* Evaluation rules

These rules update the underlying quantum state.

$$\cancel{(\varphi, L, (\text{force } U)(l_i \cdots l_j))}$$

$$\frac{\varphi' = U_{i\cdots j}(\varphi)}{(\varphi, L, (\text{force } U)(l_i, \cdots, l_j)) \longrightarrow (\varphi', L, (l_i, \cdots l_j))}$$

$$\frac{L' = L, l_{n+1}.}{(\varphi, L, (\text{force Init})()) \longrightarrow (\varphi \otimes |0\rangle, L', l_{n+1})}$$

$$\frac{\varphi = a\psi + b\gamma \quad \psi' = \psi/|0\rangle \quad \psi_{l_i} = |0\rangle \quad \gamma_{l_i} = |1\rangle \quad \gamma' = \gamma/|1\rangle}{(\varphi, L, (\text{force meas})\, l_i) \xrightarrow{|a|^2} (\psi', L/l_i, \text{False})}$$

$$\frac{\cdots}{(\varphi, L, (\text{force meas})\, l_i) \xrightarrow{|b|^2} (\gamma', L/l_i, \text{True})}$$

$$\frac{}{(\varphi, L, (\lambda x.M)V) \longrightarrow (\varphi, L, [V/x]M)}$$

$$\frac{(\varphi, L, M) \to (\varphi', L', M')}{(\varphi, L, MN) \to (\varphi', L', M'N)}$$

$$\frac{}{(\varphi, L, \text{force}(\text{lift } M)) \to (\varphi, L, M)}$$

$$\frac{}{(\varphi, L, \text{let}(x,y) = (V_1, V_2) \text{ in } M) \longrightarrow (\varphi, L, [V_1/x, V_2/y]M)}$$

$$\frac{(\varphi, L, N) \to (\varphi', L', N')}{(\varphi, L, V N) \to (\varphi, L', V N')}$$

$$\frac{(\varphi, L, M) \to (\varphi', L', M')}{(\varphi, L, (M, N)) \to (\varphi', L', (M', N))}$$

$$\frac{(\varphi, L, N) \to (\varphi', L, N')}{(\varphi, L, (V, N)) \to (\varphi', L', (V, N'))}$$

$$\frac{(\varphi, L, M) \to (\varphi', L', M')}{(\varphi, L, \mathrm{Left}\ M) \leadsto (\varphi', L', \mathrm{Left}\ M')}$$

right is similar.

$$\frac{}{\left(\varphi, L, \begin{array}{l} \mathrm{case}\ (\mathrm{Left}\ V)\ \mathrm{of} \\ \quad \mathrm{Left}\ x \to N_1 \\ \quad \mathrm{Right}\ y \to N_2 \end{array}\right) \to (\varphi, L, [V/x] N_1)}$$

right is similar.

$$\frac{(\varphi, L, M) \to (\varphi', L', M')}{\left(\varphi, L, \begin{array}{l} \mathrm{case}\ M\ \mathrm{of} \\ \quad \mathrm{Left}\ x \to N_1 \\ \quad \mathrm{Right}\ y \to N_2 \end{array}\right) \to \left(\varphi', L', \begin{array}{l} \mathrm{case}\ M'\ \mathrm{of} \\ \quad \mathrm{left}\ x \to N_1 \\ \quad \mathrm{Right}\ y \to N_2 \end{array}\right)}$$

$$\frac{(\varphi, L, M) \to (\varphi', L', M')}{(\varphi, L, \text{force } M) \to (\varphi', L', \text{force } M')}$$

$$\frac{(\varphi, L, M) \to (\varphi', L', M')}{(\varphi, L, \text{let } (x,y)=M \text{ in } N) \to (\varphi', L', \text{let } (x,y)=M' \text{ in } N)}$$

Remarks:

① The evaluation is nonderministic due to ~~the~~ measurement.

$$(\varphi, L, M) \begin{matrix} \rightsquigarrow^* V_1 \\ \rightsquigarrow^* V_2 \end{matrix} \quad \text{where } V_1 \neq V_2.$$

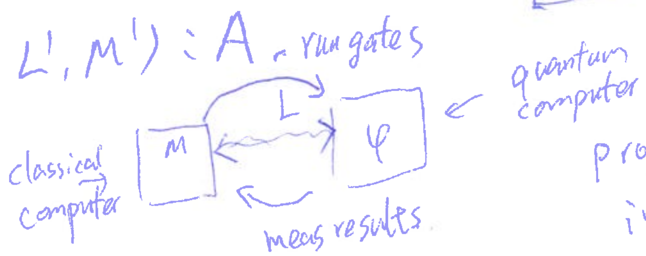
② Typer preservation is still true. Let $\Sigma_L = l_1 : Qubit \dots l_n : Qubit$ where $[l_1, \dots l_n] = L$. ← Label context.

We write a well-typed configuration as $\vdash (\varphi, L, M) : A$ if ~~Σ and L have the same~~ ~~set of labels and~~ $\Sigma_L \vdash M : A$

initial config: $(1, [\,], M)$, where $\vdash M : A$.

Thm: If $\vdash (\varphi, L, M) : A$ and $(\varphi, L, M) \rightsquigarrow (\varphi', L', M')$, then $\vdash (\varphi', L', M') : A$.

③ (φ, L, M)



programmer can not inspect φ.

* Examples: we ~~wrigh~~ write '&' for 'force'

$$\text{bell00} : \text{Qubit} \otimes \text{Qubit}.$$

$$\text{bell00} = \&CX(\&H(\&Init()), \&Init()).$$

$$(1, [\,], \text{bell00}) = (1, [\,], \cancel{\text{bell00}}\ \&CX(\&H(\&Init()), \&init()))$$

$$\to (|0\rangle, [l_1], \&CX(\&H\ l_1, \&init()))$$

$$\to (|+\rangle, [l_1], \&CX(l_1, \&Init()))$$

$$\to (|+\rangle|0\rangle, [l_1, l_2], \&CX(l_1, l_2))$$

$$\to \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}, [l_1, l_2], (l_1, l_2)\right)$$

$$\text{bell-meas} : \text{Bool} \otimes \text{Bool}$$

$$\text{bell-meas} = \text{let } (x, y) = \text{bell00 in } (\&Meas\ x, \&Meas\ y)$$

$$(1, [\,], \text{bell-meas}) = (1, [\,], \text{let } (x,y) = \text{bell00 in } (\&Meas\ x, \&Meas\ y))$$

$$\to \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}, [l_1, l_2], \text{let } (x, y) = (l_1, l_2) \text{ in } (\&Meas\ x, \&Meas\ y)\right)$$

$$\to \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}, [l_1, l_2], (\&Meas\ l_1, \&Meas\ l_2)\right)$$

$$\xrightarrow{\frac{1}{2}} (|0\rangle, [l_2], (False, \&Meas\ l_2)) \xrightarrow{1} (1, [\,], (False, False))$$

$$\xrightarrow{\frac{1}{2}} (|1\rangle, [l_2], (True, \&Meas\ l_2)) \xrightarrow{1} (1, [\,], (True, True))$$