



# Searching, Sorting, Complexity Part 02

# Efficiency

- Efficiency
  - Producing desired results with little to no waste
  - Well organized and prevents wasteful use of a resource
- Resources
  - Time
  - Space
- How do we measure efficiency?
  - Algorithms do not require computers

# Complexity

- Complexity
  - Classifies Computational Problems based on inherent difficulty
  - Relates problems to each other
  - Time and Space
- Asymptotic Analysis
  - A way to describe a *limiting* behavior / function
  - Limits in math are a value that a function *approaches* as the input *approaches* some value
  - Time and Space Complexity

# Big O Notation

- Theoretical upper bound of an algorithm
- The “Worst Case” scenario
- Let  $f$  and  $g$  be functions defined on some subset of real numbers

$$***f(n) = O(g(n)) where n \in \mathbb{R} as n \rightarrow \infty***$$

- Let  $M$  be a constant that's sufficiently large then we can say

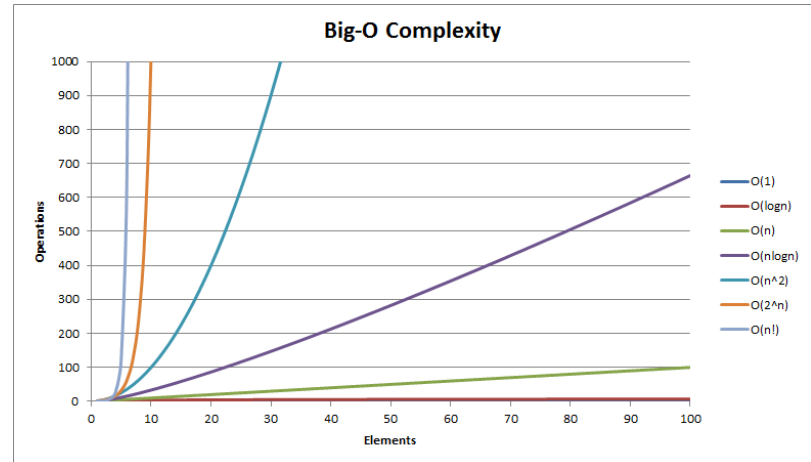
$$***|f(n)| \leq M|g(n)| for all n \geq n_0***$$

# Big O Notation

- Common Big O Complexities

- $O(1)$  – Constant
- $O(\log(n))$  – Logarithmic
- $O(n)$  – Linear
- $O(n \log n)$  – Linearithmic
- $O(n^2)$  – Quadratic
- $O(2^n)$  – Exponential “Bad”
- $O(n!)$  – Factorial “Really Bad”

## Big O



# Sorting Algorithms

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## Example

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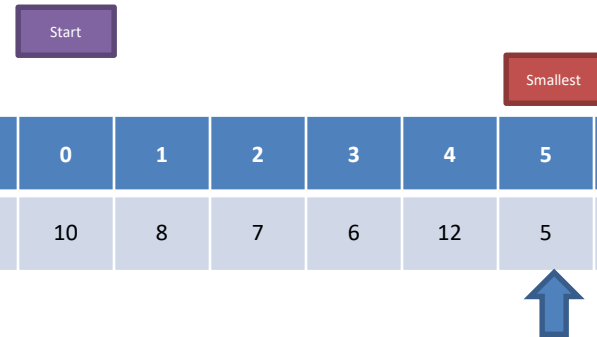
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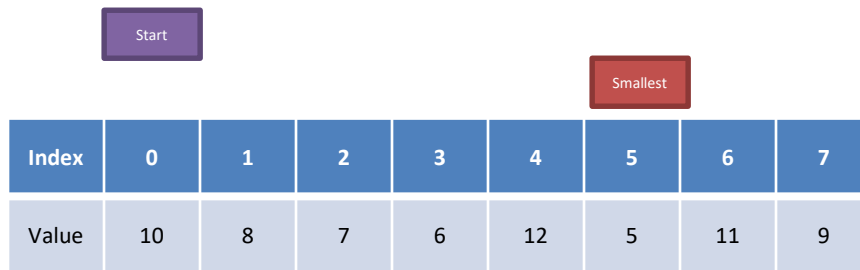
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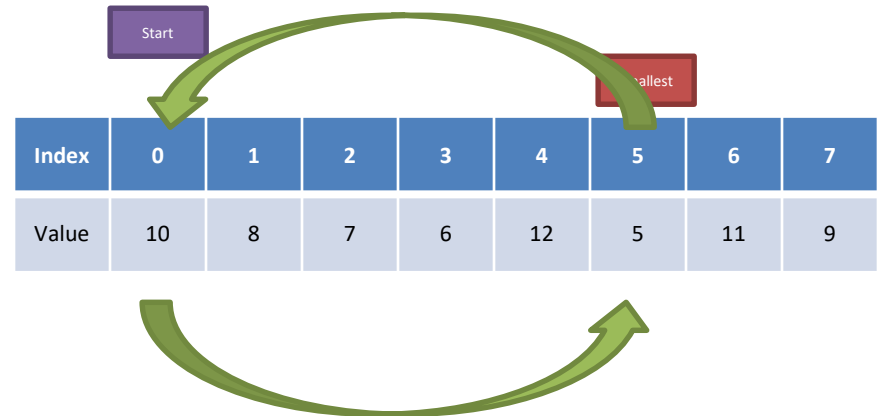


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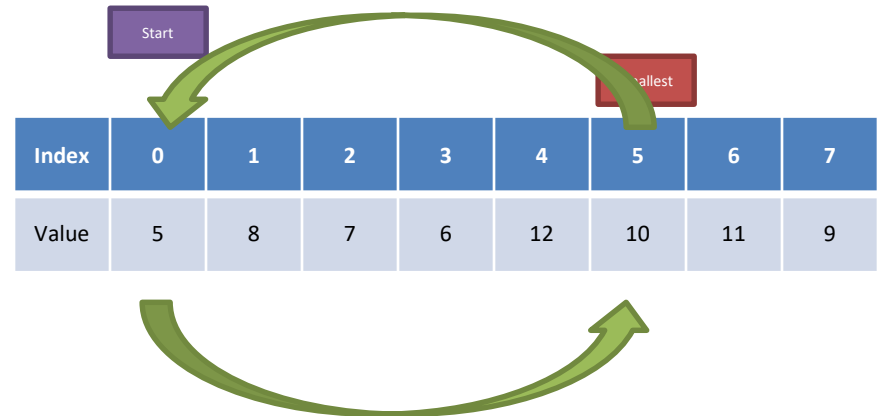
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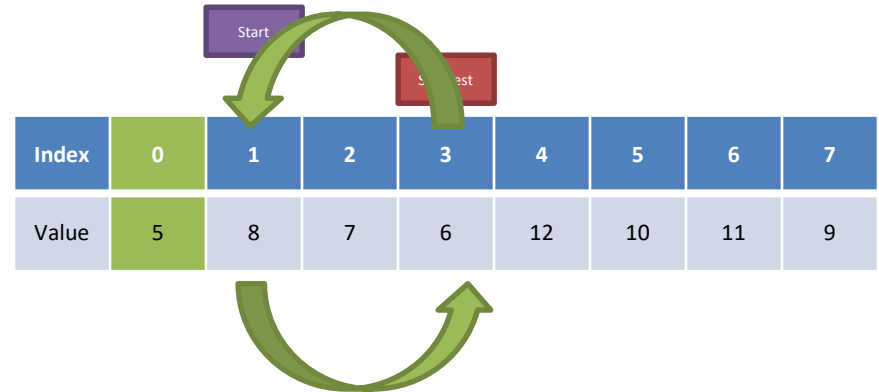
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# Selection Sort Complexity

- Worst Case
  - Sorted in Descending Order
- Operations
  - Search for smallest value =  $n$
  - Search for the next smallest value =  $n - 1$
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  - Search for the largest element =  $1$

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$O(n^2)$

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2. Check each index with its neighbor (index+1)
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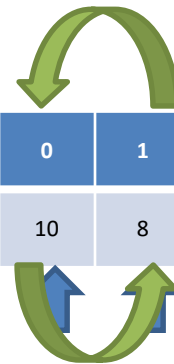


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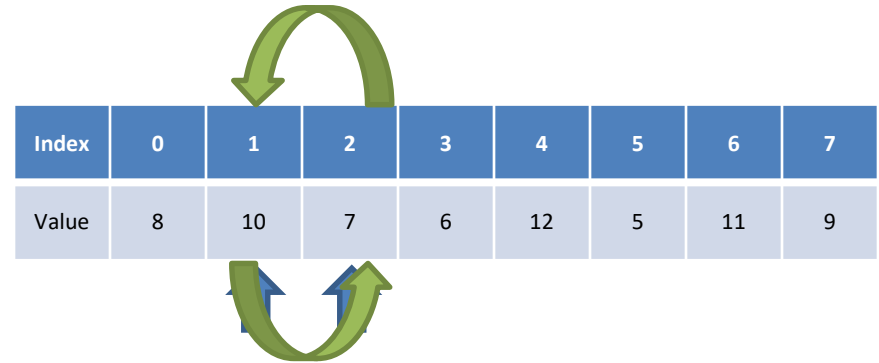


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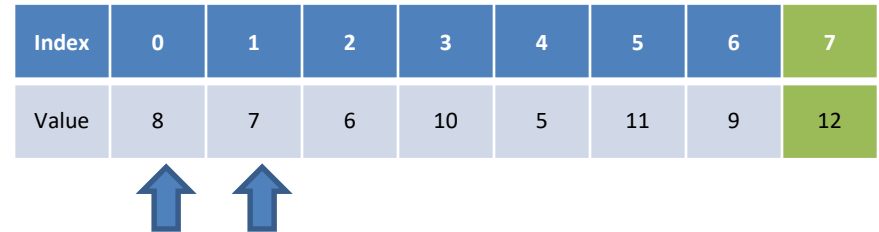
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- Operations
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Complexity

$$O(n^2)$$

Can we do better?

