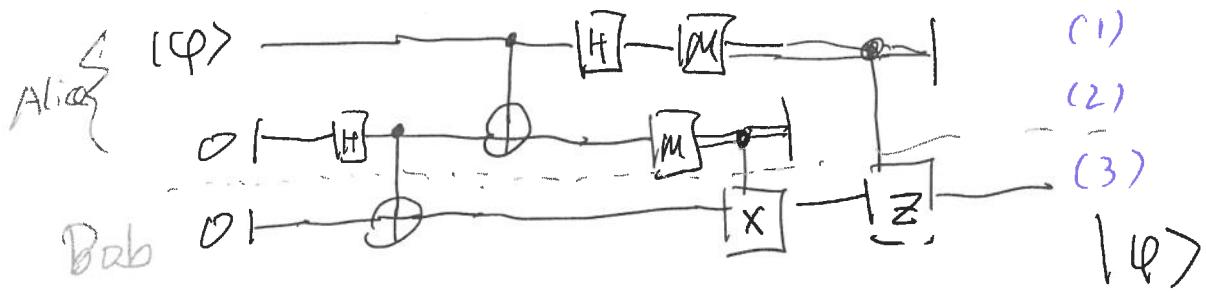


# \* Teleportation Circuit.



Teleportation protocol :

How would we verify this claim ?

$$|\psi_{00}\rangle \xrightarrow{\beta} \frac{1}{\sqrt{2}}(|\phi\rangle|00\rangle + |\phi\rangle|11\rangle)$$

$$\begin{aligned} \text{Let } |\phi\rangle &= a|0\rangle + b|1\rangle \\ &= \frac{1}{\sqrt{2}}(a|000\rangle + b|100\rangle + \\ &\quad a|011\rangle + b|111\rangle) \end{aligned}$$

$$\begin{aligned} \text{CNOT}_{1 \rightarrow 2} \rightarrow & \frac{1}{\sqrt{2}}(a|000\rangle + b|110\rangle + \\ &\quad a|011\rangle + b|101\rangle) \end{aligned}$$

$$\begin{aligned} \xrightarrow{H_1} & \frac{1}{\sqrt{2}}(a|+\rangle|00\rangle + b|-\rangle|10\rangle + \\ &\quad a|+\rangle|11\rangle + b|-\rangle|01\rangle) \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2}(a|\underline{000}\rangle + a|\underline{100}\rangle + b|\underline{010}\rangle - b|\underline{110}\rangle \\ &\quad + a|\underline{011}\rangle + a|\underline{111}\rangle + b|\underline{001}\rangle - b|\underline{110}\rangle) \end{aligned}$$

$$= \frac{1}{2} ( |00\rangle (a|0\rangle + b|1\rangle) + \\ |01\rangle (a|1\rangle + b|0\rangle) + \\ |10\rangle (a|0\rangle - b|1\rangle) + \\ |11\rangle (a|1\rangle - b|0\rangle) )$$

So  $M_1 = M_2 = 0$ . then we are done.

{ if  $M_1 = 0$ , then we apply X gate  
 $M_2 = 1$  to correct.

if  $M_1 = 1$ , then we apply Z gate  
 $M_2 = 0$  to correct.

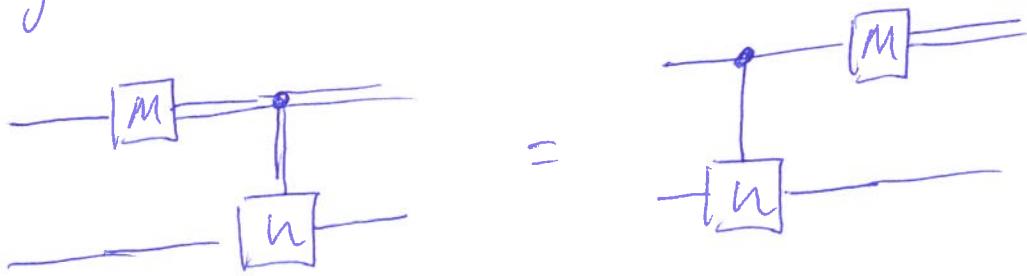
otherwise, we apply ~~XZ~~ gate.

All these happen with equal probability.

\* Principle of deferred measurement.

Measurements done in the middle of the circuit can be deferred to the end.

