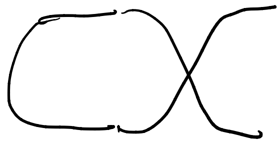
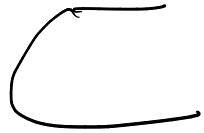
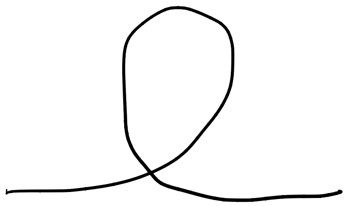
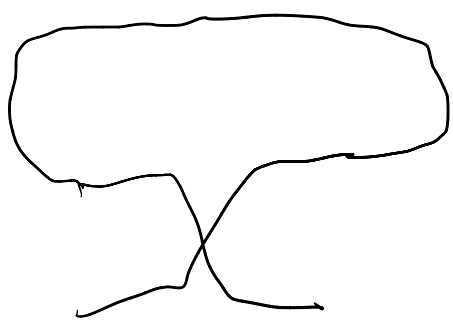
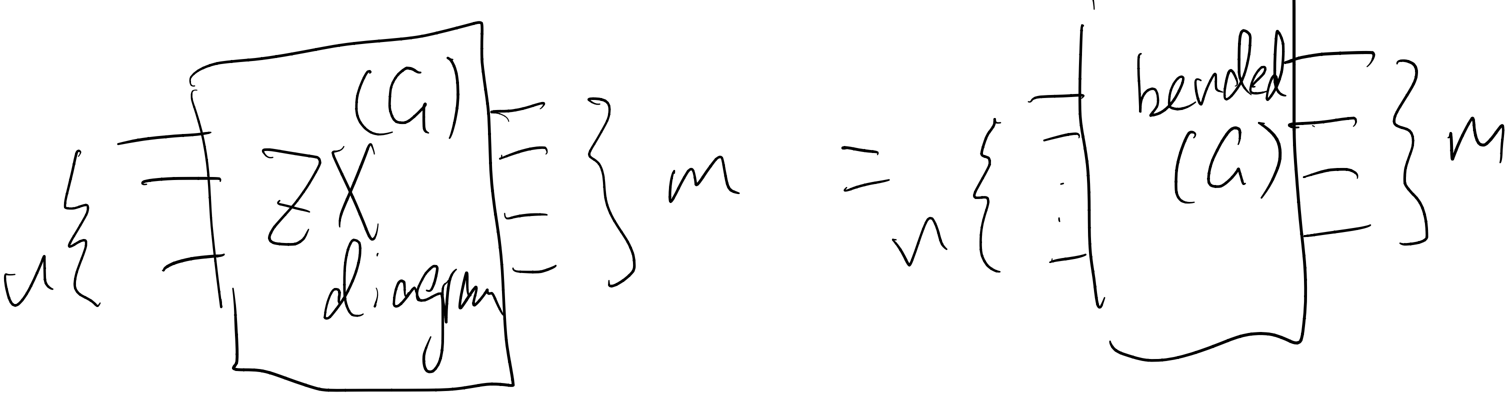
*  = 