# Sorting Algorithms

- Merge Sort

1. Recursively split the array in half until single elements remain
2. Merge two smaller arrays and return the sorted result
  1. Create an array of combined size
  2. Add elements from the two smaller arrays into the combined array in sorted order
3. Repeat Step 2 until the final array is reached

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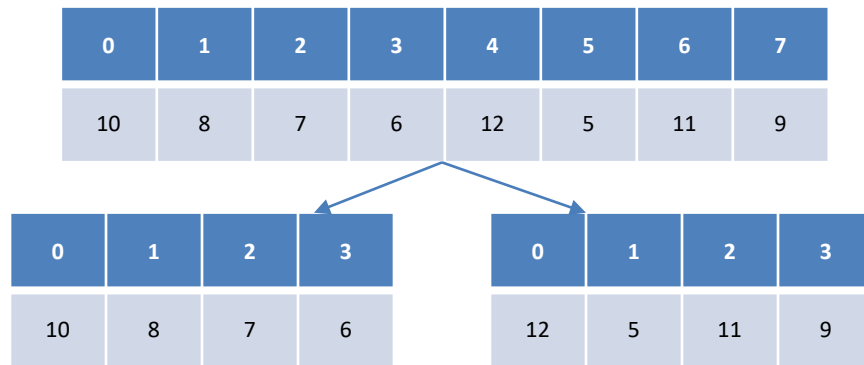
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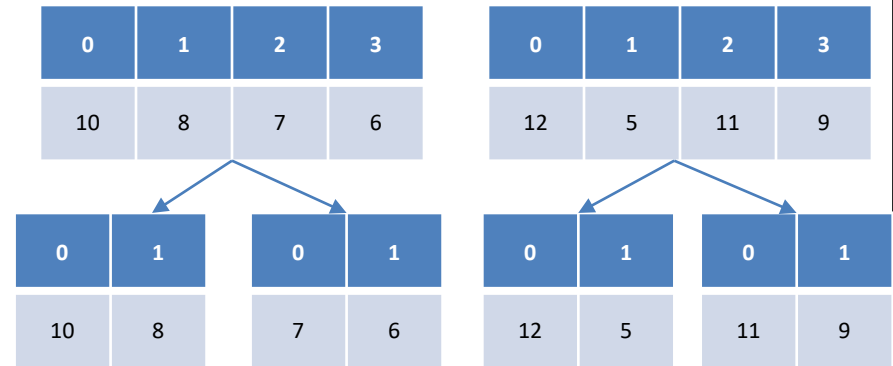


# Sorting Algorithms

- Merge Sort

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2. Merge two smaller arrays and return the sorted result
  1. Create an array of combined size
  2. Add elements from the two smaller arrays into the combined array in sorted order
3. Repeat Step 2 until the final array is reached

## Example



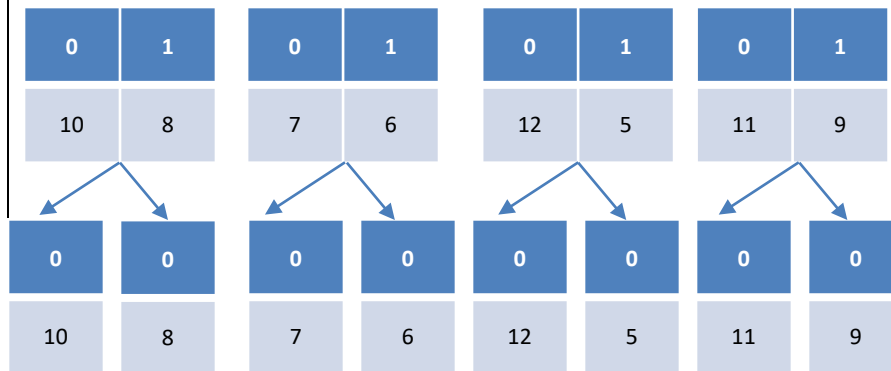


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## Example

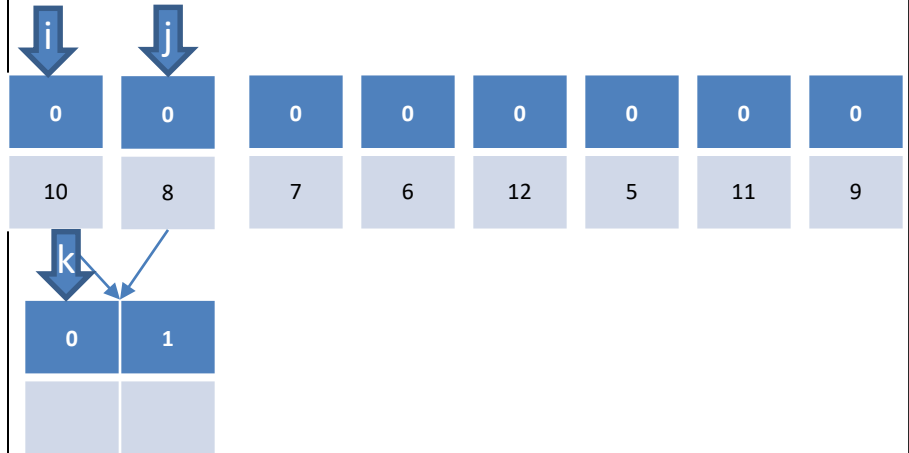
0	0	0	0	0	0	0	0
10	8	7	6	12	5	11	9

# Sorting Algorithms

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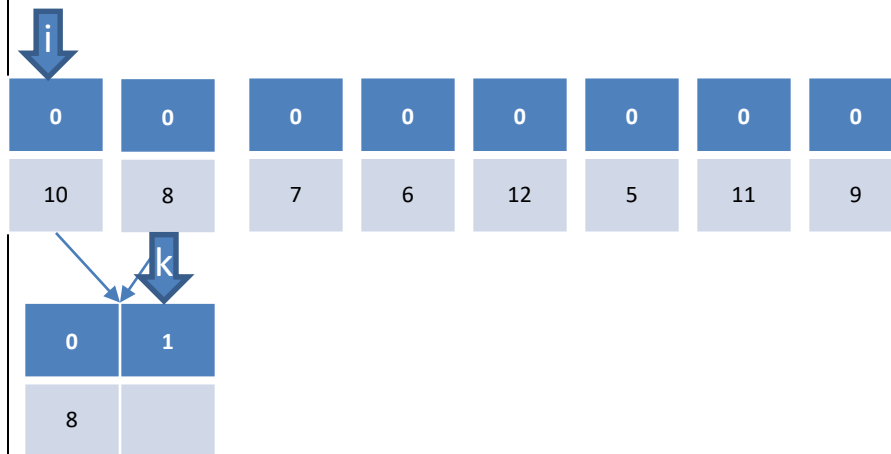


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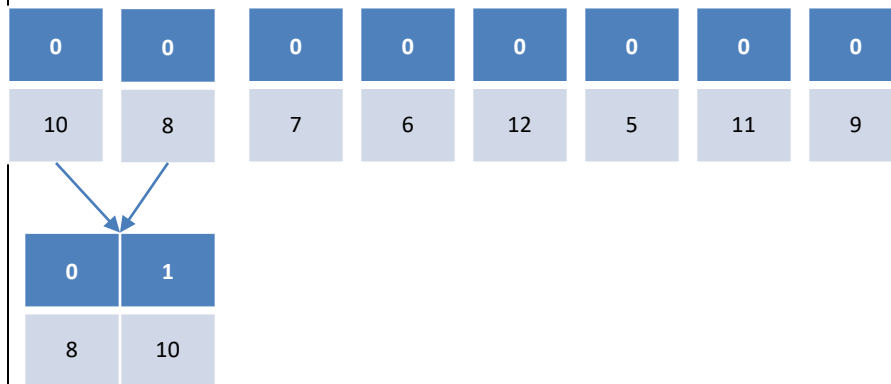


