



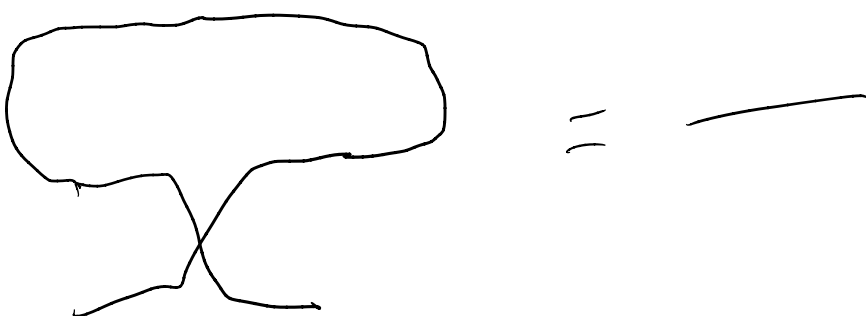
\*  $\alpha = C$



\*  $\text{loop} = \text{line}$

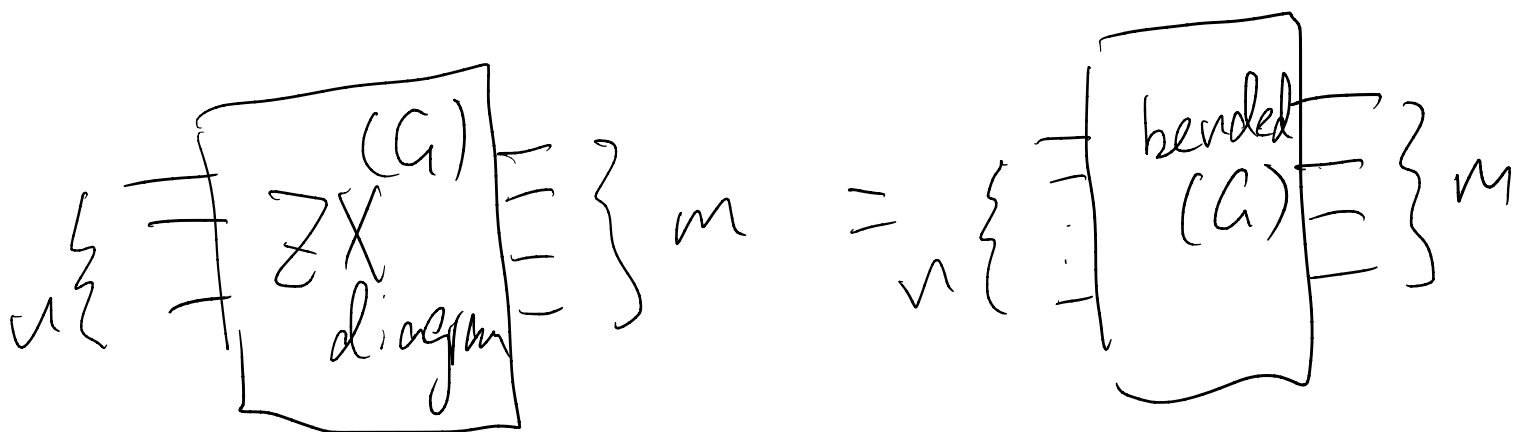


LHS =  $\text{crossing} = \text{line}$

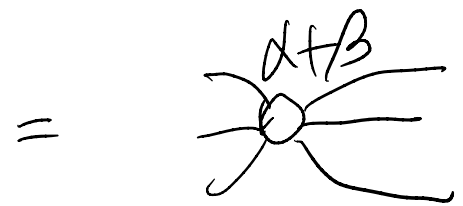
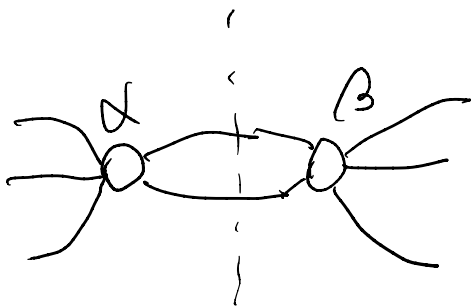
