Proto-Quipper with dynamic lifting

Frank Fu

Joint work with K. Kishida, N.J. Ross and P. Selinger

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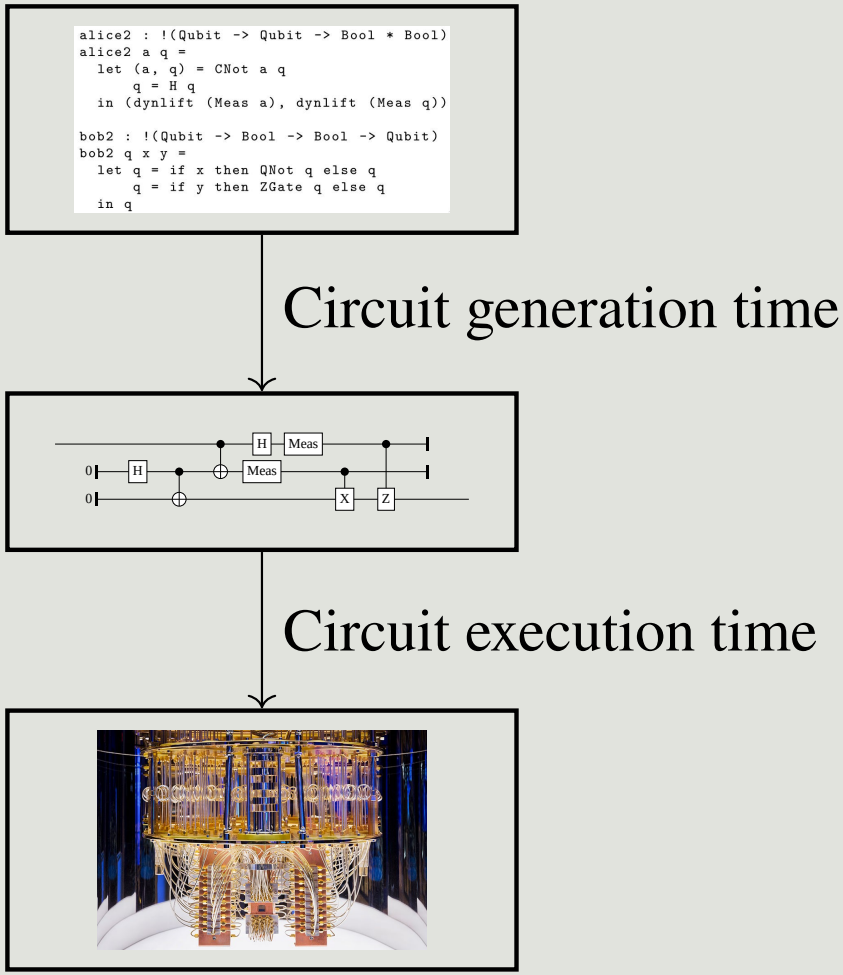
Background and motivation

- Quipper and Proto-Quipper.
- Extend Proto-Quipper with dynamic lifting.
  - Categorical semantics for dynamic lifting.
  - Type system and operational semantics for dynamic lifting.

Quipper/Proto-Quipper's two runtimes

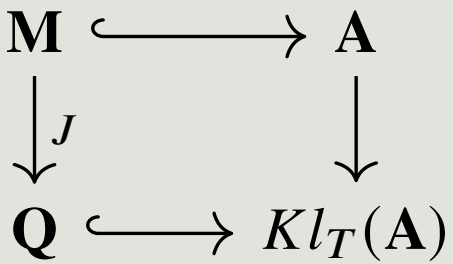


Values in the two runtimes

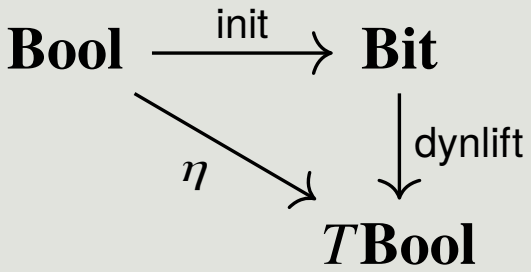
- Parameters (e.g., **Nat**, **Bool**).
- States (e.g., **Qubit**, **Bit**).
- Measurement: **Qubit** $\rightarrow$ **Bit**.
- Dynamic lifting: an *operation* that "lifts" a **Bit** to **Bool**.
- Why dynamic lifting?

A category for the two runtimes

- Category of quantum circuits **M**.
- Category of quantum operations **Q**.
- $Kl_T(\mathbf{A})$: Kleisli category of T, where T is commutative.



A diagram for dynamic lifting



Modalities for dynamic lifting

- Modality: $\alpha = 0 \mid 1$.
- Typing judgments: $\Gamma \vdash_\alpha M : A$.
  - $[\![M]\!] : [\![\Gamma]\!] \to \alpha[\![A]\!]$
- Types: $!_\alpha A$ and $A \multimap_\alpha B$.
  - $[\![!_\alpha A]\!] = !\alpha[\![A]\!]$
  - $[\![A \multimap_\alpha B]\!] = [\![A]\!] \multimap \alpha[\![B]\!]$.
- What is the point of these modalities?

Type system

$$\frac{\Gamma \vdash_\alpha M : \mathbf{Bit}}{\Gamma \vdash_0 \mathsf{dynlift}\ M : \mathbf{Bool}} \qquad \frac{\Gamma \vdash_\alpha M : !_1(S \multimap_1 U)}{\Gamma \vdash_\alpha \mathsf{box}_S M : \mathbf{Circ}(S, U)}$$

Operational Semantics

- Circuit generation time: $(C, M) \Downarrow (C', V)$
- Circuit execution time: $(Q, M) \Downarrow \sum_{i \in [n]} p_i(Q_i, V_i)$

$$\frac{(Q, M) \Downarrow (Q', \ell) \quad \text{read}(Q', \ell)}{(Q, \mathsf{dynlift}\, M) \Downarrow p_1(Q_1, \text{True}) + p_2(Q_2, \text{False})}$$

Main results

- A general categorical model for dynamic lifting.
- A type system and operational semantics that are sound w.r.t. the categorical model.
- Demo.