Quantum Programming Languages
## CSCE 790 Section 008 Homework 1
Due: January 19, Friday, 2pm

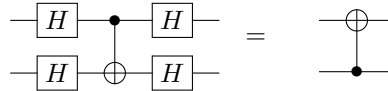1. **No cloning property**

   (a) (2 points) Prove that we do not have $U(|\phi\rangle \otimes |0\rangle) = |\phi\rangle \otimes |\phi\rangle$ for all $|\phi\rangle \in$ **Qubit**, where U is a 2-qubit unitary.

   (b) (2 points) Under what circumstances we can copy a qubit?

2. (3 points) Recall that Bell states are $\beta_{xy} = \frac{1}{\sqrt{2}}(|0y\rangle + (-1)^x |1\bar{y}\rangle)$, where $x, y \in \{0, 1\}$ and $\bar{y}$ means boolean negation on y. Prove that for any $\phi \in$ **Qubit** $\otimes$ **Qubit**, there exists $a_1, a_2, a_3, a_4 \in \mathbb{C}$ such that $\phi = a_1\beta_{00} + a_2\beta_{01} + a_3\beta_{10} + a_4\beta_{11}$ and $|a_1|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2 = 1$.

3. Determine the correctness of the following circuit identities. If an identity is true, prove it; otherwise, show why it is not true.
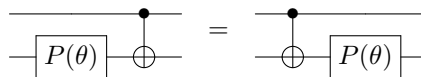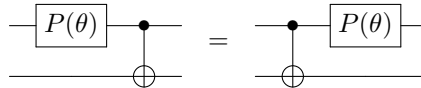
   (a) (2 points)
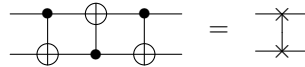
   (b) (2 points)

   (c) (2 points)

   (d) (2 points)

   (e) (2 points)