# Sorting Algorithms

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## Example

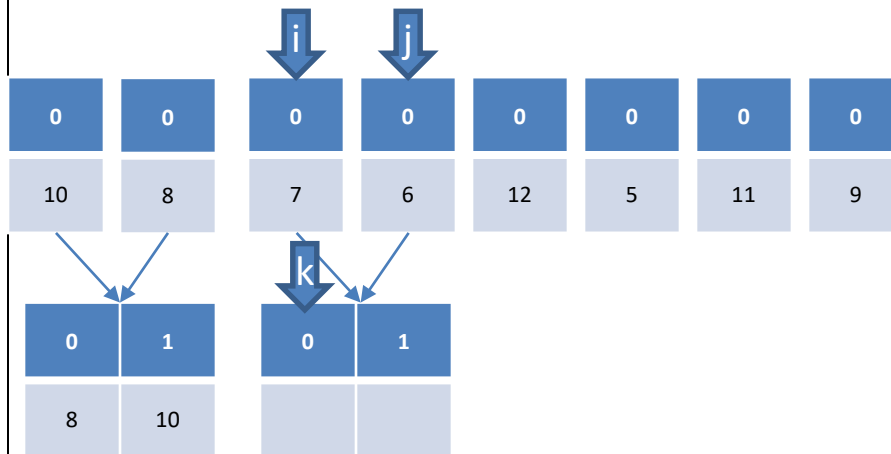


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## Example

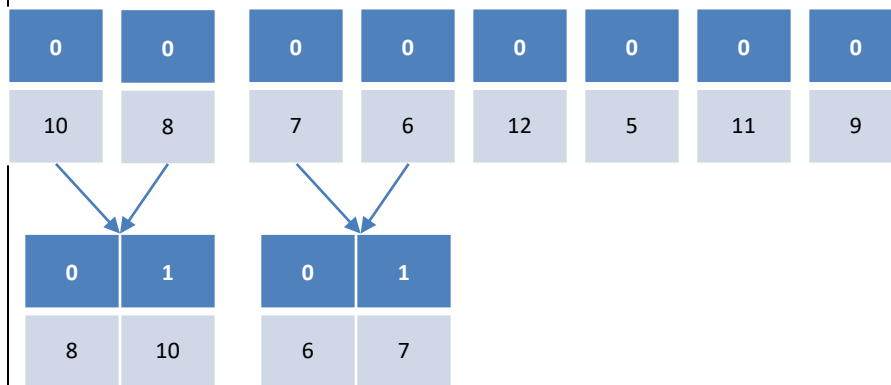


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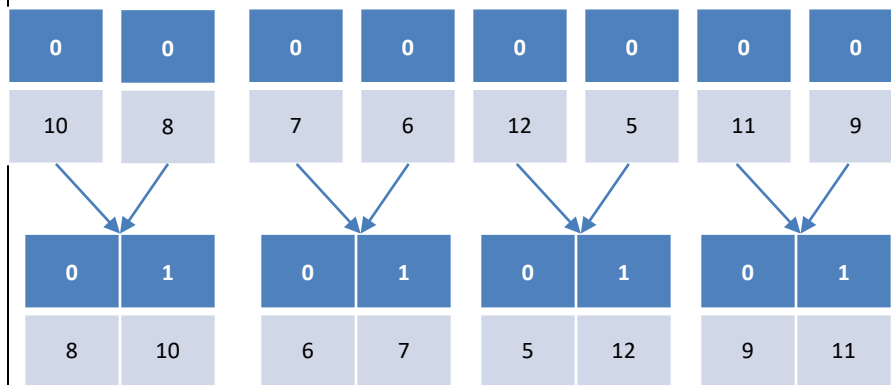


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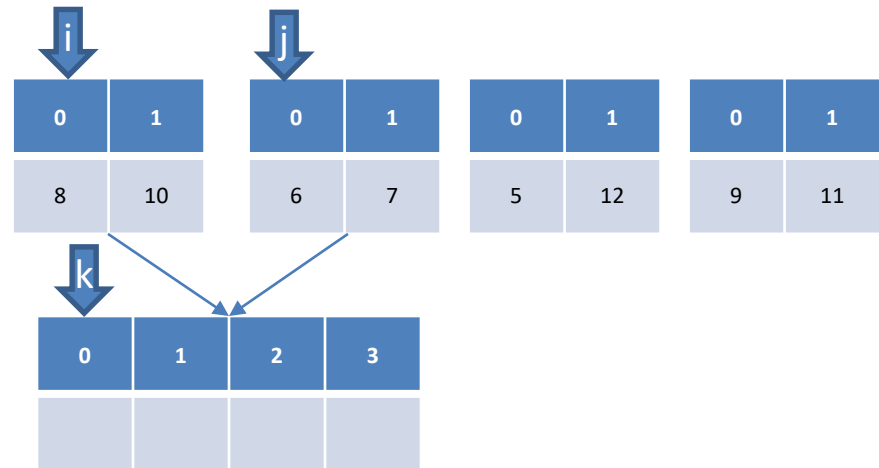
0	1	0	1	0	1	0	1
8	10	6	7	5	12	9	11

# Sorting Algorithms

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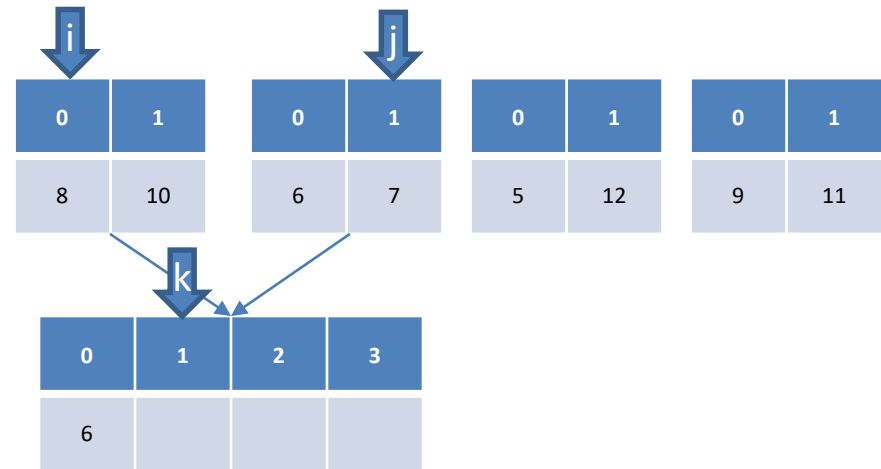


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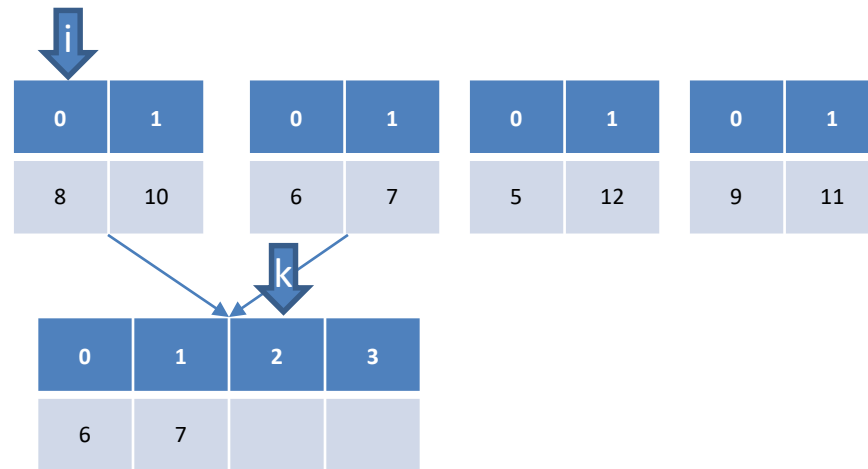


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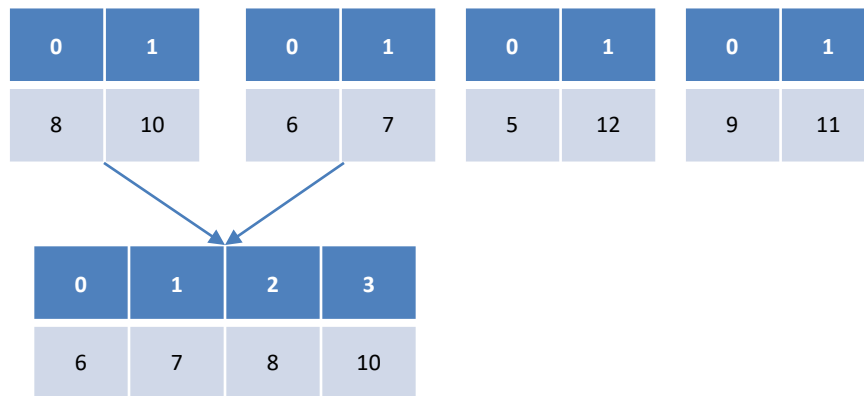