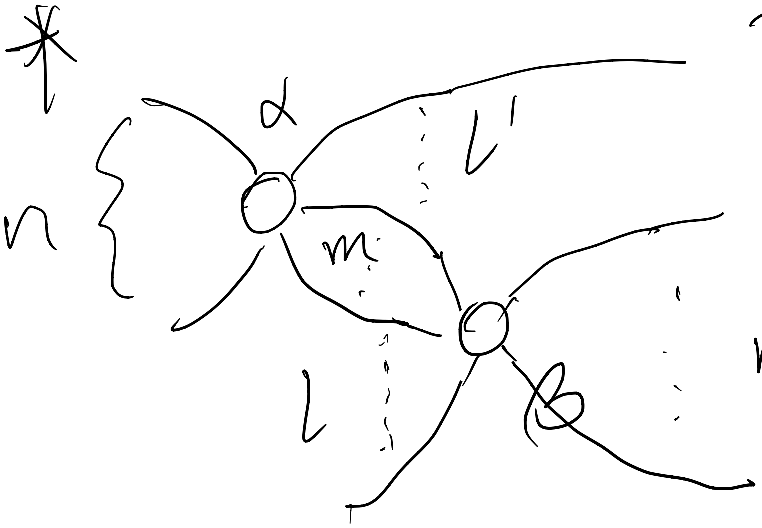
*  = 

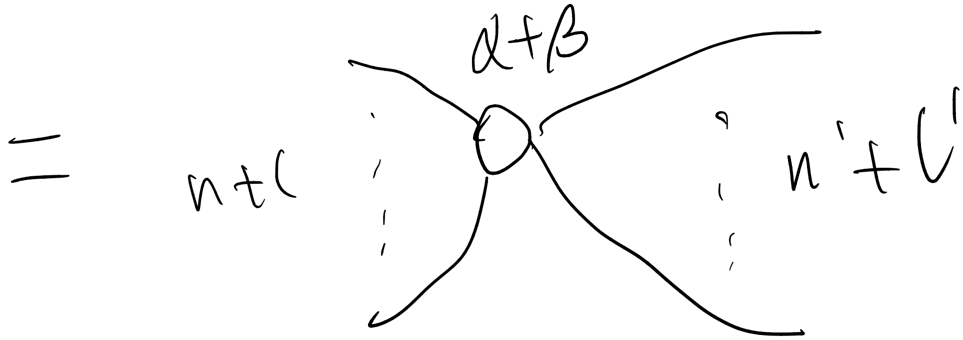
LHS =  = 

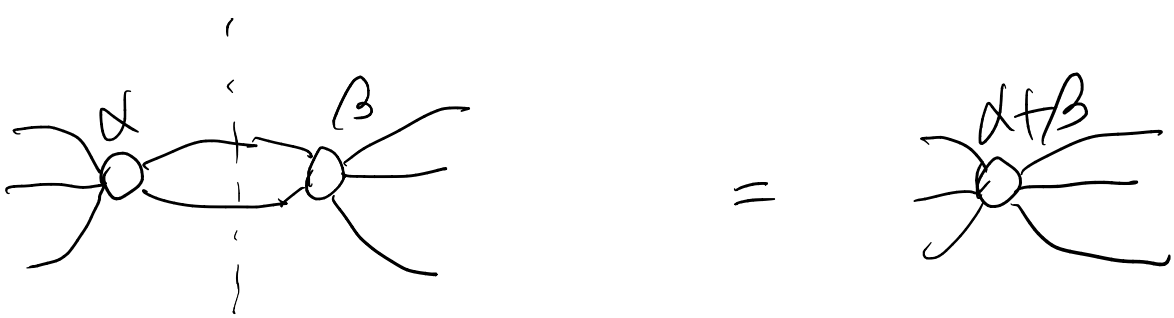
* wire bending



Spider Fusion

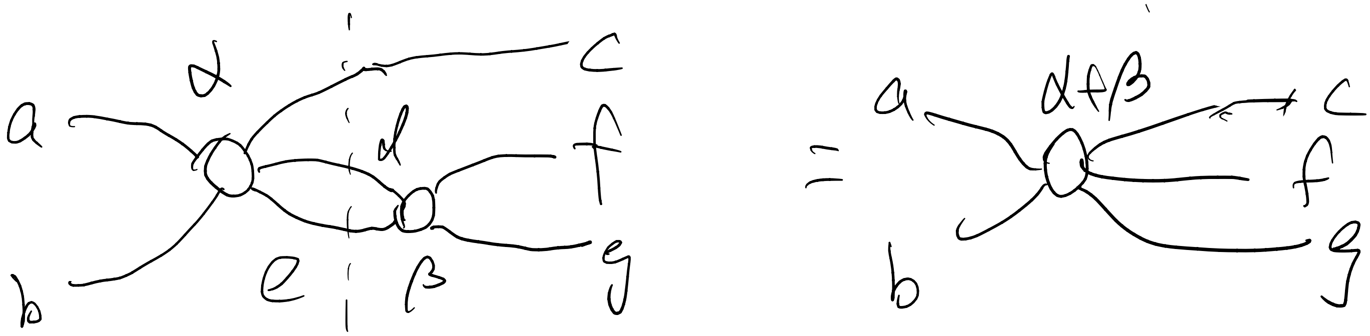






$$(|000\rangle\langle 00| + e^{i\beta}|111\rangle\langle 11|) \circ (|00\rangle\langle 000| + e^{i\alpha}|11\rangle\langle 111|)$$

