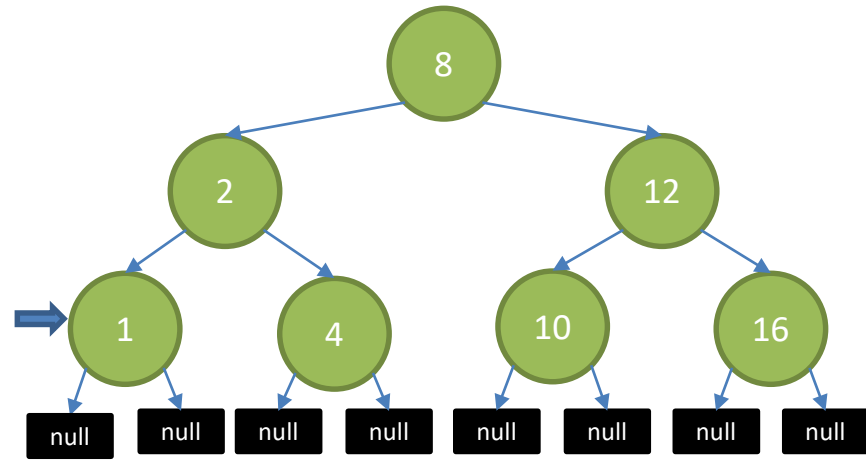
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}

```



Binary Search Tree



Console: 8 2 1

Binary Search Trees

Stack Example

```

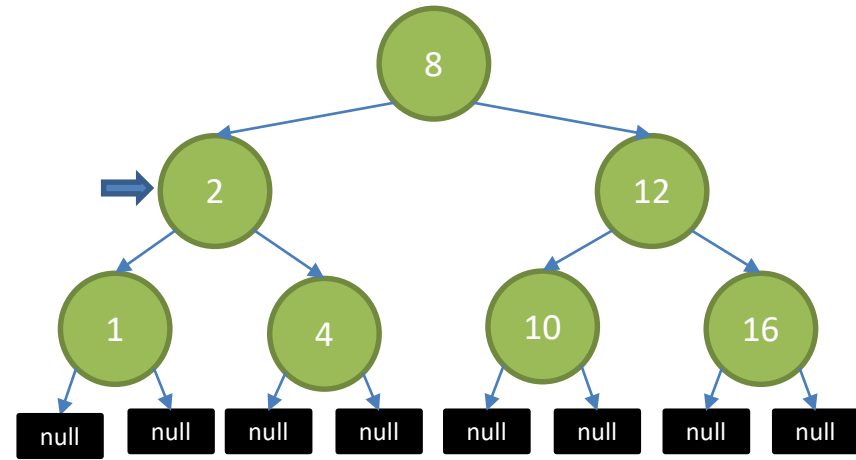
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Binary Search Tree



Console: 8 2 1

Binary Search Trees

Stack Example

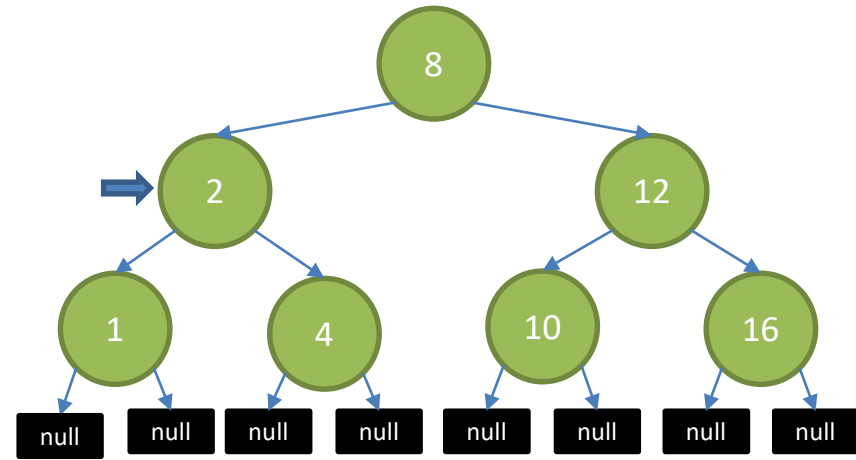
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Binary Search Tree