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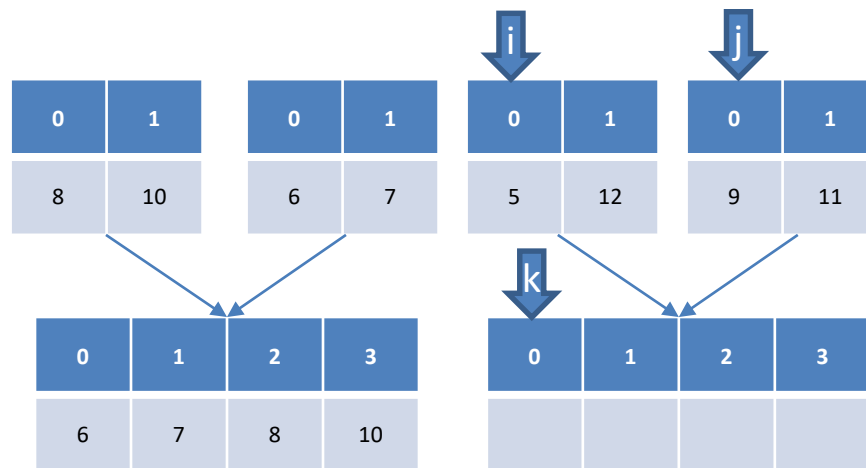


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## Example

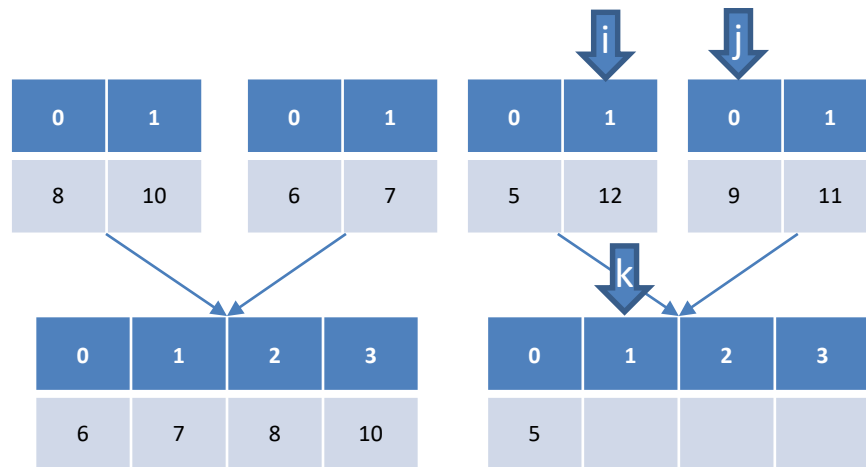


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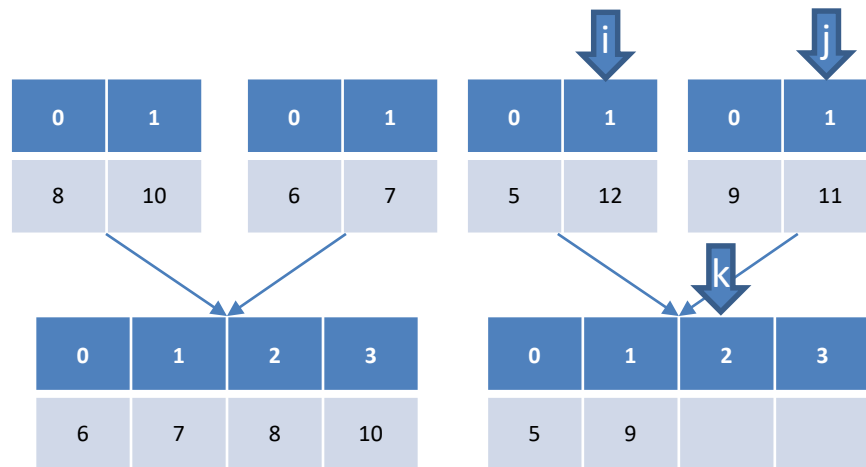


# Sorting Algorithms

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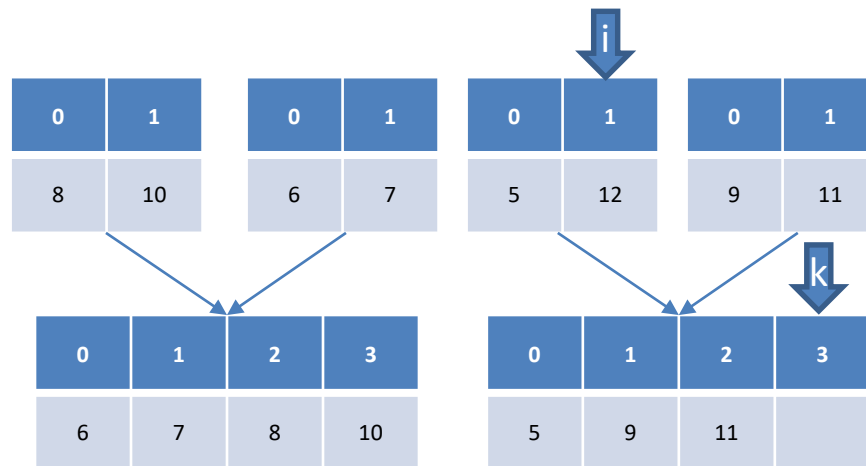


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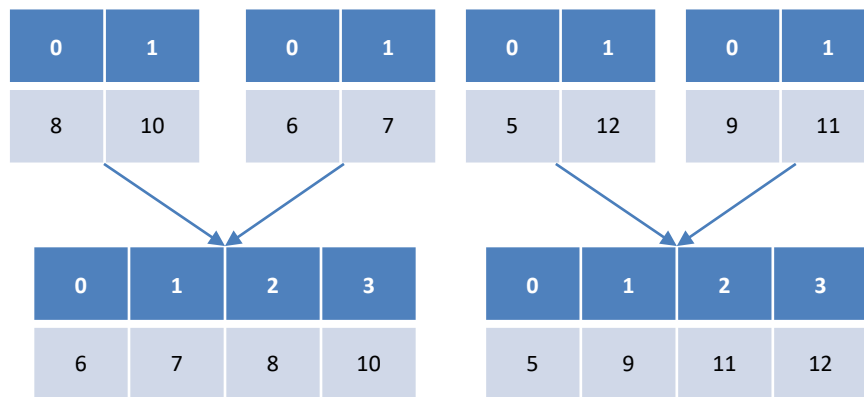


# Sorting Algorithms

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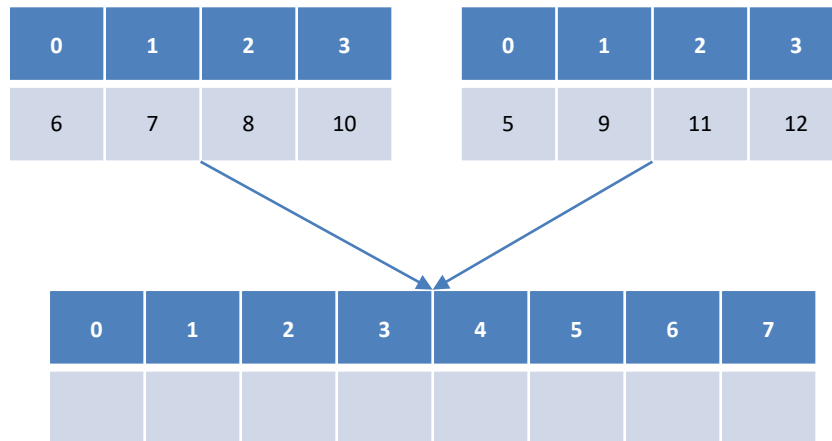


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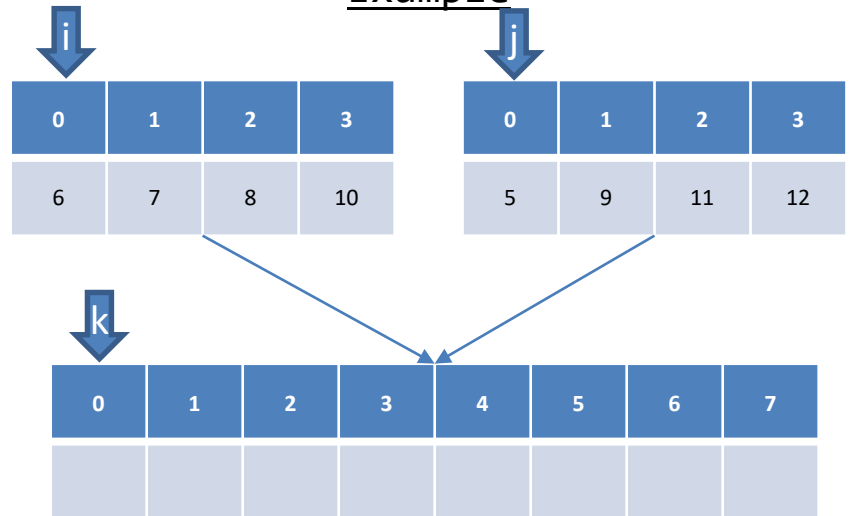