$$= |000\rangle\langle 000| + e^{i(\alpha+\beta)}|111\rangle\langle 111|$$



$$\left(\underline{|0\rangle\langle 0|+|1\rangle\langle 1|} \otimes \left(|00\rangle\langle 00| + e^{i\beta}|11\rangle\langle 11|\right)\right) \circ \left(|000\rangle\langle 00| + e^{i\alpha}|111\rangle\langle 11|\right)$$

$$= \left(|000\rangle\langle 000| + e^{i\beta}|011\rangle\langle 011| + |100\rangle\langle 100| + e^{i\beta}|111\rangle\langle 111|\right) \circ$$

$$\left(|000\rangle\langle 00| + e^{i\alpha}|111\rangle\langle 11|\right)$$

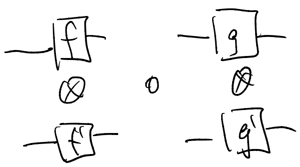
$$= |000\rangle\langle 00| + e^{i(\alpha+\beta)}|111\rangle\langle 11|$$

Note:

$$|00\rangle\langle 00|$$

$$= (|0\rangle \otimes |0\rangle) \circ (\langle 0| \otimes \langle 0|)$$

$$= |0\rangle\langle 0| \otimes |0\rangle\langle 0|$$

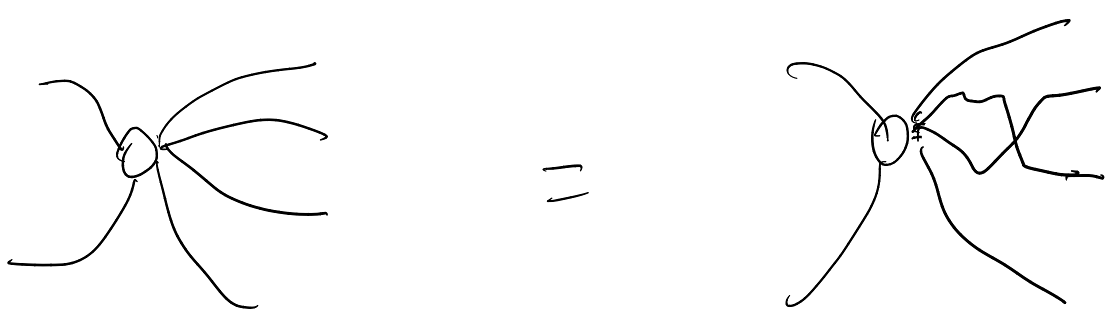


$$|0\rangle\langle 0| \otimes |00\rangle\langle 00|$$

$$= |000\rangle\langle 000|$$

$$|0\rangle\langle 0| \otimes |11\rangle\langle 11|$$

$$= |011\rangle\langle 011|$$



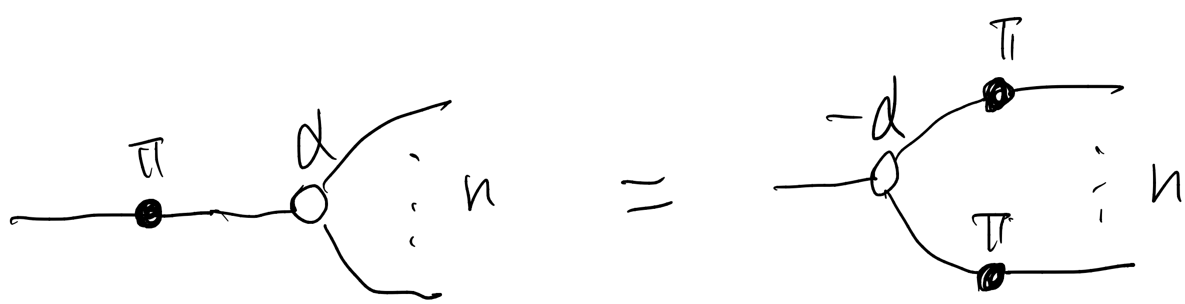
$$|0\rangle\langle 0| + e^{i\alpha} |1\rangle\langle 1|$$