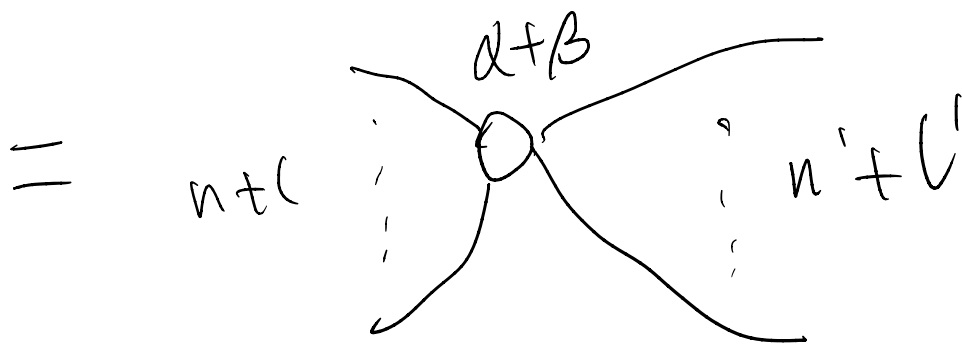
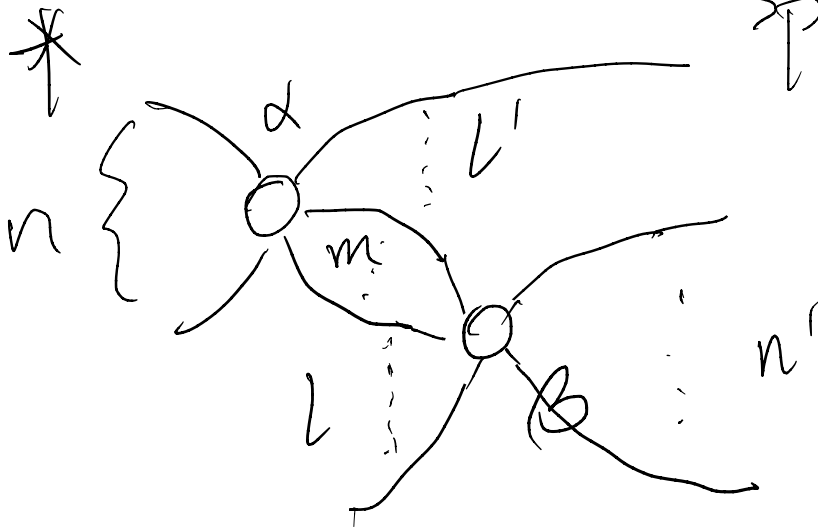


\* wire bending

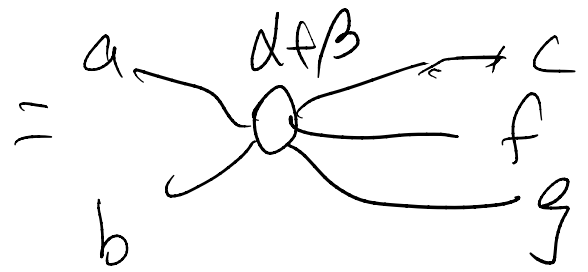
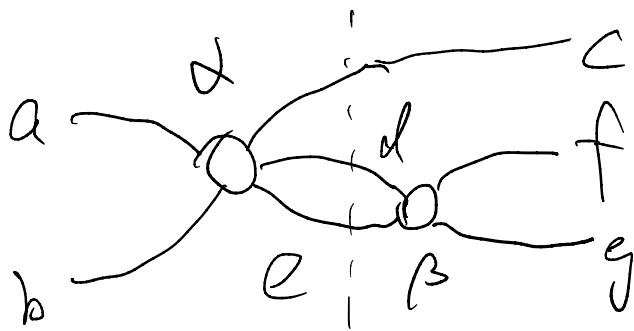
$\text{ZX diagram} (G) = \text{bended} (G)$



