* Functor category.

$$[\underline{C}, \underline{D}] \quad F \in obj([\underline{C}, \underline{D}])$$

$$F : \underline{C} \to \underline{D}.$$

morphisms are natural transformation

$$F \xrightarrow{\alpha} G \xrightarrow{\beta} H$$

$$\forall f : A \to B \qquad
\begin{tikzcd}
FA \arrow[r, "\alpha_A"] \arrow[d] \arrow[dr, phantom, "\circlearrowright"] & GA \arrow[r, "\beta_A"] \arrow[d] \arrow[dr, phantom, "\circlearrowright"] & HA \arrow[d] \\
FB \arrow[r, "\alpha_B"] & GB \arrow[r, "\beta_B"'] & HB
\end{tikzcd}$$

* $[\underline{C}, \underline{C}]$

A monoid in $[\underline{C}, \underline{C}]$ is called <u>**Monad**</u>. In other words.

A monad consists of :

$$\mu : M \circ M \to M$$

$$\eta : Id \to M.$$

such that $\forall A \in \underline{C}$

$$
\begin{tikzcd}
MMMA \arrow[r, "M\mu_A"] \arrow[d, "\mu_{MA}"'] & MMA \arrow[d, "\mu"] \\
MMA \arrow[r, "\mu"] & MA
\end{tikzcd}
$$

(associtivity)

$$
\begin{tikzcd}
MA \arrow[r, "\eta_{MA}"] \arrow[d, "M\eta_A"'] \arrow[dr, equal] & MMA \arrow[d, "\mu"] \\
MMA \arrow[r, "\mu"] & MA
\end{tikzcd}
$$

(unit)

* Examples of monad : The power set functor $P : \underline{Set} \to \underline{Set}$.

$$\eta_A : A \to PA.$$

$$\eta_A(x) = \{x\}.$$

$$\mu_A : P(P(A)) \longrightarrow P(A)$$

$$= \{ S \mid S \subseteq P(A) \}$$

$$\mu_A(S) = \bigcup S$$

$$\begin{tikzcd}
P(A) \arrow[r, "\eta_{PA}"] \arrow[dr, bend right] & PPA \arrow[d, "\mu"] \\
& PA
\end{tikzcd}$$

$$\{ S \mid S \subseteq A \}$$

$$\eta_{PA}(S) = \{ S \}$$

$$\mu(\{S\}) = S$$

$$\{ S \mid S \subseteq PPA \} = PPPA$$

$$\begin{tikzcd}
PPPA \arrow[r, "P\mu"] \arrow[d, "\mu_{PA}"'] & PPA \arrow[d, "\mu"] \\
PPA \arrow[r] & PA
\end{tikzcd}$$

$$\bigcup S$$

$$P\mu(S) = \mu(S) = \bigcup S$$