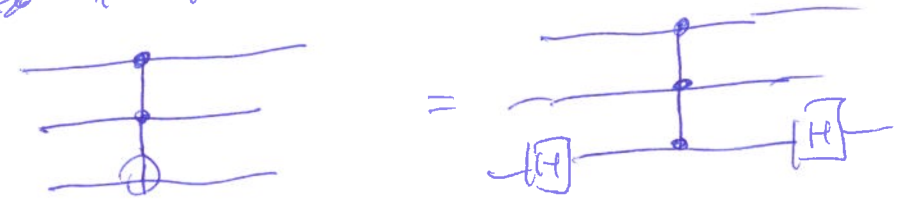
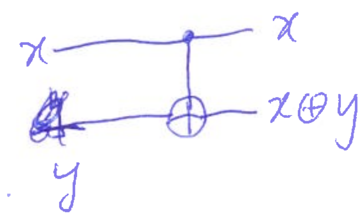
* Recall that

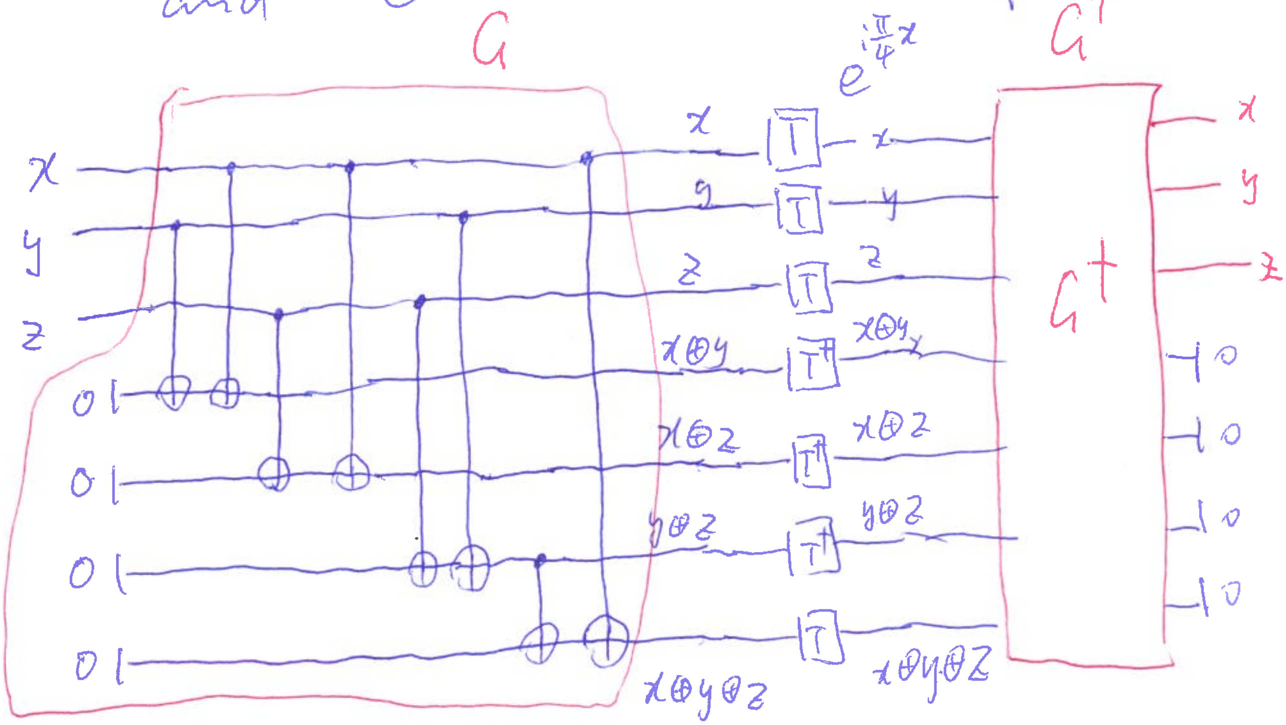


And
$$CCZ|xyz\rangle = e^{i\pi\, x\cdot y\cdot z}|xyz\rangle$$
$$= e^{i\frac{\pi}{4}(4xyz)}|xyz\rangle$$
$$= e^{i\frac{\pi}{4}(x+y+z-x\oplus y-x\oplus z-y\oplus z+x\oplus y\oplus z)}|xyz\rangle.$$
$$= e^{i\frac{\pi}{4}x}\cdot e^{i\frac{\pi}{4}y}\cdot e^{i\frac{\pi}{4}z}\cdot e^{-i\frac{\pi}{4}(x\oplus y)}\cdot e^{-i\frac{\pi}{4}(x\oplus z)}\cdot e^{-i\frac{\pi}{4}(y\oplus z)}\cdot e^{i\frac{\pi}{4}(x\oplus y\oplus z)}|xyz\rangle.$$

Note that
$$T^\dagger|a\rangle = e^{-i\frac{\pi}{4}a}|a\rangle.$$
$$T|a\rangle = e^{i\frac{\pi}{4}a}|a\rangle$$
and
$$CX|ab\rangle = |a, a\oplus b\rangle$$

Note that





* Note that it's important we apply $G^{\dagger}$ in the end to return all the ~~tem~~ ancilla (Tempory variables) back to state $|0\rangle$.

~~As~~ ~~If~~ As we don't want ancilla to be entangle with x, y, z accidentally when we use ~~CCZ~~ the constructed CCZ.

* Our construction of CCZ uses roughly 7 T-gates