# Spider Fusion



$$\begin{aligned}
 & (|000\rangle\langle 00| + e^{i\beta} |111\rangle\langle 11|) \circ (|00\rangle\langle 000| + e^{i\alpha} |11\rangle\langle 111|) \\
 & = |000\rangle\langle 000| + e^{i(\alpha+\beta)} |111\rangle\langle 111|
 \end{aligned}$$



$$\left( \underline{|0\rangle\langle 0| + |1\rangle\langle 1|} \otimes (|00\rangle\langle 00| + e^{i\beta} |11\rangle\langle 11|) \right) \otimes (|000\rangle\langle 000| + e^{i\alpha} |111\rangle\langle 111|)$$

$$= (|0000\rangle\langle 0000| + e^{i\beta} |0111\rangle\langle 0111|$$

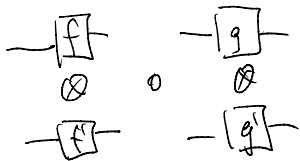
$$+ |1000\rangle\langle 1000| + e^{i\beta} |1111\rangle\langle 1111|) \otimes$$

$$(|000\rangle\langle 000| + e^{i\alpha} |111\rangle\langle 111|)$$

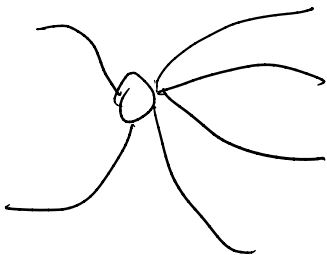
$$= |0000\rangle\langle 0000| + e^{i(\alpha+\beta)} |1111\rangle\langle 1111|$$

Note :

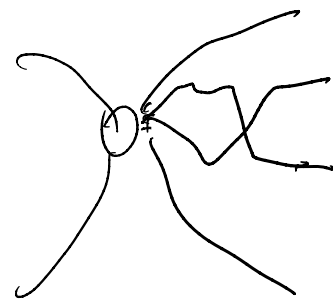
$$\begin{aligned}
 & |00\rangle \langle 00| \\
 &= (|0\rangle \otimes |0\rangle) \cdot (\langle 0| \otimes \langle 0|) \\
 &= |0\rangle \langle 0| \otimes |0\rangle \langle 0|
 \end{aligned}$$



$$\begin{aligned}
 & |0\rangle \langle 0| \otimes |00\rangle \langle 00| \\
 &= |000\rangle \langle 000| \\
 & |0\rangle \langle 0| \otimes |11\rangle \langle 11| \\
 &= |011\rangle \langle 011|
 \end{aligned}$$

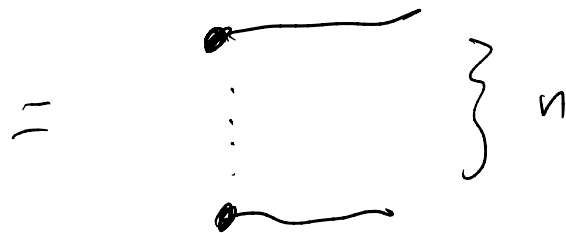
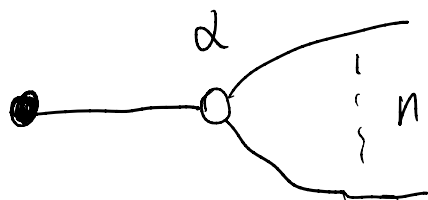


