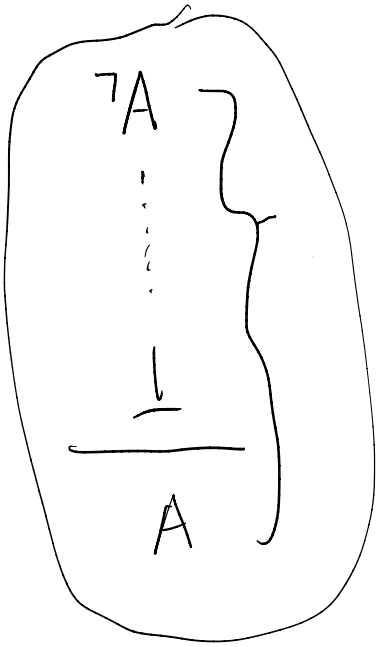
2/11/2025

$\neg A \equiv A \to \bot$.



(M)
There does not exist a term s.t.

$\phi \vdash M : (\neg A \to \bot) \to A$

$= ((A \to \bot) \to \bot) \to A$

$\vdash M : A + \neg A$

$Bool + (Bool \to \bot)$

$x : (A \to \bot) \to \bot \vdash M : A$

$\vdash \lambda x.M : ((A \to \bot) \to \bot) \to A$

$\phi \vdash N : (A \to \bot) \to \neg A$

$= (A \to \bot) \to (A \to \bot)$

$N = \lambda x.x$.

* $\phi \vdash \lambda x.(x,x) : A \to A \times A$

$\phi \vdash \lambda x.() : A \to Unit$

* Linear Types.

$A \times B$

$A ::= Qubit \mid Bool \mid A \multimap B \mid A \otimes B \mid !A$

Unit.

promotion from resource to non-resource.

$!(A \multimap B)$

$\Gamma \vdash M : A$, Parameter Types:

$P ::= Bool \mid !A \mid P \otimes P' \mid Unit$

Parameter context: $\Phi = x_1 : P_1, \ldots x_n : P_n$.

* Typing Rules

$$\frac{}{(\Phi, x : A) \vdash x : A}\ \text{var} \qquad \frac{\Gamma, x : A \vdash M : B}{\Gamma \vdash \lambda x.M : A \multimap B}$$

$$\frac{\Phi, \Gamma_1 \vdash M : A \multimap B \qquad \Phi, \Gamma_2 \vdash N : A}{\Phi, \Gamma_1, \Gamma_2 \vdash M\,N : B}$$

Φ: nonlinear, Γ_1, Γ_2: linear typing context

$$\frac{\Phi, \Gamma_1 \vdash M : A \qquad \Phi, \Gamma_2 \vdash N : B}{\Phi, \Gamma_1, \Gamma_2 \vdash (M, N) : A \otimes B}$$

Φ: nonlinear, Γ_1, Γ_2: linear typing context

$$\frac{\Phi, \Gamma_1 \vdash M : A \otimes B \qquad \Phi, \Gamma_2, x:A, y:B \vdash N : C}{\Phi, \Gamma_1, \Gamma_2 \vdash \underline{\text{let}}\ (x,y) = M\ \underline{\text{in}}\ N : C}$$

$$\frac{\dfrac{\vdash \quad \times}{x:\text{Qubit} \vdash (x,x) : \text{Qubit} \otimes \text{Qubit}}}{\phi \vdash \lambda x.(x,x) : \text{Qubit} \multimap \text{Qubit} \otimes \text{Qubit}}$$

Stuck!

$$\frac{\dfrac{\times}{x:\text{Qubit} \vdash () : \text{Unit}}}{\phi \vdash \lambda x.() : \text{Qubit} \multimap \text{Unit}}$$

← not typable

$$\frac{}{\Phi \vdash () : \text{Unit}}$$

$$\dfrac{\dfrac{}{x : \mathrm{Qubit} \otimes \mathrm{Qubit} \vdash x : \mathrm{Qubit} \otimes \mathrm{Qubit}}\,\mathrm{Var} \qquad \dfrac{}{z : \mathrm{Qubit}, w : \mathrm{Qubit} \vdash z : \mathrm{Qubit}}\,\mathrm{Var}}{x : \mathrm{Qubit} \otimes \mathrm{Qubit} \vdash \underline{\mathrm{let}}\ (z,w) = x\ \underline{\mathrm{in}}\ z : \mathrm{Qubit}}$$

$$\emptyset \vdash \lambda x.\ M : \mathrm{Qubit} \otimes \mathrm{Qubit} \multimap \mathrm{Qubit}$$

M = $\mathrm{let}\ (z,w) = x\ \mathrm{in}\ z$

$$\frac{\Phi \vdash M : A}{\Phi \vdash \underline{\mathrm{lift}}\ M : !A} \qquad \frac{\Gamma \vdash M : !A}{\Gamma \vdash \mathrm{force}\ M : A}$$

$$\frac{\emptyset \vdash \lambda x.\ x : \mathrm{Qubit} \multimap \mathrm{Qubit}}{\emptyset \vdash \mathrm{lift}\,(\lambda x.\ x) : !(\mathrm{Qubit} \multimap \mathrm{Qubit})}$$

* Remarks $\emptyset \not\vdash M : \mathrm{Qubit} \multimap \mathrm{Qubit} \otimes \mathrm{Qubit}$

$$\frac{\dfrac{\Phi \vdash x : !A}{\Phi \vdash x : !A} \qquad \dfrac{\Phi \vdash x : !A}{\Phi \vdash x : !A}}{\Phi, x : !A \vdash (x,x) : !A \otimes !A}$$

$$\emptyset \vdash \lambda x.(x,x) : !A \multimap !A \otimes !A$$