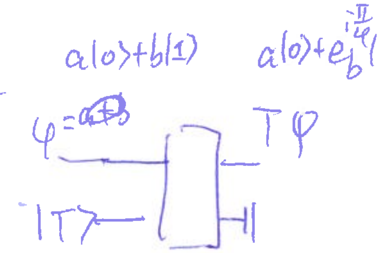
In the Clifford + T paradigm, T-gates are consider expensive, whereas Clifford gates are cheap (Transrversal, fault tolerant etc.).

* The best known construction of CCX/CCZ uses 4 T-gates.

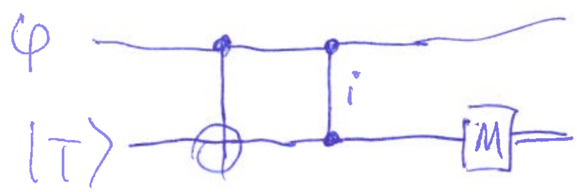
* Perform T-gate via T-state.

Suppose we can prepare T-state $|T\rangle = (|0\rangle + e^{i\frac{\pi}{4}}|1\rangle)/\sqrt{2}$.

We can actually perform T-gate with $|T\rangle$ and Clifford gates.



Consider the following circuit



Note that

CS-gate is not Clifford!

$i = e^{i\frac{\pi}{2}}$

Suppose $\varphi = a|0\rangle + b|1\rangle$.

$$|\varphi\rangle|T\rangle = \frac{1}{\sqrt{2}}(a|0\rangle + b|1\rangle)\otimes(|0\rangle + e^{i\frac{\pi}{4}}|1\rangle)$$

$$= \frac{1}{\sqrt{2}}\left\{a|00\rangle + ae^{i\frac{\pi}{4}}|01\rangle + b|10\rangle + be^{i\frac{\pi}{4}}|11\rangle\right\}$$

$$\xmapsto{CX_{1\to 2}} \frac{1}{\sqrt{2}}\left\{a|00\rangle + ae^{i\frac{\pi}{4}}|01\rangle + b|11\rangle + be^{i\frac{\pi}{4}}|10\rangle\right\}$$