Console: 8 2 1

Binary Search Trees

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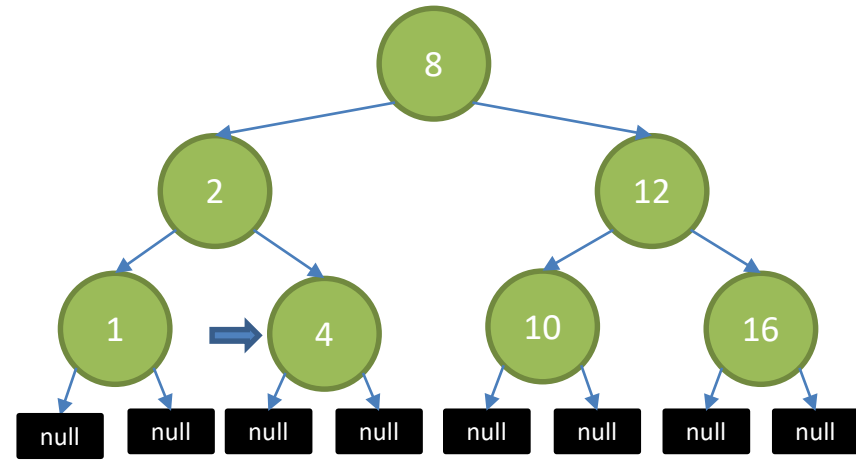
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Binary Search Tree

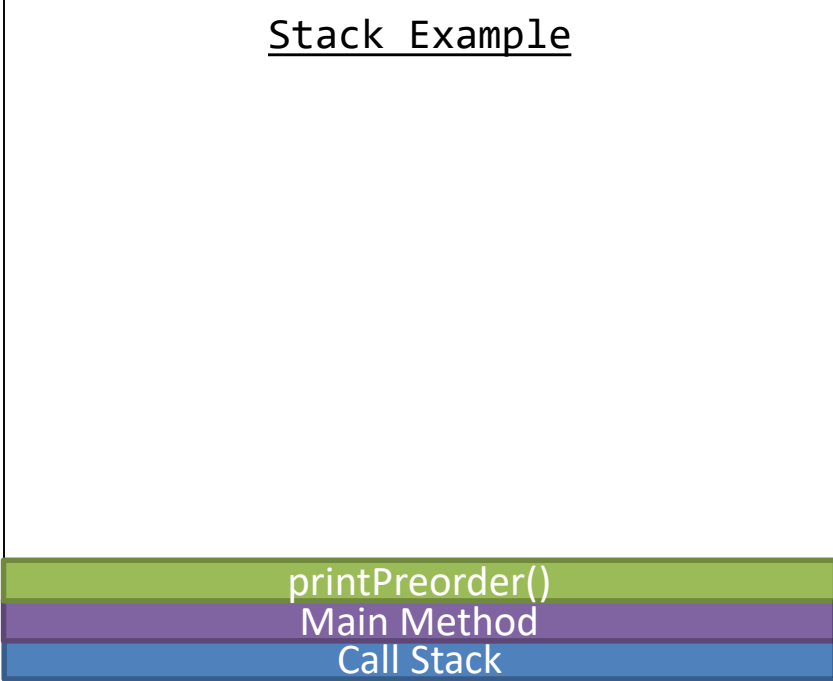


Console: 8 2 1

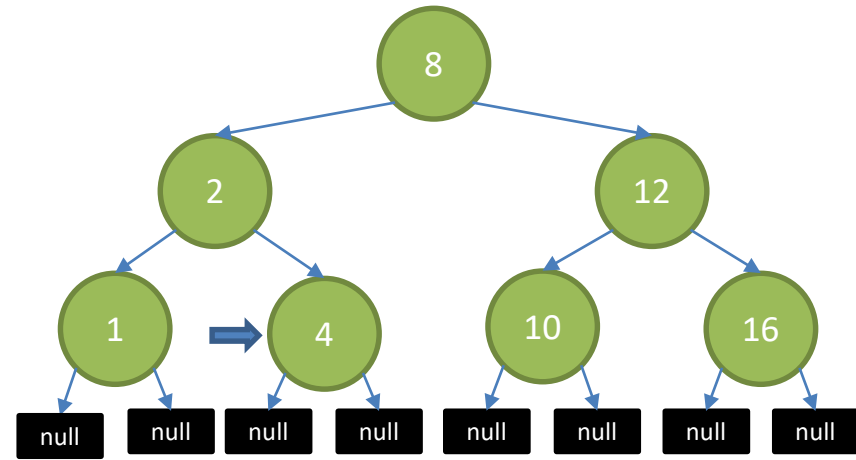
A Few Traversals Later

Binary Search Trees

Stack Example



Binary Search Tree



Console: 8 2 1 4 12 10 16

Binary Search Trees

- Breadth-First Traversal
 1. Add the Root node to a queue
 2. Dequeue a Node
 3. Process the Node's Value
 4. Enqueue the Node's Children
 5. Repeat Step 2 until the queue is empty

Binary Search Tree