In particular, we have the following identity.



How to show this?

Suppose we have an input state

$$\psi = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle.$$

LHS:  
(left hand side)

$$|\alpha|^2 + |\beta|^2 \quad \psi$$

$$a|0\rangle + b|1\rangle \quad \xrightarrow{\alpha^2 + \beta^2} \quad c|0\rangle + d|1\rangle$$

$$c|0\rangle + d|1\rangle \quad \xrightarrow{U} \quad cU|0\rangle + dU|1\rangle.$$

so LHS  
= RHS.

RHS:

(right hand side)

$$|\alpha|^2 + |\beta|^2 \quad \psi$$

$$a|0\rangle + b|1\rangle \quad \xrightarrow{CU} \quad cU|0\rangle + dU|1\rangle$$

$$a|0\rangle + b|1\rangle \quad \xrightarrow{|\alpha|^2 + |\beta|^2} \quad cU|0\rangle + dU|1\rangle.$$

\* Construction of Toffoli gate.  
 Recall that Clifford gate set is  
 $\{S, H, CNOT\}$ .

And  $T|0\rangle = |0\rangle$   
 $T|1\rangle = e^{i\frac{\pi}{4}}|1\rangle$ .

or  $T|\alpha\rangle = e^{i\frac{\pi}{4}\alpha}|\alpha\rangle$ , where  $\alpha \in \{0, 1\}$ .

So  $T$  moves the "state"  $\alpha$  into the  
 "Phase"  $e^{i\frac{\pi}{4}\alpha}$ .

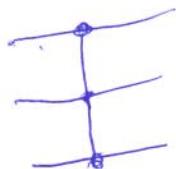
$CNOT|\alpha, b\rangle = |\alpha, \alpha \oplus b\rangle$ ,  $\alpha, b \in \{0, 1\}$ .

we have,

The diagram shows a single horizontal line representing a qubit. A circle with a dot inside is placed on the line. To its right, there is an equals sign followed by two sequences of gates. The first sequence consists of a Hadamard gate (H) followed by a T gate. The second sequence consists of a T gate followed by a Hadamard gate (H).

Similarly, we have

The diagram shows two horizontal lines representing two qubits. A circle with a dot inside is placed on the bottom line. To its right, there is an equals sign followed by two sequences of gates. The first sequence consists of a Hadamard gate (H) on the top line, followed by a T gate on the bottom line. The second sequence consists of a T gate on the bottom line followed by a Hadamard gate (H) on the top line.



is CCZ gate (control-controlled-Z).

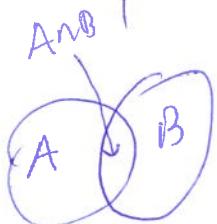
$$CCZ|\alpha, b, c\rangle = e^{i\pi(\alpha \cdot b \cdot c)}|\alpha, b, c\rangle$$

Note that  $e^{i\pi} = -1$ .

So to construct Toffoli gate using Clifford + T, we can just construct CCZ using Clifford + T.

\* Inclusion-exclusion principle. (For set).

$$|A \cap B| = |A| + |B| - |A \cup B|$$



For binary number, we have :

$$2ab = a+b - a \oplus b$$

$$\text{So } 4abc = 2(2ab)c = 2(a+b-a \oplus b) \cdot c$$

$$= 2ac + 2bc - 2(a \oplus b)c$$

$$= ac - a \oplus c + bc - b \oplus c - (a \oplus b + c - a \oplus b \oplus c)$$

$$= a+b+c - a \oplus c - b \oplus c - a \oplus b + a \oplus b \oplus c$$

$$\text{So } \cancel{e^{i\pi(abc)}} = e^{\frac{i\pi}{4}(4abc)} = e^{\frac{i\pi}{4}(a+b+c - a \oplus c - b \oplus c - a \oplus b + a \oplus b \oplus c)}$$

$$= e^{\frac{i\pi}{4}a} \cdot e^{\frac{i\pi}{4}b} \cdot e^{\frac{i\pi}{4}c} \cdot e^{-\frac{i\pi}{4}(a \oplus c)} \cdot e^{-\frac{i\pi}{4}(b \oplus c)} \cdot e^{-\frac{i\pi}{4}(a \oplus b)} \cdot e^{\frac{i\pi}{4}(a \oplus b \oplus c)}$$