* Labels. let $\mathcal{L}$ be a countable infinite set of labels. We write $l, l_1, l_2, \cdots \in \mathcal{L}$.

We extend notion of context with Labels.

$$\Gamma ::= \phi \mid \Gamma, x : A \mid \Gamma, l : \text{Qubit}$$

We write $\Sigma := [l_1 : \text{Qubit}, \cdots l_n : \text{Qubit}]$.

We call Σ Label context.

* Labels corresponds to wires, or the addresses of the qubits.

* Values. we extend the notion of value.

$$V ::= \cdots \mid l \mid \text{lift } M \mid U \mid \text{Init} \mid \text{Meas.} \mid \text{force } V.$$

* Label vs variable

A variable can be substituted by another term, but we can't substitute a label with anther term.

~~we can only rename a label to another label~~

* Evaluation

Rather than evaluating only a term likes STLC, we evaluate a "configuration".

A configuration is a triple (φ, L, M).

where L is a finite list of labels $[l_1, \cdots l_n]$.

and $\varphi \in \underbrace{Qubit \otimes \cdots \otimes Qubit}_{|L|}$.

and M is a term.

We write φ_{l_i} for the i-th ~~q~~ qubit in φ.

e.g. ① $\varphi = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

φ_{l_1} refers to the first qubit of φ.

φ_{l_2} refers to the second qubit.

② $\varphi = \beta_{00} \otimes |0\rangle$.

$\varphi_{l_3} = |0\rangle$.

* in (φ, L, M), M may contain labels in L.

~~[illegible]~~ .

* Evaluation rules

$$\frac{\varphi' = U_{i \cdots j}(\varphi)}{(\varphi, L, (\text{force } U)(l_i, \cdots, l_j)) \rightarrow (\varphi', L, (l_i, \cdots l_j))}$$

These rules update the underlying quantum state.

$$\cancel{(\varphi, L, (\text{force } U)(l_i \cdots l_j))}$$

$$\frac{L' = L, l_{n+1}}{(\varphi, L, (\text{force Init})()) \rightarrow (\varphi \otimes |0\rangle, L', l_{n+1})}$$

$$\frac{\varphi = a\psi + b\gamma \quad \psi' = \psi/|0\rangle \quad \psi_{l_i} = |0\rangle \quad \gamma_{l_i} = |1\rangle \quad \gamma' = \gamma/|1\rangle}{(\varphi, L, (\text{force meas})\, l_i) \xrightarrow{|a|^2} (\psi', L/l_i, \text{False})}$$

$$\frac{\cdots}{(\varphi, L, (\text{force meas})\, l_i) \xrightarrow{|b|^2} (\gamma', L/l_i, \text{True})}$$

$$\frac{(\varphi, L, M) \rightarrow (\varphi', \ldots}{(\varphi, L, MN) \rightarrow (\varphi', L', M' \ldots}$$

$$\frac{}{(\varphi, L, (\lambda x.M)\, V) \rightarrow (\varphi, L, [V/x]M)}$$

$$\frac{}{(\varphi, L, \text{force}(\text{lift } M)) \rightarrow (\varphi, L, M)}$$

$$\frac{}{(\varphi, L, \text{let}\,(x,y) = (V_1, V_2) \text{ in } M) \rightarrow (\varphi, L, [V_1/x, V_2/y]M)}$$