=



$$|0\rangle \langle 0| + e^{i\alpha} |1\rangle \langle 1|$$

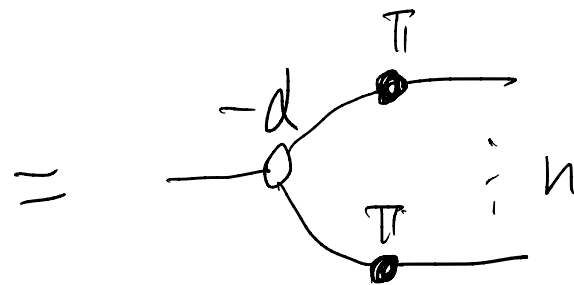
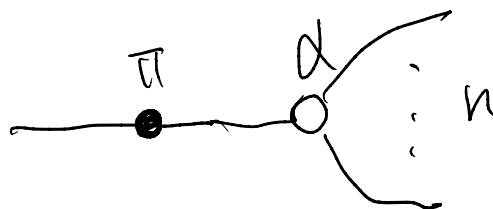
\*



$$|+\rangle + |-\rangle \sim |0\rangle$$

$$(|00\rangle\langle 0| + e^{i\alpha}|11\rangle\langle 1|)|0\rangle = |00\rangle$$

\*



$$(|00\rangle\langle 0| + e^{i\alpha}|11\rangle\langle 1|) \circ X = |00\rangle\langle 0| \circ X +$$