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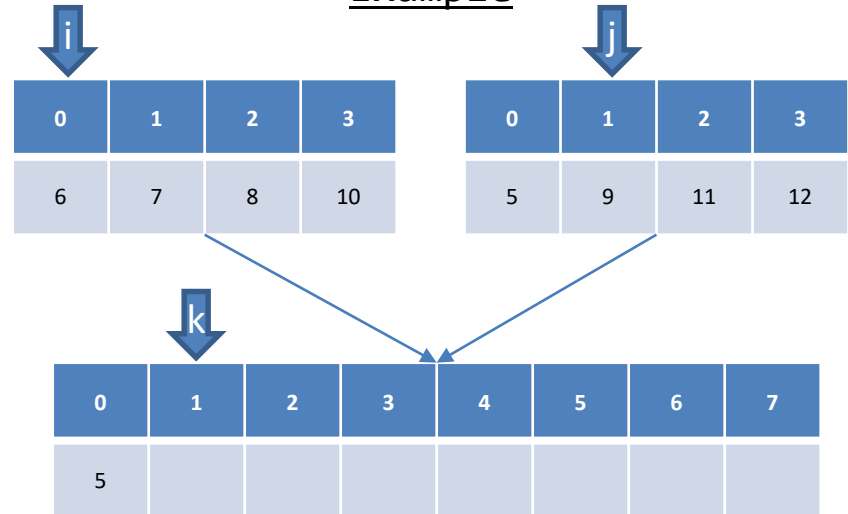


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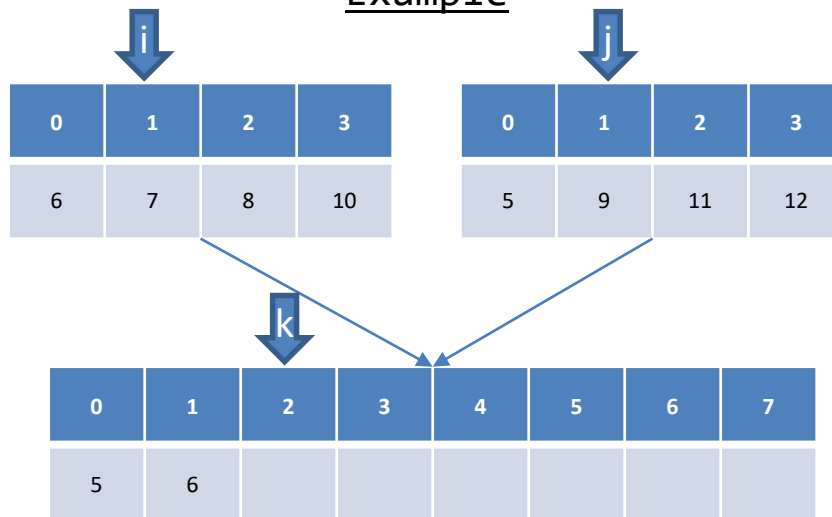


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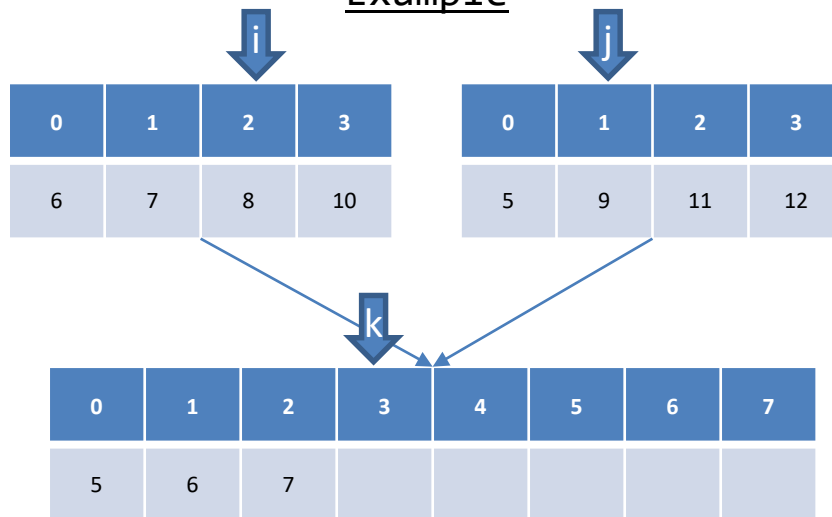


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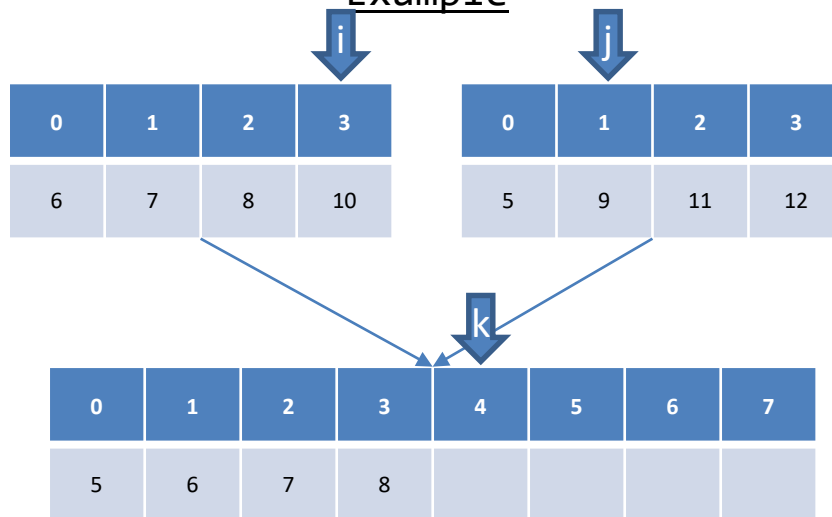


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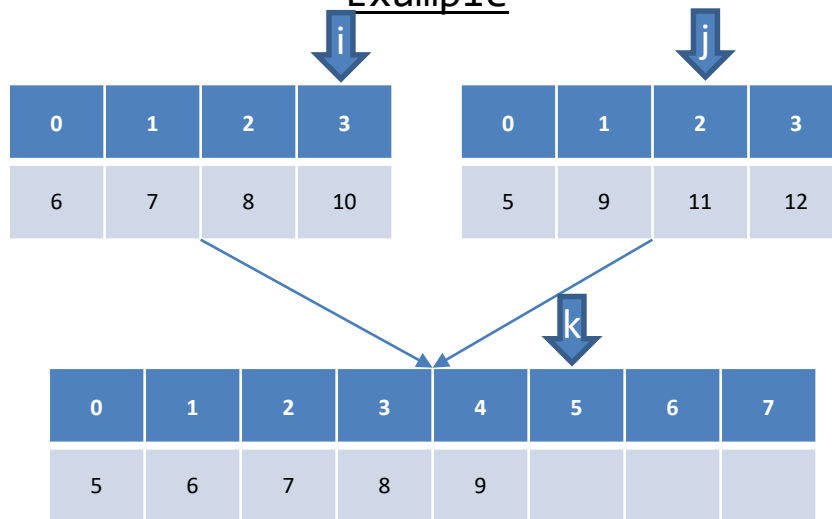


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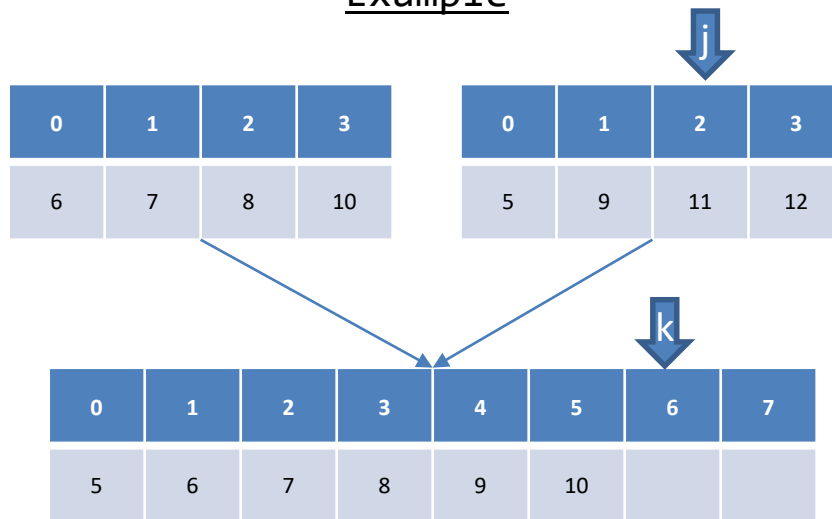


