* Set and functions.

$a \in A$

$f : A \to B$

* Monoid.

$(A, e : A, * : A \times A \to A)$

① $\forall a \in A, \quad e * a = a * e = a.$

② $\forall a, b, c \in A. \quad (a * b) * c = a * (b * c)$

e.g. $(\mathbb{N}, 0, +)$

$(\mathbb{N}, 1, \times)$

$(\text{String}, \text{""}, ++)$

$(A \Rightarrow A, \ id_A : A \Rightarrow A, \ \circ)$

$f, g \in A \Rightarrow A$

$(f \circ g)(a) = f(g(a))$

* Monoid homomorphism.

$f : A \to B$ s.t.

$f(e_A) = e_B$

$\forall a, b \in A. \ f(a *_A b) = f(a) *_B f(b)$ $\left(\begin{array}{l} \text{Note:} \\ f(a), f(b) \in B \end{array} \right)$

ex1. $f : (\mathbb{N}, 0, +) \to (\mathbb{N}, 1, *)$

$$f(0) = 1$$
$$f(a+b) = f(a) * f(b)$$

$$f(x) = 2^x$$

ex2 : $\text{length} : [a] \to \mathbb{N}$

* Category. $\mathcal{C}$

① A collection of objects. $A \in \text{obj}(\mathcal{C})$

$A \in \mathcal{C}$

② $\forall A, B \in \mathcal{C}$, there is a set

$\text{Hom}(A, B)$.

$f \in \text{Hom}(A, B)$,

f is called a morphism from A to B.

denoted by $f : A \to B$

③ $\forall f : A \to B$, $g : B \to C$, $h : C \to D$

$\exists\, g \circ f : A \to C$ s.t.

$$h \circ (g \circ f) = (h \circ g) \circ f.$$

④ $\forall A$, $id_A : A \to A$ s.t.

$$f \circ id_A = f \quad \forall f : A \to B.$$

and $$id_A \circ g = g \quad \forall g : C \to A$$

$$f : A \to B$$

domain of f codomain of f.

F

* Functors

A "functor" from $\mathcal{C} \to \mathcal{D}$

denoted by $F : \mathcal{C} \to \mathcal{D}$

$\forall A \in \mathcal{C}$, $F(A) \in \mathcal{D}$

$\forall f : A \to B$ in $\mathcal{C}$, we have a

morphism $F(f) : F(A) \to F(B)$ s.t.

$$F(id_A) = id_{F(A)} : F(A) \to F(A)$$

$$F(f \circ g) = F(f) \circ F(g).$$

$$F_{AB} : \mathrm{Hom}(A, B) \longrightarrow \mathrm{Hom}(F(A), F(B))$$