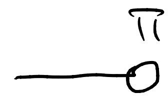
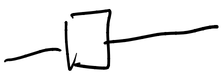
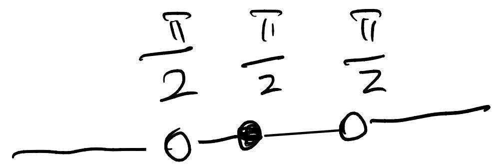
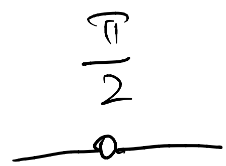
 $\langle 0|$  $\langle 1|$

 $\langle +|$  $\langle -|$

 $\cong$  (Euler Decomposition)

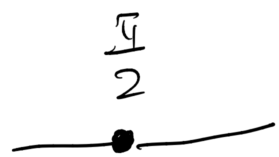
$$\boxed{H|0\rangle = |+\rangle}$$

$$H|1\rangle = |-\rangle$$



$$|0\rangle\langle 0| + |1\rangle\langle 1| e^{i\frac{\pi}{2}}$$

$$= |0\rangle\langle 0| + i|1\rangle\langle 1| = S$$



$$R_x(\tfrac{\pi}{2}) = |+\rangle\langle +| + i|-\rangle\langle -|$$

$$S\, R_x(\tfrac{\pi}{2}) S\, |0\rangle = \underline{S\, R_x(\tfrac{\pi}{2})\, |0\rangle}$$

$$= S\left(\left(|+\rangle\langle +| + i|-\rangle\langle -|\right) |0\rangle \right)$$

$$= S\left(|+\rangle\langle +|0\rangle + i|-\rangle\langle -|0\rangle \right)$$

$$\langle + | 0 \rangle = \langle - | 0 \rangle = \frac{1}{\sqrt{2}}$$

$$\langle + | 1 \rangle = \frac{\langle 0 | 1 \rangle + \langle 1 | 1 \rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\langle - | 1 \rangle = \frac{\langle 0 | 1 \rangle - \langle 1 | 1 \rangle}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

$$= S\left(\frac{|0\rangle + |1\rangle}{2} + i\left(\frac{|0\rangle - |1\rangle}{2}\right) \right)$$

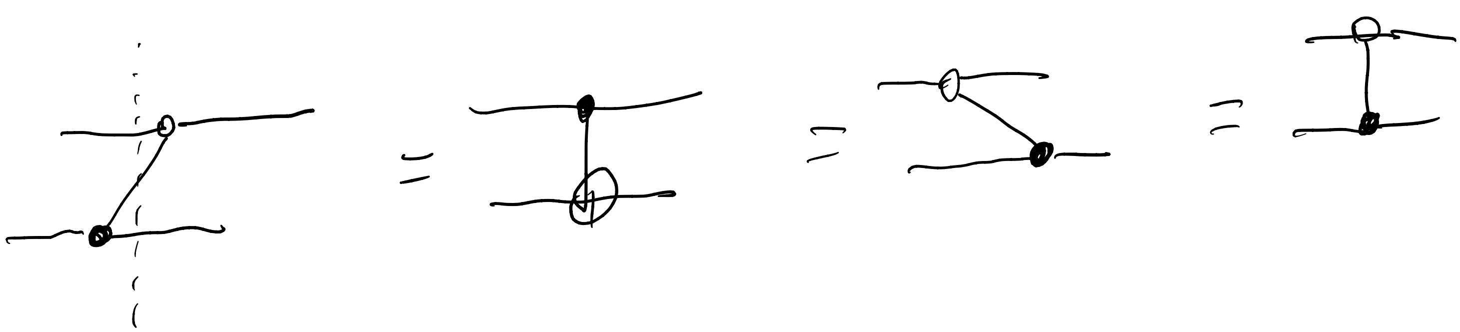
$$= \frac{1}{2}\left(|0\rangle + i|1\rangle + i|0\rangle + |1\rangle \right)$$

$$= \frac{1+i}{2}|0\rangle + \frac{1+i}{2}|1\rangle$$

$$= \frac{1+i}{2}\left(|0\rangle + |1\rangle \right)$$

$$\cong |+\rangle$$

*what about CNOT?

