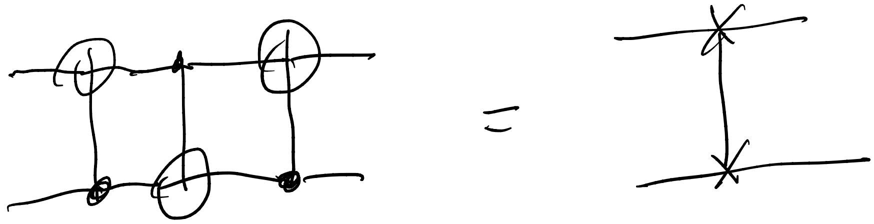
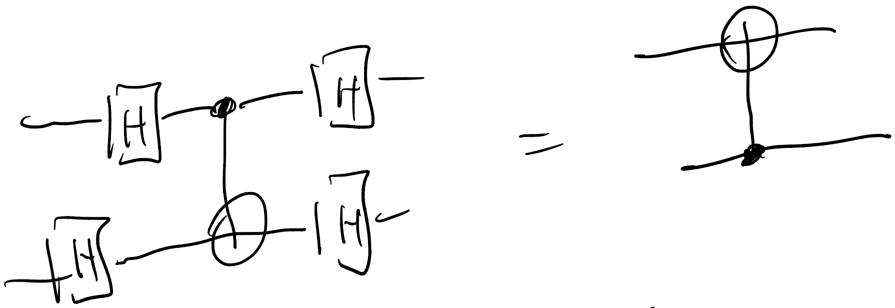
Apr 8

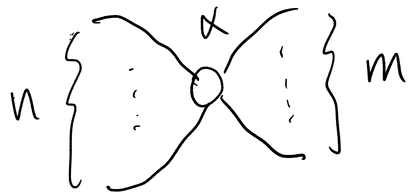
ZX calculus/ diagrams.





* what is a ZX diagram?

$\alpha \in [0, 2\pi)$ linear function



$|\underbrace{0\cdots0}_{m}\rangle\langle\underbrace{0\cdots0}_{n}| + e^{i\alpha}|\underbrace{1\cdots1}_{m}\rangle\langle\underbrace{1\cdots1}_{n}|$

Z-spiders.

$\langle 0| : Q \to \text{Unit} = \mathbb{C}$

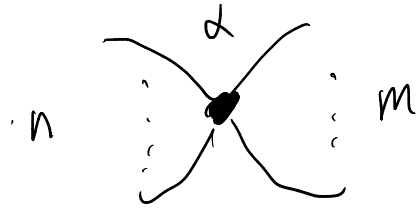
$\langle 0|0\rangle = \langle 0|(|0\rangle) = 1$

$\langle 0|1\rangle = \langle 0|(|1\rangle) = 0$

$\langle 000| : Q^{\otimes 3} \to \text{Unit}$

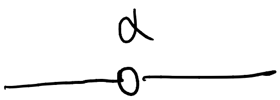
$\langle 000|000\rangle = 1$

$\langle 000 | abc \rangle = 0$ otherwise.



$$|\underbrace{+\cdots+}_{m}\rangle\langle\underbrace{+\cdots+}_{n}| + e^{i\alpha}|\underbrace{-\cdots-}_{m}\rangle\langle\underbrace{-\cdots-}_{n}|$$

X- spiders.



$P(\alpha) = (|0\rangle\langle 0| + e^{i\alpha}|1\rangle\langle 1|)$

$$P(\alpha)|0\rangle = (|0\rangle\langle 0| + e^{i\alpha}|1\rangle\langle 1|)|0\rangle$$
$$= |0\rangle\langle 0|0\rangle + e^{i\alpha}|1\rangle\langle 1|0\rangle$$
$$= |0\rangle$$

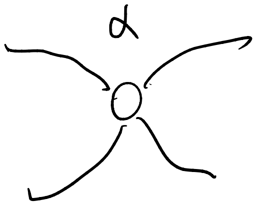
$$P(\alpha)|1\rangle = \cdots = e^{i\alpha}|1\rangle.$$