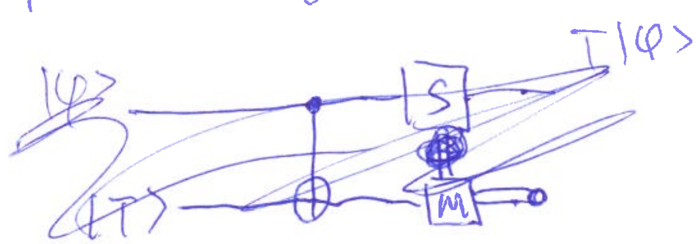
$$\xmapsto{CS} \frac{1}{\sqrt{2}}\left\{\underline{a|00\rangle} + ae^{i\frac{\pi}{4}}|01\rangle + be^{i\frac{\pi}{2}}|11\rangle + \underline{be^{i\frac{\pi}{4}}|10\rangle}\right\}$$

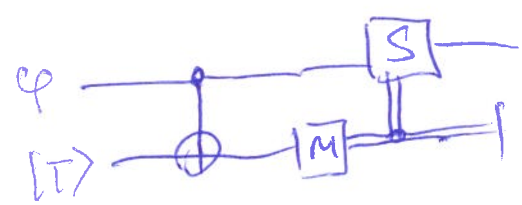
$$= \frac{1}{\sqrt{2}}\left\{(a|0\rangle + be^{i\frac{\pi}{4}}|1\rangle)\otimes|0\rangle + e^{i\frac{\pi}{4}}(a|0\rangle + be^{i\frac{\pi}{4}}|1\rangle)\otimes|1\rangle\right\}$$

So if measurement is $|0\rangle$.
then we have $T|\varphi\rangle$.
if measurement is $|1\rangle$,
we have $e^{i\frac{\pi}{4}} T|\varphi\rangle \simeq T|\varphi\rangle$
up-to- a global phase $e^{i\frac{\pi}{4}}$. ~~up to~~

Note: 1. $\forall \theta \in [0, 2\pi)$, $e^{i\theta}(a|0\rangle + b|1\rangle)$ have the same measurement statistic as $a|0\rangle + b|1\rangle$.

2. CS gate is not Clifford. but we can use deferred measurement principle to get the following.





~~*. It is believed that to support~~

3. $|T\rangle$ state is "consumed" after the above circuit.

* Qubit assertion.

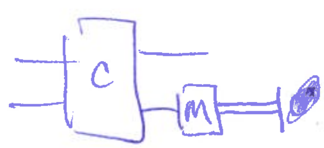
* x —| 0   assert that a qubit is in $|0\rangle$ state

In practice, programmer will need to ensure ~~that~~ x is actually $|0\rangle$. if it is not, ~~then~~ "—| 0" is undefined / throw an error.

For those who know $\langle 0|$, "—| 0" is ~~a notation~~ ~~for $\langle 0|$~~. can be seen as a notation for $\langle 0|$.

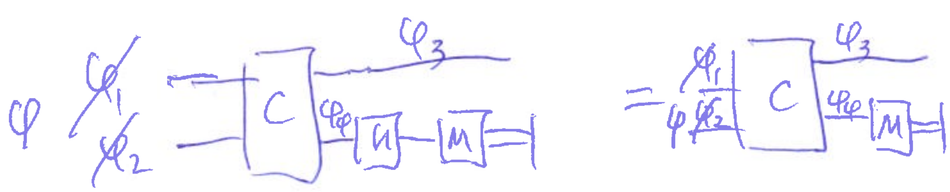
* if we have

and we are not ~~plan~~ plan on ~~using it~~, reversing the whole circuit, then we can ~~replace the above by~~ instead use the following.



i.e. measure and the discard. ~~otherwise undefined~~

thm: if $C(\cancel{|\varphi_1\rangle\otimes|\varphi_2\rangle}\ |\varphi\rangle) = |\varphi_3\rangle \otimes |\varphi_4\rangle$, then



$\forall$ unitary U.

Justification: $|\varphi_3\rangle$ and $|\varphi_4\rangle$ are not entangle. therefore how we throw away $|\varphi_4\rangle$ does not impact $|\varphi_3\rangle$.