C: 

$$R_x = |{+}{+}\rangle\langle + | + |{-}{-}\rangle\langle - |$$

$$R_z = |0\rangle\langle 00| + |1\rangle\langle 11|$$

$$C = (R_z \otimes Id) \circ (Id \otimes R_x)$$

$$C|10\rangle = (R_z \otimes Id)\circ(Id \otimes R_x)|10\rangle$$

$$= (R_z \otimes Id)(|1\rangle \otimes R_x|0\rangle)$$

$$= (R_z \otimes Id)(|1\rangle \otimes (|++\rangle\langle +|0\rangle + |--\rangle\langle -|0\rangle))$$

$$= \frac{1}{\sqrt{2}}(R_z \otimes Id)(|1\rangle \otimes |++\rangle + |1\rangle \otimes |--\rangle)$$

$$= \frac{1}{\sqrt{2}}(R_z(|1\rangle \otimes |+\rangle) \otimes |+\rangle + R_z(|1\rangle \otimes |-\rangle) \otimes |-\rangle)$$

$$= \frac{1}{\sqrt{2}}((|0\rangle\langle 00|\underline{1,+\rangle} + |1\rangle\langle \underline{11|1,+\rangle})\otimes|+\rangle$$

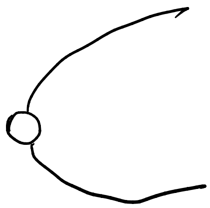
$$+ (|0\rangle\langle \underline{00|1}-\rangle + |1\rangle\langle \underline{11|1,-\rangle})\otimes|-\rangle)$$

$$\boxed{\begin{aligned} |1\rangle \otimes |+\rangle &= |10\rangle + |11\rangle \\ |1\rangle \otimes |-\rangle &= |10\rangle - |11\rangle \\ \langle 00| \end{aligned}}$$

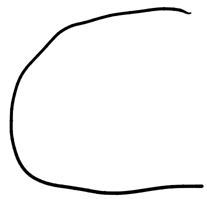
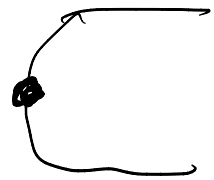
$$\left(\frac{1}{2}\right) = (|1\rangle \otimes |+\rangle - |1\rangle \otimes |-\rangle)$$

$$\left(\frac{1}{2}\right) = \left(\frac{|10\rangle + |11\rangle}{\sqrt{2}} - \frac{|10\rangle - |11\rangle}{\sqrt{2}}\right)$$

$$\left(\frac{1}{\sqrt{2}}\right) = \left(\sqrt{2}\,|11\rangle\right)$$

* 

$$|00\rangle + |11\rangle$$

$=$  $=$ 

"cup"

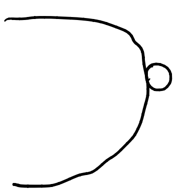
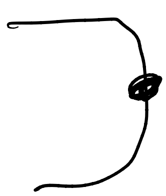
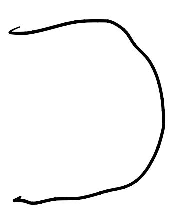
$$|{+}{+}\rangle + |{-}{-}\rangle$$

$$= \frac{1}{2}\Big(|00\rangle + \cancel{|01\rangle} + \cancel{|10\rangle} + |11\rangle$$

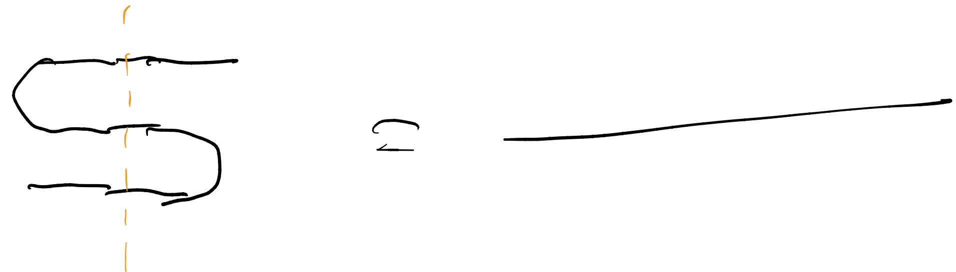
$$+ |00\rangle - \cancel{|01\rangle} - \cancel{|10\rangle} + |11\rangle\Big)$$

$$= |00\rangle + |11\rangle$$

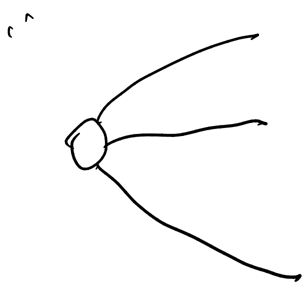
similarly,

 $=$  $\overset{def}{=}$  "cap"