$|0\rangle^{\dagger} = \langle 0|$ $\qquad (f \circ g)^{\dagger} = g^{\dagger} \circ f^{\dagger}$

$\langle 1|^{\dagger} = |1\rangle$ $\qquad (f + g)^{\dagger} = f^{\dagger} + g^{\dagger}$

$$(|0\rangle\langle 0|)^{\dagger} = \langle 0|^{\dagger} \circ |0\rangle^{\dagger} = |0\rangle\langle 0|.$$

$$P(\alpha)^{\dagger} = P(-\alpha)$$

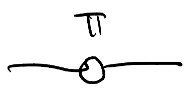


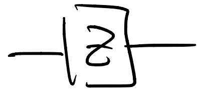
$$A(\alpha) = |00\rangle\langle 00| + e^{i\alpha}|11\rangle\langle 11|$$

$$A(\alpha)^\dagger = |00\rangle\langle 00| + e^{-i\alpha}|11\rangle\langle 11|$$

$$A(\alpha)^\dagger \circ A(\alpha) = \left(|00\rangle\langle 00| + e^{-i\alpha}|11\rangle\langle 11|\right)\circ \left(|00\rangle\langle 00| + e^{i\alpha}|11\rangle\langle 11|\right)$$

$$= |00\rangle\langle 00| + |11\rangle\langle 11|$$

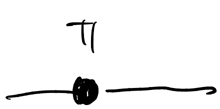
Note: ① 

= 

②  = 

③  $= \sqrt{2}\cdot|+\rangle$

$$R_x(\alpha) = |+\rangle\langle +| + e^{i\alpha}|-\rangle\langle -|$$

① 

$$R_x(\pi) = |+\rangle\langle +| - |-\rangle\langle -|$$

$$R_x(\pi)|0\rangle = |+\rangle\langle +|0\rangle - |-\rangle\langle -|0\rangle$$

$$= \frac{|0\rangle + |1\rangle}{\sqrt{2}} \cdot \frac{\langle 0|0\rangle + \langle 1|0\rangle}{\sqrt{2}}$$

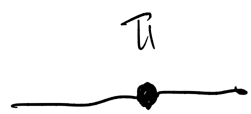
$$- \frac{|0\rangle-|1\rangle}{\sqrt{2}} \cdot \frac{\langle 0|0\rangle-\langle 1|0\rangle}{\sqrt{2}}$$

$$= \frac{|0\rangle+|1\rangle}{2} - \frac{|0\rangle-|1\rangle}{2}$$

$$= \frac{2|1\rangle}{2} = |1\rangle.$$

similarly

$$R_X(\pi)|1\rangle = |0\rangle.$$

So  $= \boxed{X}$

②  $= (|+\rangle\langle+| + |-\rangle\langle-|)\,|0\rangle$

$\langle+|0\rangle = \langle-|0\rangle = \frac{1}{\sqrt{2}}$

$$= \frac{|0\rangle+|1\rangle}{2} + \frac{|0\rangle-|1\rangle}{2}$$

$\langle+|1\rangle = \frac{\langle 0|1\rangle+\langle 1|1\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$$= \frac{2|0\rangle}{2} = |0\rangle$$

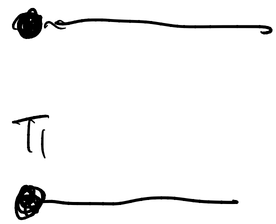
$\langle-|1\rangle = \frac{\langle 0|1\rangle-\langle 1|1\rangle}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$

So $(|+\rangle\langle+| + |-\rangle\langle-|)\,|1\rangle$

$$= \frac{|0\rangle+|1\rangle}{2} - \frac{|0\rangle-|1\rangle}{2}$$

$$= \frac{2|1\rangle}{2} = |1\rangle.$$

3



$$|+\rangle + |-\rangle = \frac{2|0\rangle}{\sqrt{2}} = \sqrt{2}\cdot|0\rangle$$