Introduction to Quantum circuit

* Bit : $|0\rangle, |1\rangle$

* Qubit $|\varphi\rangle = a|0\rangle + b|1\rangle$ where

magnitude $a, b \in \mathbb{C}$, $\|a\|^2 + \|b\|^2 = 1$.

"absolute value" $\|a\| = \sqrt{x^2+y^2}$ for $a = x+iy$.

we say $|\varphi\rangle$ is a linear combination of $|0\rangle$ and $|1\rangle$

* Multiple qubits via tensor product.

Qubit $\otimes$ Qubit.

$$|\varphi_1\rangle \otimes |\varphi_2\rangle = (a_1|0\rangle + b_1|1\rangle) \otimes (a_2|0\rangle + b_2|1\rangle)$$

$$= a_1a_2 |0\rangle \otimes |0\rangle + a_1b_2|0\rangle \otimes |1\rangle +$$
$$a_2b_1|1\rangle \otimes |0\rangle + b_1b_2|1\rangle \otimes |1\rangle$$

$$= a_1a_2|00\rangle + a_1b_2|01\rangle + a_2b_1|10\rangle$$
$$+ b_1b_2|11\rangle.$$

So a 2-qubit state is simultaneously in $|00\rangle, |01\rangle, |10\rangle, |11\rangle$ bit state.

* Some common states.

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

like $\{|0\rangle, |1\rangle\}$

$\{|+\rangle, |-\rangle\}$ is also a set of basis states.

i.e. $|\varphi\rangle = a|+\rangle + b|-\rangle$

$\forall\, a, b \in \mathbb{C},\ |a|^2 + |b|^2 = 1$

* unitary operations.

We can update 1/2 qubit state using unitary operations. Unitary operations are very special:

① They are "linear". i.e.

$$U(a|\varphi_1\rangle + b|\varphi_2\rangle) = aU|\varphi_1\rangle + bU|\varphi_2\rangle.$$

② They are "reversable". i.e.

$$\exists\, U^\dagger \text{ s.t. } U \circ U^\dagger = U^\dagger \circ U = I.$$

quantum circuit notation.

* Pauli operations. $\{X, Y, Z, I \ : Q \to Q\}$.

$X|0\rangle = |1\rangle$

$X|1\rangle = |0\rangle$

$Y|0\rangle = i|1\rangle$

$Y|1\rangle = -i|0\rangle$

$Y\overset{|0\rangle}{X} = -i|0\rangle$

$Y|0\rangle = i|1\rangle$

$Z|0\rangle = |0\rangle$

$Z|1\rangle = -|1\rangle$.

$I|\varphi\rangle = |\varphi\rangle$

Note: $X^2 = I = Y^2 = Z^2$.

$ZX = iY$.

$XZ = -ZX$

So $X^\dagger = X$

$Y^\dagger = Y$

$Z^\dagger = Z$

* Hadamard operation (gate)

$H|0\rangle = |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$H|1\rangle = |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$.

$H^2 = I$

$HXH = Z$

$HZH = X$.

* Phase gate $P(\theta)$

$$P(\theta)|0\rangle = |0\rangle$$
$$P(\theta)|1\rangle = e^{i\theta}|1\rangle$$

phase

Note that $e^{i\theta} = \cos\theta + i\sin\theta$

and $\cos^2\theta + \sin^2\theta = 1$

** Some important phase gates

$$P(\pi) = Z$$

$T = P(\frac{\pi}{4})$ and $S = P(\frac{\pi}{2})$

Note: $T^2 = S$. $S^2 = Z$. $Z^2 = I$.

...

* 2-qubit gates