2/13/2025

$$\frac{}{(\Phi, x : A) \vdash x : A}\ \text{var} \qquad \frac{\Gamma, x : A \vdash M : B}{\Gamma \vdash \lambda x. M : A \multimap B}$$

$$\frac{\Phi, \Gamma_1 \vdash M : A \multimap B \qquad \Phi, \Gamma_2 \vdash N : A}{\Phi, \Gamma_1, \Gamma_2 \vdash M N : B}$$

$$\frac{\Phi, \Gamma_1 \vdash M : A \qquad \Phi, \Gamma_2 \vdash N : B}{\Phi, \Gamma_1, \Gamma_2 \vdash (M, N) : A \otimes B}$$

$$\frac{\Phi, \Gamma_1 \vdash M : A \otimes B \qquad \Phi, \Gamma_2, x : A, y : B \vdash N : C}{\Phi, \Gamma_1, \Gamma_2 \vdash \underline{\text{let}}\ \underbrace{(x, y) = M\ \underline{\text{in}}\ N} : C}$$

$$\frac{\Phi \vdash M : A}{\Phi \vdash \text{lift}\ M : !A} \qquad \frac{\Gamma \vdash M : !A}{\Gamma \vdash \text{force}\ M : A}$$

$$\frac{}{\Phi \vdash () : \text{Unit}}$$

$\lambda x : ()$: Qubit $\cancel{\multimap}$ Unit

$$\frac{}{\Phi \vdash H : \,!(\mathrm{Qubit} \multimap \mathrm{Qubit})}$$

$$\frac{}{\Phi \vdash \mathrm{CNot} : \,!(\mathrm{Qubit} \otimes \mathrm{Qubit} \multimap \mathrm{Qubit} \otimes \mathrm{Qubit})}$$

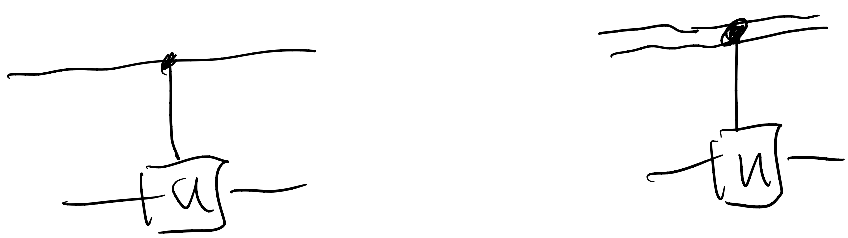
$$\frac{}{\Phi \vdash \mathrm{Init0} : \,!(\mathrm{Unit} \multimap \mathrm{Qubit})}$$

$$\frac{}{\Phi \vdash \mathrm{Term0} : \,!(\mathrm{Qubit} \multimap \mathrm{Unit})}$$

"——|0"

$$\frac{}{\Phi \vdash \mathrm{Meas} : \,!(\mathrm{Qubit} \multimap \mathrm{Bool})}$$

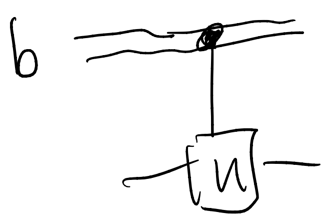
"1|——



$$\frac{}{\Phi \vdash \text{True} : \text{Bool}}$$

False :

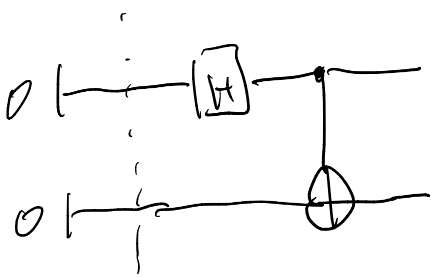
$$\frac{\Phi, \Gamma_1 \vdash M : \text{Bool} \qquad \begin{array}{c} \Phi, \Gamma_2 \vdash N_1 : C \\ \Phi, \Gamma_2 \vdash N_2 : C \end{array}}{\Phi, \Gamma_1, \Gamma_2 \vdash \text{if } \underbrace{M \text{ then } N_1 \text{ else } N_2}_{} : C}$$



if b then $\underset{\sim}{U}$ else(lift id)