*

$$|+\rangle + |-\rangle \sim |0\rangle$$

$$(|00\rangle\langle 0| + e^{i\alpha}|11\rangle\langle 1|)\,|0\rangle = |00\rangle$$

*



$$(|00\rangle\langle 0| + e^{i\alpha}|11\rangle\langle 1|) \circ X = |00\rangle\langle 0| \circ X + e^{i\alpha}|11\rangle\langle 1| \circ X$$

$$= |00\rangle\langle 1| + e^{i\alpha}|11\rangle\langle 0|$$

$$\overset{e^{-i\alpha}}{\sim} e^{-i\alpha}|00\rangle\langle 1| + |11\rangle\langle 0|$$

RHS

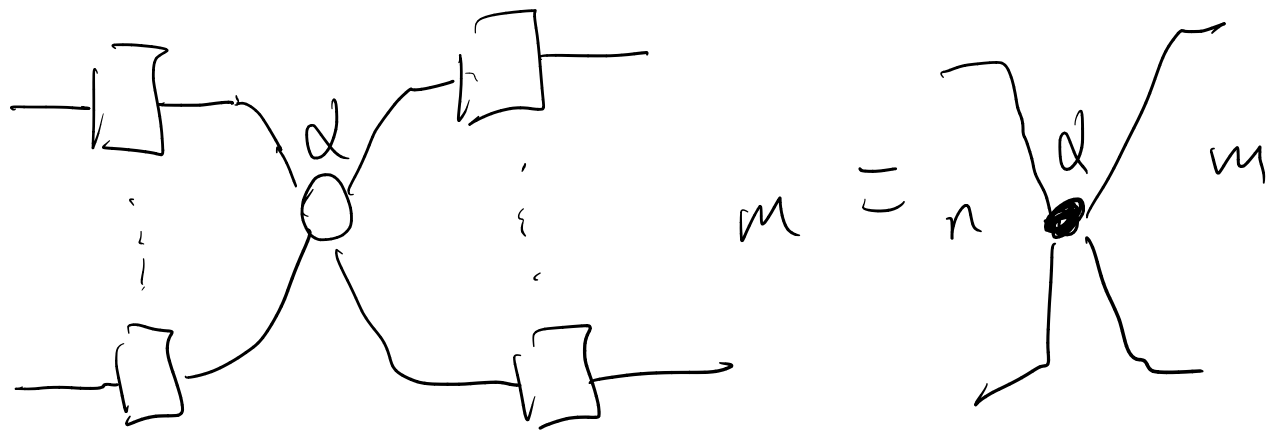
$$(X \otimes X) \circ \left(|00\rangle\langle 0| + e^{i\beta} |11\rangle\langle 1| \right)$$

$$= |11\rangle\langle 0| + e^{i\beta} |00\rangle\langle 1|$$

$$= e^{i\beta} |00\rangle\langle 1| + |11\rangle\langle 0|$$

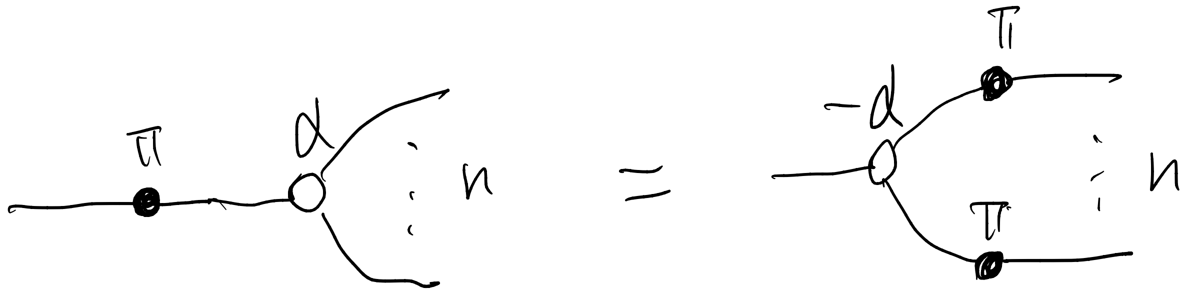
so $\beta = -\alpha$.