$$e^{i\alpha}|11\rangle\langle 1| \circ X$$

$$= |00\rangle\langle 1| + e^{i\alpha}|11\rangle\langle 0|$$

$$e^{-i\alpha}$$

$$e^{-i\alpha}|00\rangle\langle 1| + |11\rangle\langle 0|$$

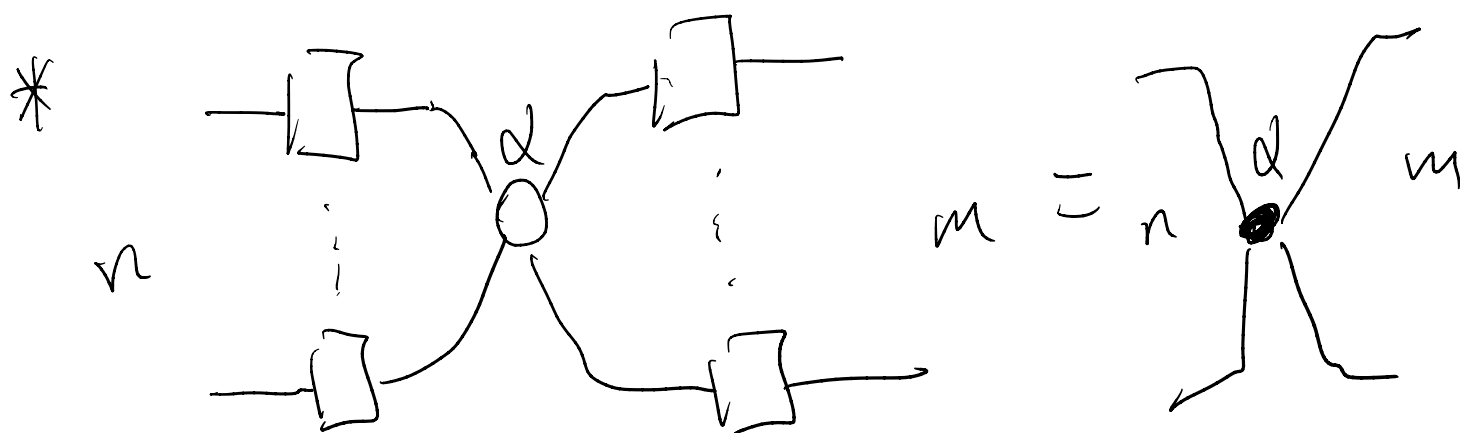
RHS

$$(X \otimes X)_0 (|00\rangle\langle 0| + e^{i\beta} |11\rangle\langle 1|)$$

$$= |11\rangle\langle 0| + e^{i\beta} |00\rangle\langle 1|$$

$$= e^{i\beta} |00\rangle\langle 1| + |11\rangle\langle 0|$$

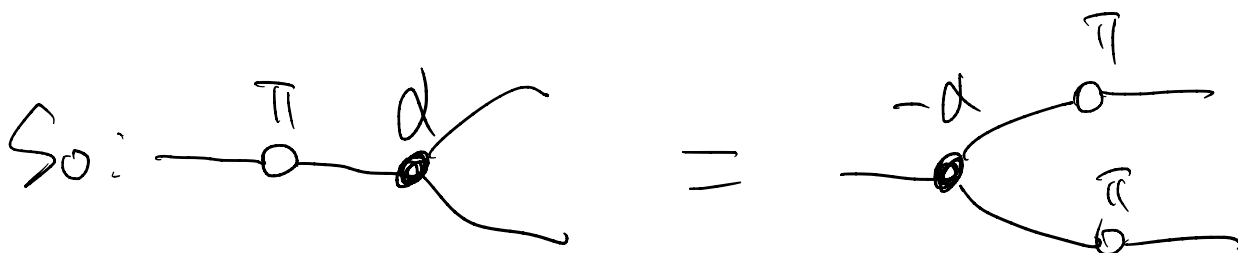
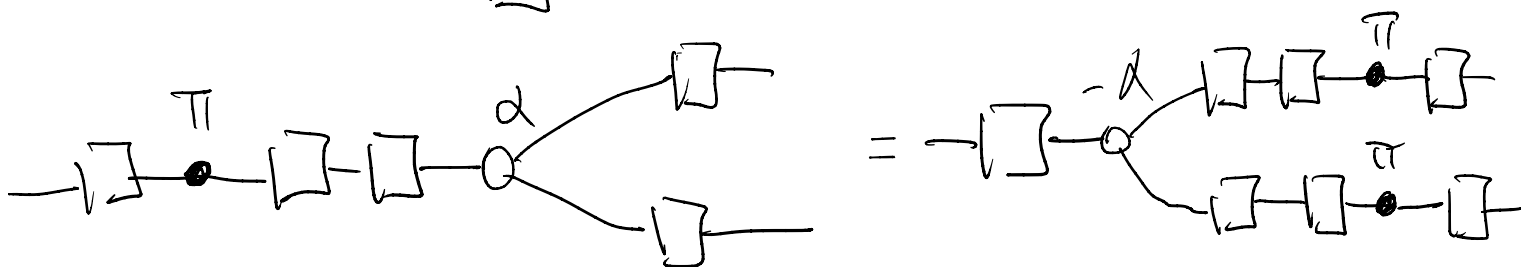
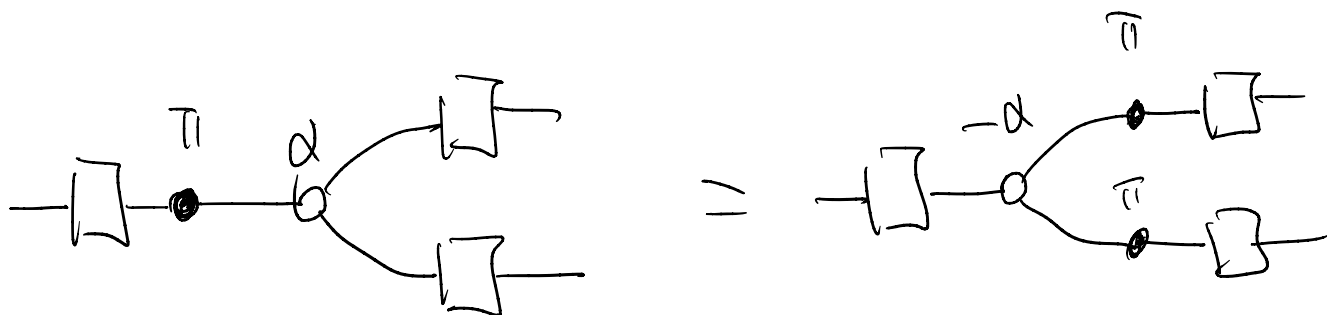
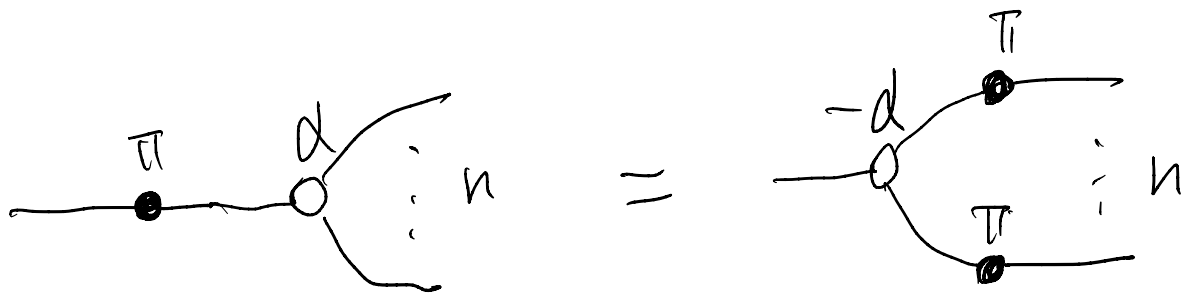
$$\text{so } \beta = -\alpha.$$