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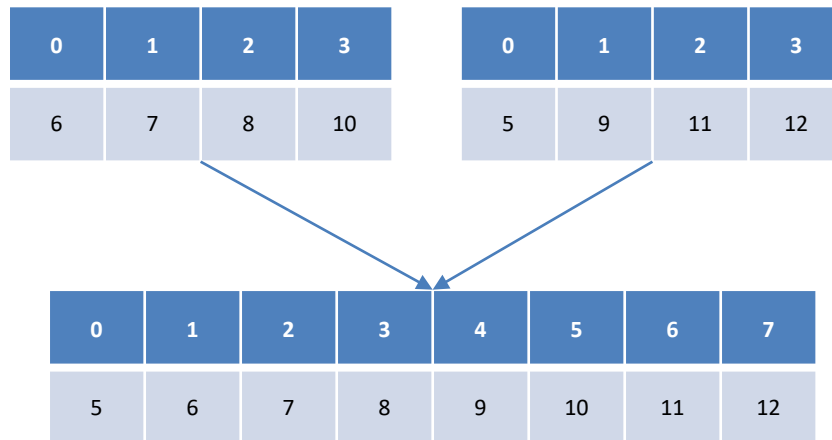


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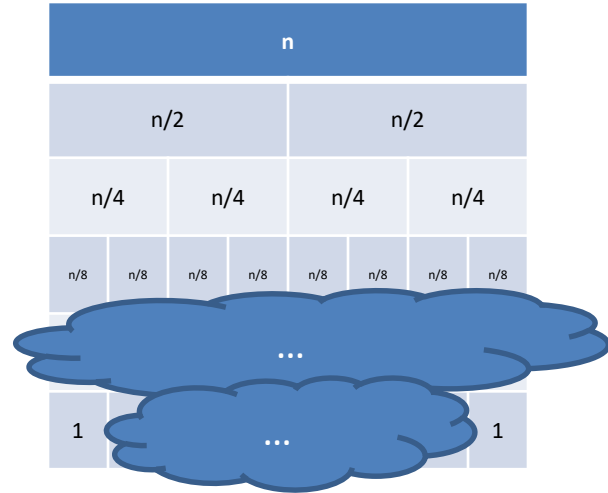
## Example

0	1	2	3	4	5	6	7
5	6	7	8	9	10	11	12

# Merge Sort Complexity

- Worst Case
  - Sorted in Descending Order
- Operations
  - Split
  - Merge

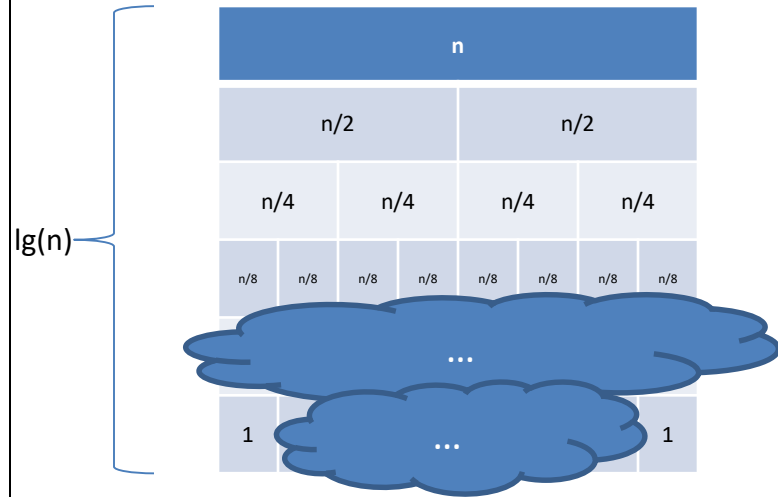
## Complexity Visual



# Merge Sort Complexity

- Worst Case
  - Sorted in Descending Order
- Operations
  - Split
  - Merge

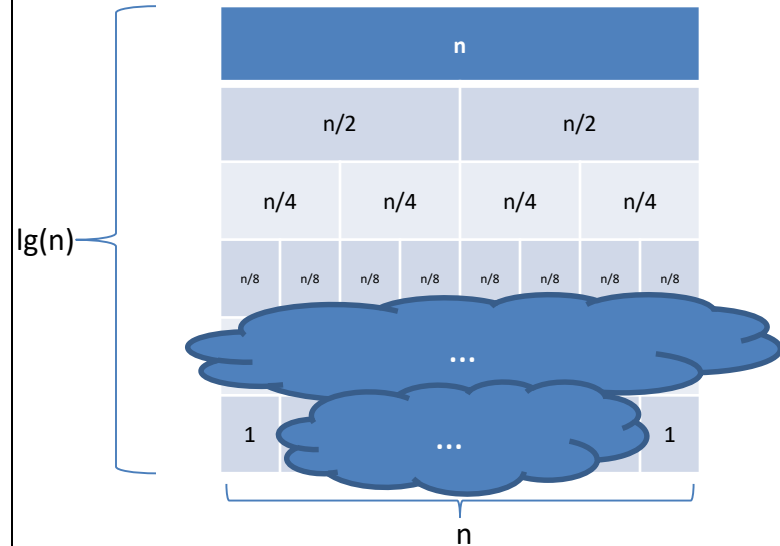
## Complexity Visual



# Merge Sort Complexity

- Worst Case
  - Sorted in Descending Order
- Operations
  - Split
  - Merge

## Complexity Visual



# Merge Sort Complexity

- Worst Case
  - Sorted in Descending Order
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Complexity

$$O(n \lg(n))$$



# Sorting Algorithms

- Quick Sort

1. Pick an arbitrary value called a “pivot” from the array
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  1. Reorder the array where smaller values are to the left of the pivot, and large / equal values are to the right
  2. Once it has been partitioned the pivot value is where it should be
3. Recursively continue now with the array to the left, and the array to the right of the pivot

## Example

Index	0	1	2	3	4	5	6	7
Value	10	8	7	6	12	5	11	9

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## Example



Index	0	1	2	3	4	5	6	7
Value	10	8	7	6	12	5	11	9 pivot

If value at  $j$  is smaller than the pivot then swap values at  $i$  and  $j$  and increase  $i$  by 1

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Value	10	8	7	6	12	5	11	9 pivot

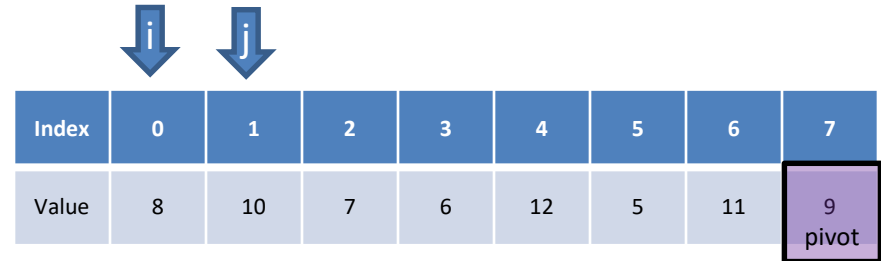
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## Example



The diagram shows two blue arrows pointing downwards to the array. The left arrow is labeled 'i' and points to the element at index 1 (value 10). The right arrow is labeled 'j' and points to the element at index 2 (value 7).