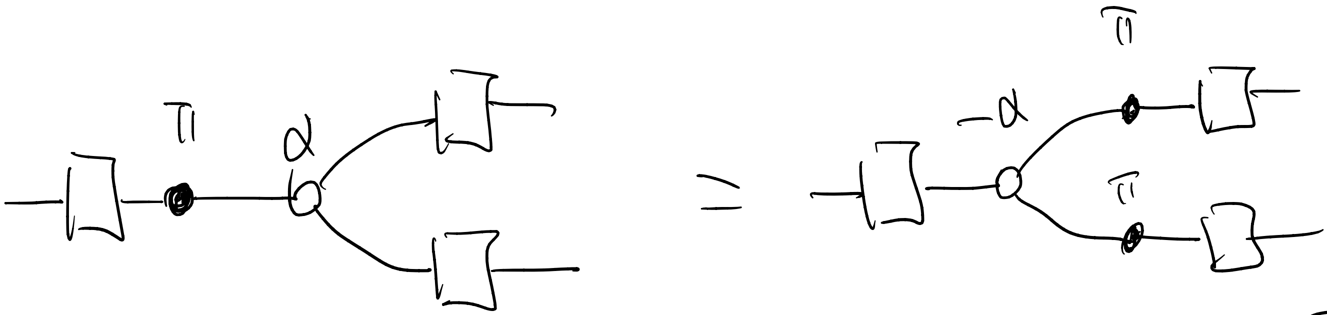
*

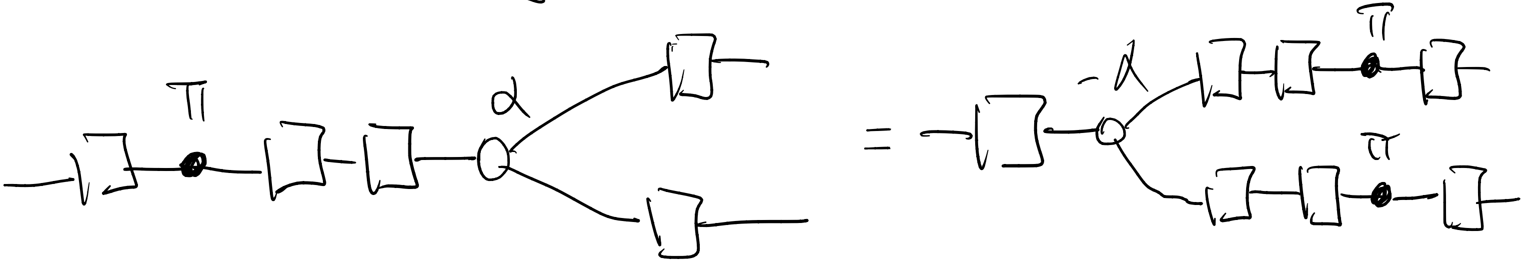


$$H^{\otimes n} \circ |0\ldots0\rangle\langle 0 \ldots 0| H^{\otimes n} + e^{i\alpha} H^{\otimes n} |1 \ldots 1\rangle\langle 1 \ldots 1| H^{\otimes n}$$

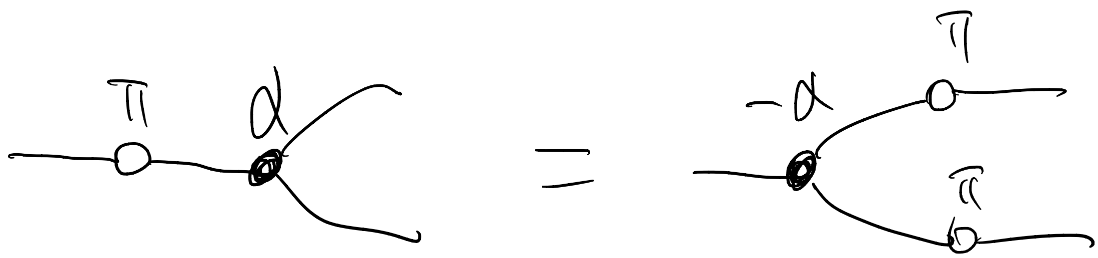
$$= |+\ldots+\rangle\langle +\ldots+| + e^{i\alpha} |-\ldots-\rangle\langle -\ldots-|$$



