* In general, reflection about the mean is

$2|\psi\rangle\langle\psi| - I$ , where $|\psi\rangle = \underbrace{|+\rangle \otimes |+\rangle \cdots \otimes |+\rangle}_{n}$

$$2|\psi\rangle\langle\psi| - I = 2\left(\vec{H}|\vec{0}\rangle\langle\overleftarrow{0}|\overleftarrow{H}\right) - \vec{H}\overleftarrow{H}$$
$$= \vec{H}\circ\left(2|\vec{0}\rangle\langle\overleftarrow{0}| - I\right)\circ\overleftarrow{H}$$

So we just need to implement

$2|\vec{0}\rangle\langle\overleftarrow{0}| - I$. when $n=2$.

$$(2|00\rangle\langle 00| - I)|00\rangle = |00\rangle$$
$$\cdots \quad |01\rangle = -|01\rangle$$
$$\cdots \quad |10\rangle = -|10\rangle$$
$$\cdots \quad |11\rangle = -|11\rangle.$$

Up to a global phase (−1), $2|00\rangle\langle 00| - I$ is the same unitary as $I - 2|00\rangle\langle 00|$, and $(I - 2|00\rangle\langle 00|)|00\rangle = -|00\rangle$

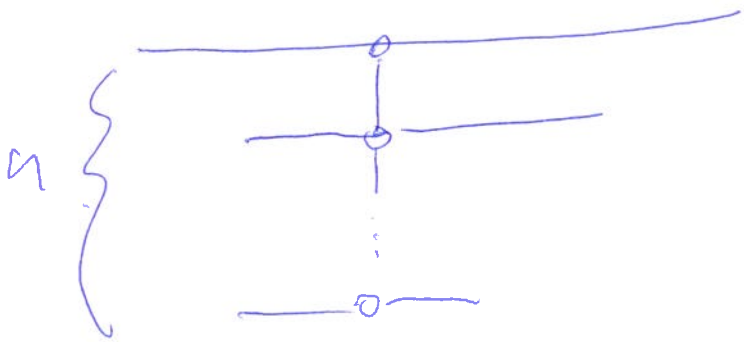
$$\cdots \quad |01\rangle = |01\rangle$$
$$\cdots \quad |10\rangle = |10\rangle$$
$$\cdots \quad |11\rangle = |11\rangle.$$

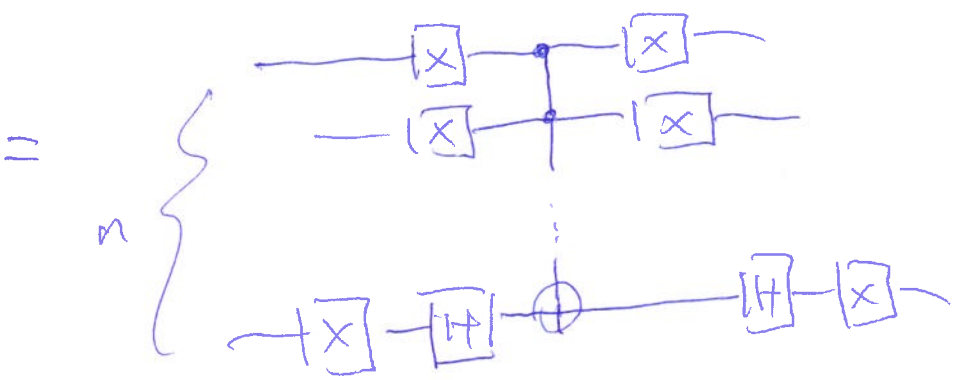
So $I-2|00\rangle\langle 00|$ looks a lot like CZ-gate if we can pretend 0 as 1.

In fact, $I-2|00\rangle\langle 00|$ is "negative CZ".

i.e.

So in general, $I-2|\overrightarrow{0}\rangle\langle\overleftarrow{0}|$ is n-fold negative CZ.





and we know how to implement n-fold CX.