Index	0	1	2	3	4	5	6	7
Value	8	10	7	6	12	5	11	9 pivot

If value at  $j$  is smaller than the pivot then swap values at  $i$  and  $j$  and increase  $i$  by 1

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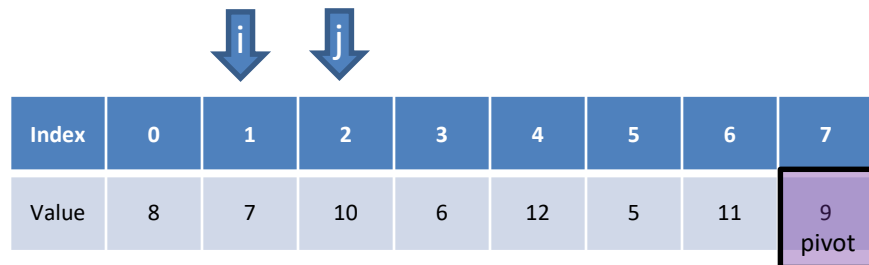


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Value	8	7	10	6	12	5	11	9 pivot

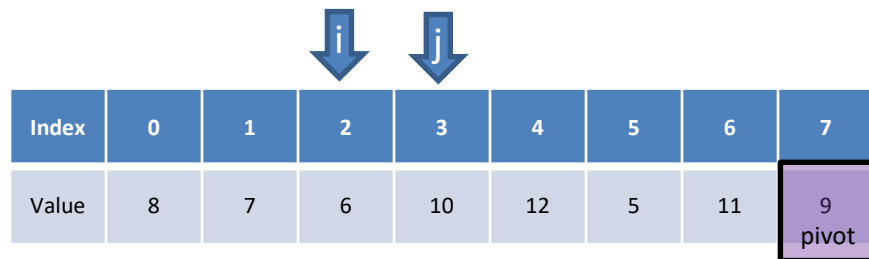
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Index	0	1	2	3	4	5	6	7
Value	8	7	6	10	12	5	11	9 pivot

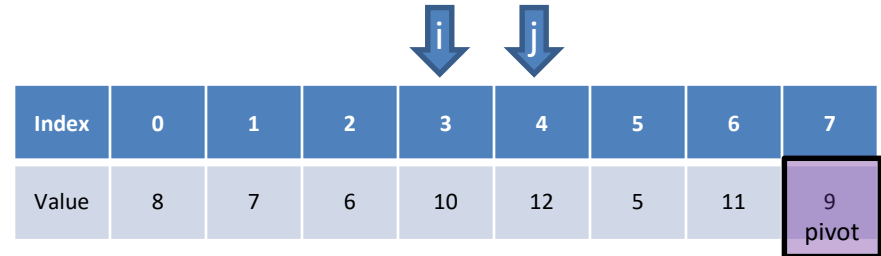
If value at  $j$  is smaller than the pivot then swap values at  $i$  and  $j$  and increase  $i$  by 1

# Sorting Algorithms

- Quick Sort

1. Pick an arbitrary value called a “pivot” from the array
2. Using the pivot value “partition” the array
  1. Reorder the array where smaller values are to the left of the pivot, and large / equal values are to the right
  2. Once it has been partitioned the pivot value is where it should be
3. Recursively continue now with the array to the left, and the array to the right of the pivot

## Example



Index	0	1	2	3	4	5	6	7
Value	8	7	6	10	12	5	11	9 pivot

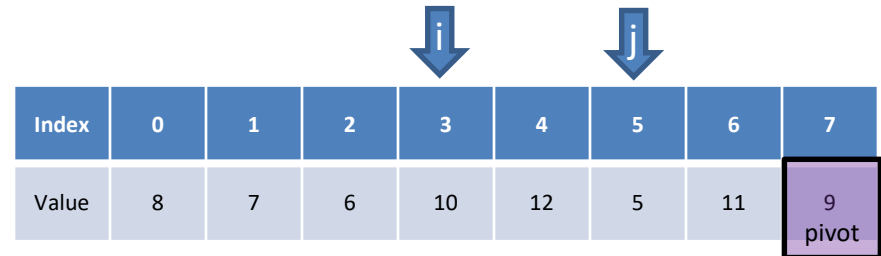
If value at  $j$  is smaller than the pivot then swap values at  $i$  and  $j$  and increase  $i$  by 1

# Sorting Algorithms

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1. Pick an arbitrary value called a “pivot” from the array
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Index	0	1	2	3	4	5	6	7
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## Example

Index	0	1	2	3	4	5	6	7
Value	8	7	6	5	12	10	11	9 pivot

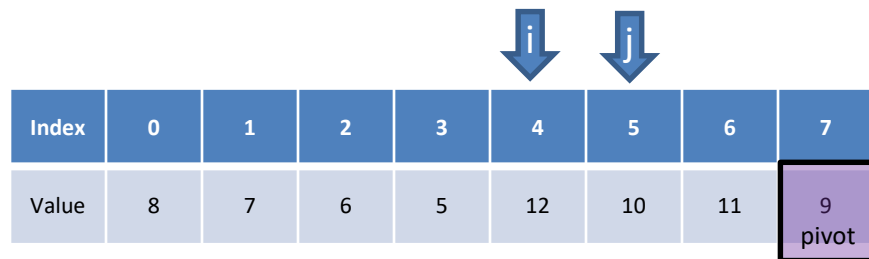
If value at  $j$  is smaller than the pivot then swap values at  $i$  and  $j$  and increase  $i$  by 1

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- Quick Sort

1. Pick an arbitrary value called a “pivot” from the array
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Index	0	1	2	3	4	5	6	7
Value	8	7	6	5	12	10	11	9 pivot

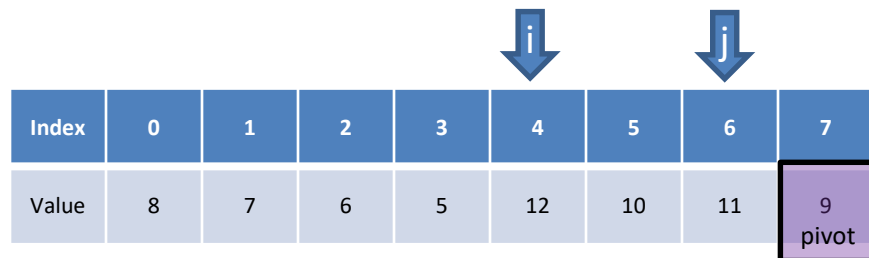
If value at  $j$  is smaller than the pivot then swap values at  $i$  and  $j$  and increase  $i$  by 1

# Sorting Algorithms

- Quick Sort

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  2. Once it has been partitioned the pivot value is where it should be
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## Example



Index	0	1	2	3	4	5	6	7
Value	8	7	6	5	12	10	11	9 pivot

Swap values at  $i$  and the pivot

# Sorting Algorithms

- Quick Sort

1. Pick an arbitrary value called a “pivot” from the array
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## Example

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1. Pick an arbitrary value called a “pivot” from the array
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## Example



Index	0	1	2	3	4	5	6	7
Value	8	7	6	5	9	10	11	12

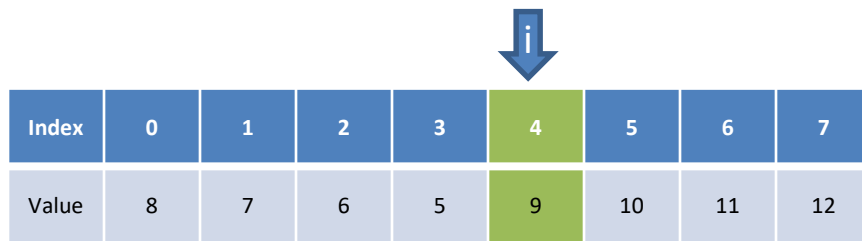
Swap values at  $i$  and the pivot

# Sorting Algorithms

- Quick Sort

1. Pick an arbitrary value called a “pivot” from the array
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  1. Reorder the array where smaller values are to the left of the pivot, and large / equal values are to the right
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3. Recursively continue now with the array to the left, and the array to the right of the pivot

## Example



Index	0	1	2	3	4	5	6	7
Value	8	7	6	5	9	10	11	12

Recursively do the same for the values to the left of the partition and to the right of the partition



# Sorting Algorithms

- Quick Sort

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## Example

Index	0	1	2	3	4	5	6	7
Value	8	7	6	5 pivot	9	10	11	12 pivot

Recursively do the same for the values to the left of the partition and to the right of the partition

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Index	0	1	2	3	4	5	6	7
Value	5	7	6	8	9	10	11	12 pivot

Recursively do the same for the values to the left of the partition and to the right of the partition

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Index	0	1	2	3	4	5	6	7
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Recursively do the same for the values to the left of the partition and to the right of the partition

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Recursively do the same for the values to the left of the partition and to the right of the partition



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## Example

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Value	5	7	6	8	9	10	11	12 pivot

Recursively do the same for the values to the left of the partition and to the right of the partition

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Index	0	1	2	3	4	5	6	7
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Value	5	7	6 pivot	8	9	10	11	12 pivot

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Value	5	6	7 pivot	8	9	10	11	12 pivot

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Value	5	6	7	8	9	10	11	12 pivot

Recursively do the same for the values to the left of the partition and to the right of the partition




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Value	5	6	7	8	9	10	11	12

Recursively do the same for the values to the left of the partition and to the right of the partition

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## Example

Index	0	1	2	3	4	5	6	7
Value	5	6	7	8	9	10	11 pivot	12

Recursively do the same for the values to the left of the partition and to the right of the partition

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Index	0	1	2	3	4	5	6	7
Value	5	6	7	8	9	10	11 pivot	12

Recursively do the same for the values to the left of the partition and to the right of the partition

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## Example



Index	0	1	2	3	4	5	6	7
Value	5	6	7	8	9	10	11	12

Recursively do the same for the values to the left of the partition and to the right of the partition

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3. Recursively continue now with the array to the left, and the array to the right of the pivot

## Example

Index	0	1	2	3	4	5	6	7
Value	5	6	7	8	9	10 pivot	11	12

Recursively do the same for the values to the left of the partition and to the right of the partition