$$H^{\otimes n} |0 \dots 0\rangle\langle 0 \dots 0| H^{\otimes n} + e^{i\alpha} |1 \dots 1\rangle\langle 1 \dots 1|$$

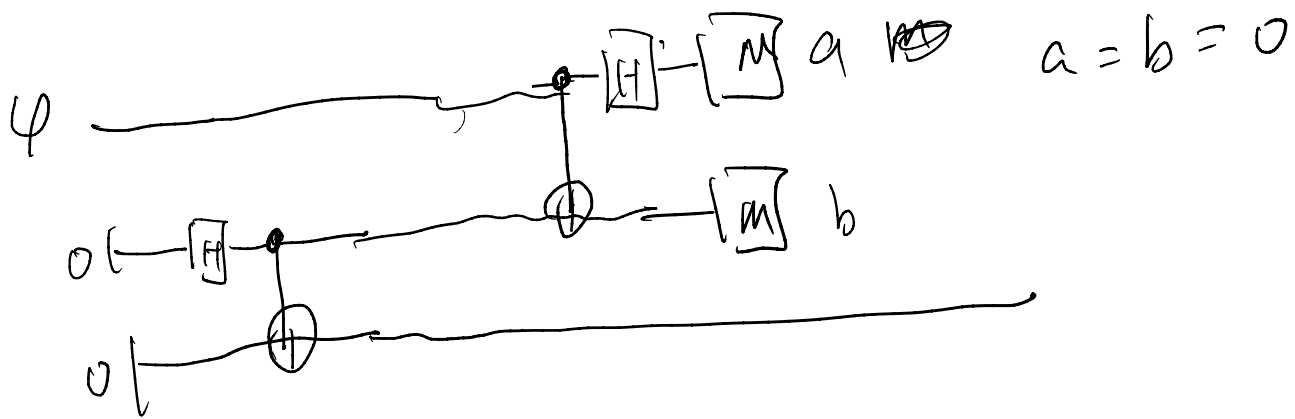
$H^{\otimes n} \quad H^{\otimes n}$

$$= |+\dots+\rangle\langle +\dots+| + e^{i\alpha} |-\dots-\rangle\langle -\dots-|$$