$U : !(\text{Qubit} \multimap \text{Qubit})$

$\text{lift id} : !(\text{Qubit} \multimap \text{Qubit})$



$\vdash (\text{force CX})\ ([(\text{force } H)\ (\text{force Init0})\ ()],\ (\text{force Init0})\ ())$

'&'

$: \text{Qubit} \otimes \text{Qubit}$

$!(\text{Unit} \multimap \text{Qubit} \otimes \text{Qubit})$

* Evaluation

Configuration $(\varphi, L, \underline{M}) \rightsquigarrow (\varphi', L', M')$

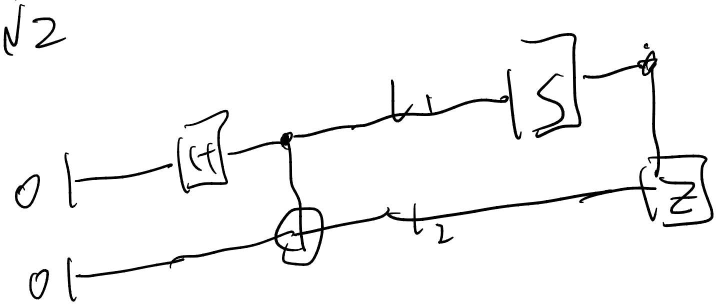
$\varphi \in \underbrace{\text{Qubit} \otimes \text{Qubit} \cdots \text{Qubit}}_{|L|}$

Labels. $l_1, l_2, \ldots$

$L = [l_1, \ldots l_n]$

$$\frac{}{l_i : \text{Qubit} \vdash l_i : \text{Qubit}}$$

$\left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}, [l_1, l_2], (l_1, l_2) \right)$



$V ::= l_i \mid \lambda x.M \mid (V_1, V_2) \mid \text{True} \mid \text{False} \mid ()$
$\quad \mid \text{lift } M \mid H \mid \text{Init0} \mid \text{force } U$

$$\frac{\text{fresh } l', \quad L' = L, l'}{(\varphi, L, \underbrace{(\text{force Init0})\,()}_{\text{Qubit}}) \rightsquigarrow (\varphi \otimes |0\rangle, L', \underset{\text{Qubit}}{l'})}$$

$$\frac{\varphi' = [\![U]\!]_{l_i \cdots l_j}(\varphi)}{(\varphi, L, (\text{force } U)(l_i, \cdots l_j)) \rightsquigarrow (\varphi', L, (l_i \cdots l_j))}$$

$$\frac{\varphi = a\psi + b\gamma \qquad \psi_{l_i} = |0\rangle \quad \gamma_{l_i} = |1\rangle}{(\varphi, L, (\text{force Meas})\, l_i) \overset{|a|^2}{\rightsquigarrow} (\psi/|0\rangle_{l_i}, L/l_i, \text{False})}$$

$$\overset{|b|^2}{\rightsquigarrow} (\gamma/|1\rangle_{l_i}, L/l_i, \text{True})$$

$$\frac{}{(\varphi, L, (\lambda x . M) V) \rightsquigarrow (\varphi, L, [V/x] M)}$$

$$\frac{}{(\varphi, L, \text{let}(x, y) = (V_1, V_2) \text{ in } N) \rightsquigarrow (\varphi, L, [V_1/x, V_2/y] N)}$$

$$\frac{}{(\varphi, L, \mathrm{force}(\mathrm{Lift}\ \underline{M})) \rightsquigarrow (\varphi, L, M)}$$

$$\frac{}{(\varphi, L, \mathrm{if\ True\ then}\ N_1\ \mathrm{else}\ N_2) \rightsquigarrow (\varphi, L, N_1)} \qquad \text{False:} \left\{ (\varphi, L, N_2) \right\}$$

$$\frac{(\varphi, L, M_1) \rightsquigarrow (\varphi', L', M_1')}{(\varphi, L, (M_1, M_2)) \rightsquigarrow (\varphi', L', (M_1', M_2))}$$

$$\frac{(\varphi, L, M_2) \rightsquigarrow (\varphi', L', M_2')}{(\varphi, L, (V_1, M_2)) \rightsquigarrow (\varphi', L', (V_1, M_2'))}$$

application is similar.