# Sorting Algorithms

- Quick Sort

1. Pick an arbitrary value called a “pivot” from the array
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  1. Reorder the array where smaller values are to the left of the pivot, and large / equal values are to the right
  2. Once it has been partitioned the pivot value is where it should be
3. Recursively continue now with the array to the left, and the array to the right of the pivot

## Example

Index	0	1	2	3	4	5	6	7
Value	5	6	7	8	9	10	11	12

Recursively do the same for the values to the left of the partition and to the right of the partition

# Quick Sort Complexity

- Worst Case
  - Sorted in Ascending Order
  - Assuming pivot is always picked from the last index
- Operations
  - The first index moves  $n$  spaces
  - The first index moves  $n-1$  spaces
  - The first index moves  $n-2$  spaces
  - ...
  - The first index moves 1 space


## Complexity Example

Index	0	1	2	3	4	5	6	7
Value	5	6	7	8	9	10	11	12

# Quick Sort Complexity

- Worst Case
  - Sorted in Ascending Order
  - Assuming pivot is always picked from the last index
- Operations
  - The first index moves  $n$  spaces
  - The first index moves  $n-1$  spaces
  - The first index moves  $n-2$  spaces
  - ...
  - The first index moves 1 space

## Complexity Example




Index	0	1	2	3	4	5	6	7
Value	5	6	7	8	9	10	11	12 pivot



# Quick Sort Complexity

- Worst Case
  - Sorted in Ascending Order
  - Assuming pivot is always picked from the last index
- Operations
  - The first index moves  $n$  spaces
  - The first index moves  $n-1$  spaces
  - The first index moves  $n-2$  spaces
  - ...
  - The first index moves 1 space

## Complexity Example



The diagram shows an array of 8 elements with indices 0 to 7. The values are 5, 6, 7, 8, 9, 10, 11, and 12. The pivot is the last element, 12. Two blue arrows labeled 'j' and 'i' point downwards to index 1, indicating the start of a partitioning step.

Index	0	1	2	3	4	5	6	7
Value	5	6	7	8	9	10	11	12 pivot

# Quick Sort Complexity

- Worst Case
  - Sorted in Ascending Order
  - Assuming pivot is always picked from the last index
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  - The first index moves  $n$  spaces
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## Complexity Example

Index	0	1	2	3	4	5	6	7
Value	5	6	7	8	9	10	11	12 pivot

# Quick Sort Complexity

- Worst Case
  - Sorted in Ascending Order
  - Assuming pivot is always picked from the last index
- Operations
  - The first index moves  $n$  spaces
  - The first index moves  $n-1$  spaces
  - The first index moves  $n-2$  spaces
  - ...
  - The first index moves 1 space

## Complexity Example

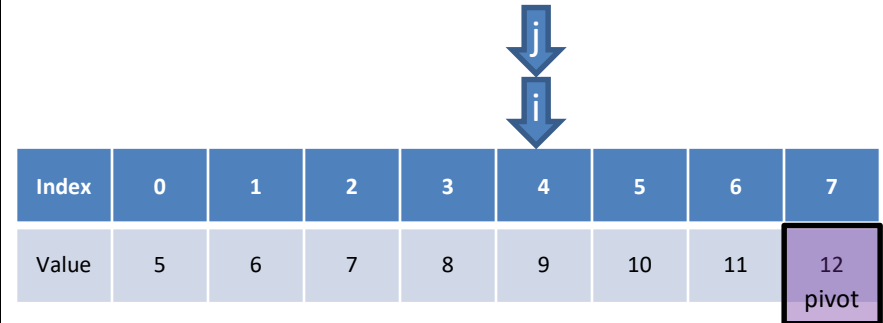


Index	0	1	2	3	4	5	6	7
Value	5	6	7	8	9	10	11	12 pivot

# Quick Sort Complexity

- Worst Case
  - Sorted in Ascending Order
  - Assuming pivot is always picked from the last index
- Operations
  - The first index moves  $n$  spaces
  - The first index moves  $n-1$  spaces
  - The first index moves  $n-2$  spaces
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  - The first index moves 1 space

## Complexity Example

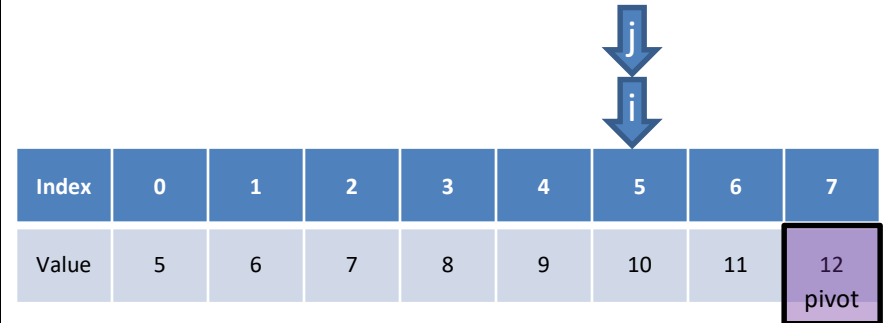


Index	0	1	2	3	4	5	6	7
Value	5	6	7	8	9	10	11	12 pivot

# Quick Sort Complexity

- Worst Case
  - Sorted in Ascending Order
  - Assuming pivot is always picked from the last index
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  - The first index moves  $n$  spaces
  - The first index moves  $n-1$  spaces
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  - The first index moves 1 space

## Complexity Example



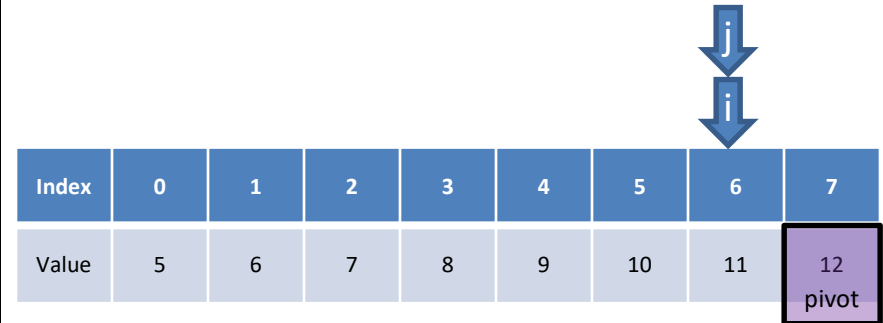
The diagram shows an array with indices 0 through 7 and corresponding values 5 through 12. The pivot is the element at index 7, which has the value 12. Two blue arrows labeled 'j' and 'i' point to the element at index 5, which has the value 10.

Index	0	1	2	3	4	5	6	7
Value	5	6	7	8	9	10	11	12 pivot

# Quick Sort Complexity

- Worst Case
  - Sorted in Ascending Order
  - Assuming pivot is always picked from the last index
- Operations
  - The first index moves  $n$  spaces
  - The first index moves  $n-1$  spaces
  - The first index moves  $n-2$  spaces
  - ...
  - The first index moves 1 space

## Complexity Example

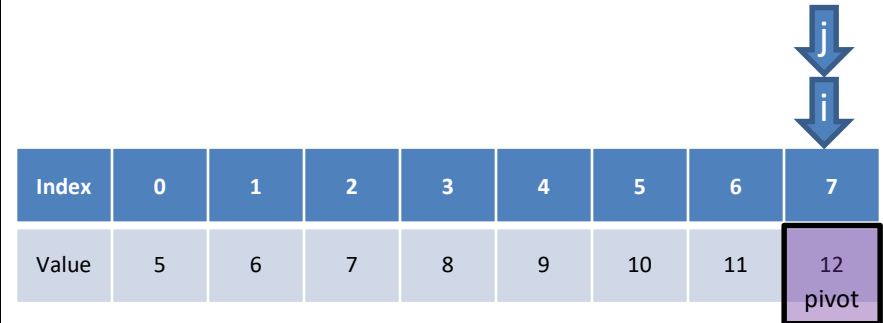


Index	0	1	2	3	4	5	6	7
Value	5	6	7	8	9	10	11	12 pivot

# Quick Sort Complexity

- Worst Case
  - Sorted in Ascending Order
  - Assuming pivot is always picked from the last index
- Operations
  - The first index moves  $n$  spaces
  - The first index moves  $n-1$  spaces
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Complexity

$$O(n^2)$$

# Quick Sort Merge Sort Compared

## Merge Sort

- Worst Time Complexity =  $O(n \lg(n))$
- Average Time Complexity =  $\Theta(n \lg(n))$
- Worst Space Complexity =  $O(n)$  additional

## Quick Sort

- Worst Time Complexity =  $O(n^2)$
- Average Time Complexity =  $\Theta(n \lg(n))$
- Worst Space Complexity =  $O(\lg(n))$  additional