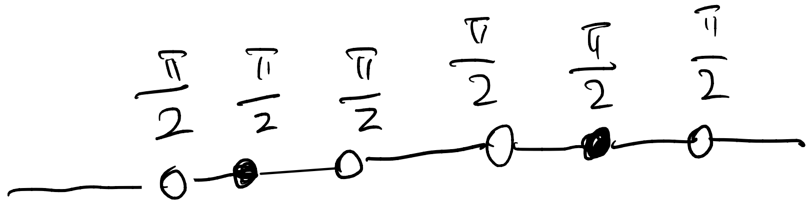




So:

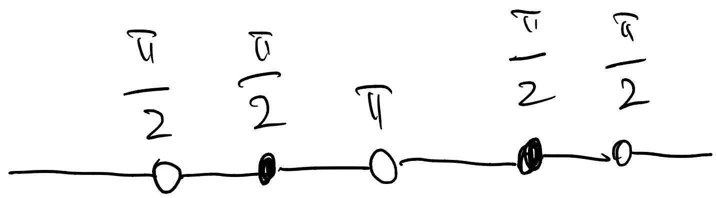


$* \quad HH = Id$



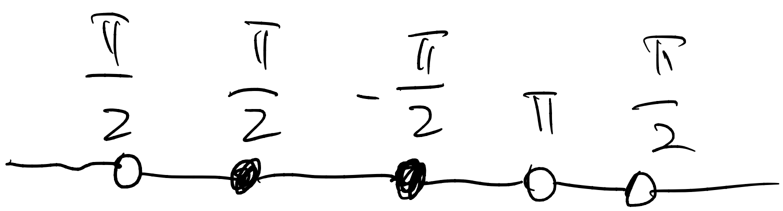
$\frac{\pi}{2}$ $\frac{\pi}{2}$ $\frac{\pi}{2}$ $\frac{\pi}{2}$ $\frac{\pi}{2}$ $\frac{\pi}{2}$

$=$



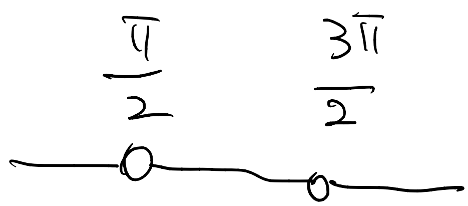
$\frac{\pi}{2}$ $\frac{\pi}{2}$ π $\frac{\pi}{2}$ $\frac{\pi}{2}$

$=$



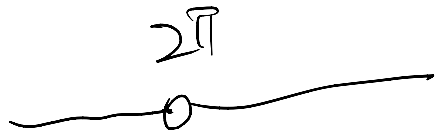
$\frac{\pi}{2}$ $\frac{\pi}{2}$ $-\frac{\pi}{2}$ π $\frac{\pi}{2}$

$=$



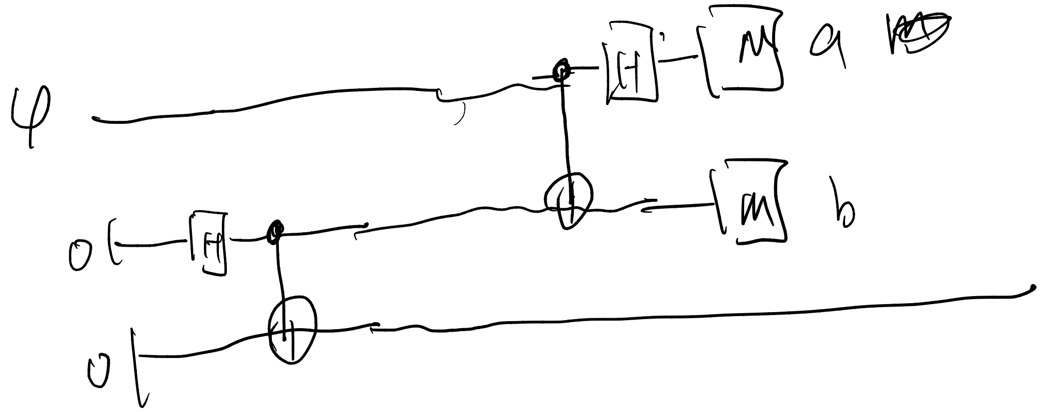
$\frac{\pi}{2}$ $\frac{3\pi}{2}$

$=$



2π

$=$



$a = b = 0$

In ZX,