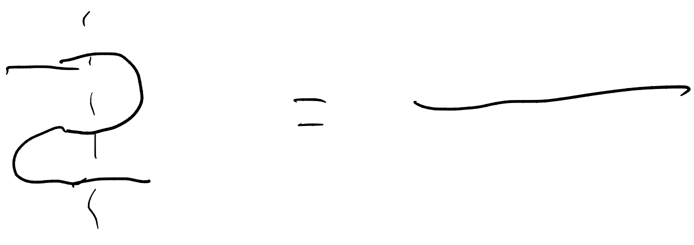
$$\langle 00| + \langle 11|$$



$$Q \cong Unit \otimes Q \to Q \otimes Q \otimes Q \to Q \otimes Unit \cong Q$$



$$|000\rangle + |111\rangle$$



"compact structure"

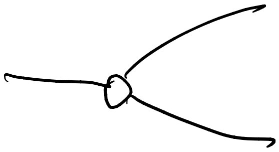
$$(id \otimes cap) \circ (cup \otimes id)(1 \otimes |0\rangle)$$

$$= (id \otimes cap) \; cup \otimes |0\rangle$$

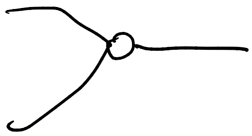
$$= (id \otimes cap)(|000\rangle + |110\rangle)$$

$$= |0\rangle \otimes cap|00\rangle + |1\rangle \otimes cap|10\rangle$$

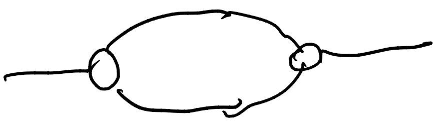
$$= |0\rangle$$



$|00\rangle\langle 0| + |11\rangle\langle 1|$



$|0\rangle\langle 00| + |1\rangle\langle 11|$

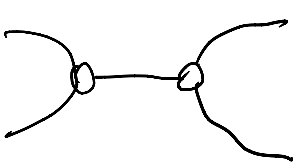


$(|0\rangle\langle 00| + |1\rangle\langle 11|) \circ (|00\rangle\langle 0| + |11\rangle\langle 1|)$

$= |0\rangle\langle 0| + |1\rangle\langle 1|$

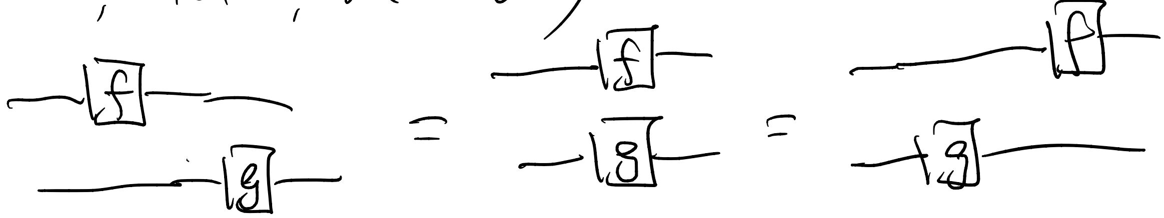
A linear function f is "isometric"
if $f^\dagger \circ f = id$.

$(|00\rangle\langle 0| + |11\rangle\langle 1|) \circ (|0\rangle\langle 00| + |1\rangle\langle 11|)$



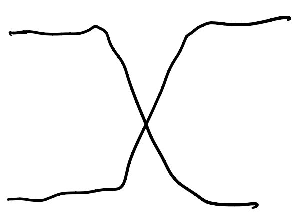
$= |00\rangle\langle 00| + |11\rangle\langle 11|$

$(\otimes, \circ, id, Unit)$



"monoidal structure"

* ZX diagram has "symmetric" monoidal structure and the compact structure.

 : $Q \otimes Q \to Q \otimes Q$.