2/18/2025

* Well-typed configuration denoted by

" $\vdash (\varphi, L, M) : A$ "

meaning: $\varphi \in \text{Qubit}^{\otimes |L|}$

$$\Sigma_L = l_1 : \text{Qubit}, \ldots\ l_n : \text{Qubit}$$

$$\Sigma_L \vdash M : A$$

$$\Gamma = x_1 : A, \ldots\ x_n : A, \ l_1 : \text{Qubit}, \ldots\ l_n : \text{Qubit}$$

* Type preservation Thm.

if $\vdash (\varphi, L, M) : A$ and

$$(\varphi, L, M) \rightsquigarrow (\varphi', L', M')$$

then $\vdash (\varphi', L', M') : A$

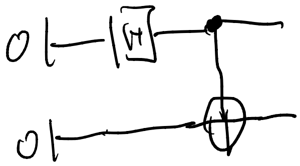
* Example of evaluation.
writing '&' for 'force'

$$\Phi \vdash H : !(\mathrm{Qubit} \multimap \mathrm{Qubit})$$

$$\Phi \vdash CX : !(\mathrm{Qubit} \otimes \mathrm{Qubit} \multimap \mathrm{Qubit} \otimes \mathrm{Qubit})$$

$$\Phi \vdash \mathrm{Init0} : !(\mathrm{Unit} \multimap \mathrm{Qubit})$$

$$\mathrm{bell00} = \&CX(\&H((\&\mathrm{Init0})()), (\&\mathrm{Init})()) : \mathrm{Qubit} \otimes \mathrm{Qubit}$$



* Initial configuration

$$(c, [\,], M)$$

$$c \in I = \mathbb{C} \quad |c|^2 = 1$$

I, Qubit, Qubit $\otimes$ Qubit ......

$= \mathbb{C}$

$$\vdash (c, [\,], \&CX(\&H(\underline{\&Init0()}), \&Init0())) : Qubit \otimes Qubit$$

$$\rightsquigarrow \vdash (\underbrace{c \otimes |0\rangle}_{\approx |0\rangle}, [l_1], \&CX(\underline{\&H\ l_1}, \&Init0())) : Q^2$$

$$\rightsquigarrow \vdash (\underbrace{H|0\rangle}_{= |+\rangle}, [l_1], \&CX(l_1, \underline{\&Init0()})) : Q^2$$

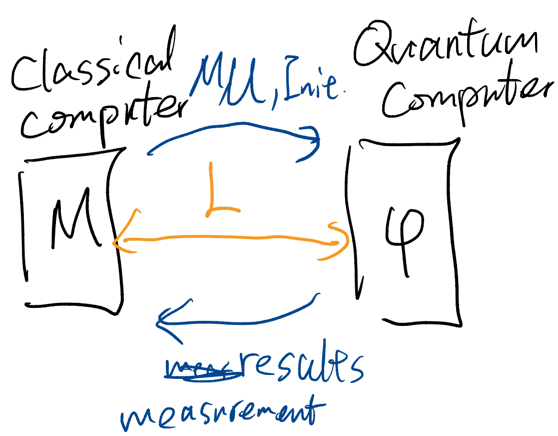
$$\rightsquigarrow \vdash (|+\rangle|0\rangle, [l_1, l_2], \&CX(l_1, l_2)) : Q^2$$

$$\rightsquigarrow (CX(|+\rangle \otimes |0\rangle), [l_1, l_2], (l_1, l_2)) : Q^2$$

$$= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$= \left(\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), [l_1, l_2], (l_1, l_2)\right)$$

$$(\varphi, L, M)$$



$$\left(l_1 \leftarrow \mathrm{Init}\,0 ; l_2 \leftarrow \mathrm{Init}\,0 \; ; \; H\, l_1 \, , \; CX(l_1, l_2) \right)$$

$$* \quad M = \mathrm{let}\ (x, y) = \underbrace{\mathrm{belloo}}_{\mathrm{Qubit} \otimes \mathrm{Qubit}}\ \mathrm{in}\ (\&\mathrm{Meas}\ x\, ,\ \&\mathrm{Meas}\ y) = \mathrm{Bool} \otimes \mathrm{Bool}$$

$$\Phi \vdash \mathrm{Meas} : !(\mathrm{Qubit} \multimap \mathrm{Bool})$$

$$(C, [\,], \mathrm{let}\ (x, y) = \mathrm{belloo}\ \mathrm{in}\ (\&\mathrm{Meas}\ x, \&\mathrm{Meas}\ y))$$

$$\overset{*}{\rightsquigarrow} \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}} , [l_1, l_2], \underline{\mathrm{let}\ (x, y) = (l_1, l_2)}\ \mathrm{in}\ \underline{(\&\mathrm{Meas}\ x\, ,\ \&\mathrm{Meas}\ y)} \right)$$

$$\rightsquigarrow \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}} , [l_1, l_2] , (\underline{\&\mathrm{Meas}\ l_1} , \&\mathrm{Meas}\ l_2) \right)$$

$$= \frac{1}{\sqrt{2}} \cancel{|0\rangle} |0\rangle + \frac{1}{\sqrt{2}} \cancel{|1\rangle} |1\rangle$$

$$① \overset{0.5}{\rightsquigarrow} \left(|0\rangle , [l_2] , (\mathrm{False}, \underline{\&\mathrm{Meas}\ l_2}) \right)$$

$$② \overset{0.5}{\rightsquigarrow} \left(|1\rangle , [l_2] , (\mathrm{True} , \&\mathrm{Meas}\ l_2) \right)$$

① $\rightsquigarrow (c, [\,], (False, False)) : Bool \otimes Boo$

② $\rightsquigarrow (c, [\,], (True, True)) : Bool \otimes Bool.$