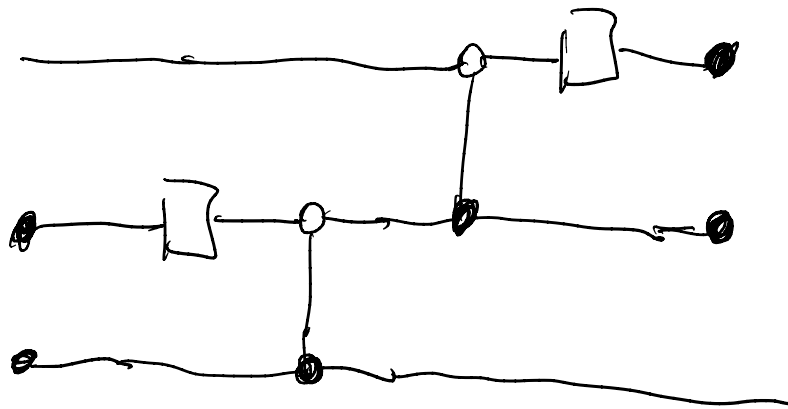


$$* \quad HH = \text{Id}$$

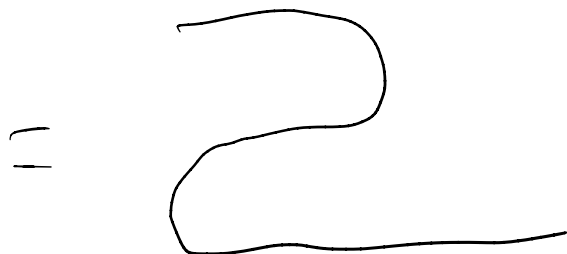
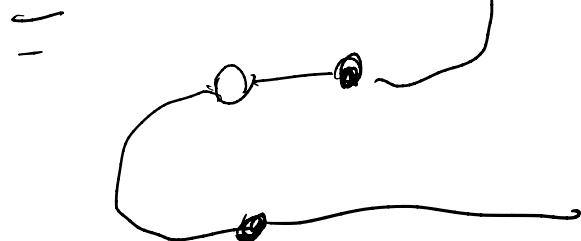
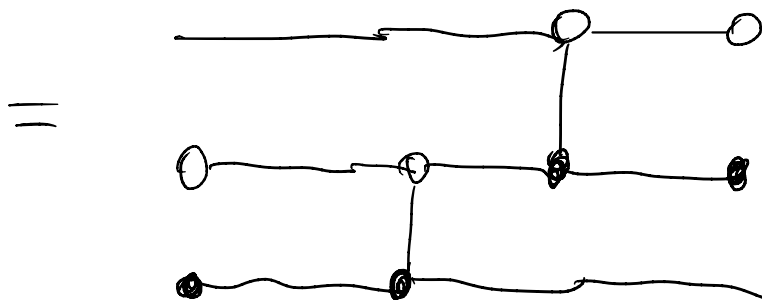
$$\begin{aligned}
 & \text{---} \underset{\circ}{\overset{\frac{\pi}{2}}{\quad}} \text{---} \underset{\bullet}{\overset{\frac{\pi}{2}}{\quad}} \text{---} \underset{\circ}{\overset{\frac{\pi}{2}}{\quad}} \text{---} \underset{\circ}{\overset{\frac{\pi}{2}}{\quad}} \text{---} \underset{\bullet}{\overset{\frac{\pi}{2}}{\quad}} \text{---} \underset{\circ}{\overset{\frac{\pi}{2}}{\quad}} \text{---} \\
 = & \text{---} \underset{\circ}{\overset{\frac{\pi}{2}}{\quad}} \text{---} \underset{\bullet}{\overset{\frac{\pi}{2}}{\quad}} \text{---} \underset{\circ}{\overset{\pi}{\quad}} \text{---} \underset{\bullet}{\overset{\frac{\pi}{2}}{\quad}} \text{---} \underset{\circ}{\overset{\pi}{\quad}} \text{---} \\
 = & \text{---} \underset{\circ}{\overset{\frac{\pi}{2}}{\quad}} \text{---} \underset{\bullet}{\overset{\frac{\pi}{2}}{\quad}} \text{---} \underset{\bullet}{\overset{-\frac{\pi}{2}}{\quad}} \text{---} \underset{\circ}{\overset{\pi}{\quad}} \text{---} \underset{\circ}{\overset{\frac{\pi}{2}}{\quad}} \text{---} \\
 = & \text{---} \underset{\circ}{\overset{\frac{\pi}{2}}{\quad}} \text{---} \underset{\circ}{\overset{3\pi}{\quad}} \text{---} \\
 = & \text{---} \underset{\circ}{\overset{2\pi}{\quad}} \text{---} = \text{---}
 \end{aligned}$$



In ZX,



$\equiv$



$\equiv$

