Proto-Quipper with dynamic lifting

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Background and motivation

- Quipper and Proto-Quipper.
- Extend Proto-Quipper with dynamic lifting.
  - Categorical semantics for dynamic lifting.
  - Type system and operational semantics for dynamic lifting.
Quipper/Proto-Quipper’s two runtimes

Circuit generation time

Circuit execution time
Values in the two runtimes

- Parameters (e.g., \texttt{Nat}, \texttt{Bool}).
- States (e.g., \texttt{Qubit}, \texttt{Bit}).
- Measurement: \texttt{Qubit} $\rightarrow$ \texttt{Bit}.
- Dynamic lifting: an \emph{operation} that “lifts” a \texttt{Bit} to \texttt{Bool}.
- Why dynamic lifting?
A category for the two runtimes

- Category of quantum circuits $\mathbf{M}$.
- Category of quantum operations $\mathbf{Q}$.
- $\mathcal{K}_T(A)$: Kleisli category of $T$, where $T$ is commutative.
A diagram for dynamic lifting
Modalities for dynamic lifting

- Modality: $\alpha = 0 \mid 1$.
- Typing judgments: $\Gamma \vdash_{\alpha} M : A$.
  - $\left\llbracket M \right\rrbracket : \left\llbracket \Gamma \right\rrbracket \rightarrow \alpha \left\llbracket A \right\rrbracket$
- Types: $!_{\alpha} A$ and $A \rightarrow_{\alpha} B$.
  - $\left\llbracket !_{\alpha} A \right\rrbracket = !_{\alpha} \left\llbracket A \right\rrbracket$
  - $\left\llbracket A \rightarrow_{\alpha} B \right\rrbracket = \left\llbracket A \right\rrbracket \rightarrow_{\alpha} \left\llbracket B \right\rrbracket$.
- What is the point of these modalities?
Type system

\[
\begin{align*}
\Gamma \vdash_\alpha M : \text{Bit} & \quad \Gamma \vdash_\alpha M : !_1(S \rightarrow_1 U) \\
\Gamma \vdash_0 \text{dynlift } M : \text{Bool} & \quad \Gamma \vdash_\alpha \text{box}_SM : \text{Circ}(S, U)
\end{align*}
\]
Operational Semantics

- Circuit generation time: \((C, M) \downarrow (C', V)\)
- Circuit execution time: \((Q, M) \downarrow \sum_{i \in [n]} p_i(Q_i, V_i)\)

\[
(Q, M) \downarrow (Q', \ell) \quad \text{read}(Q', \ell)
\]

\[
(Q, \text{dynlift } M) \downarrow p_1(Q_1, \text{True}) + p_2(Q_2, \text{False})
\]
Main results

- A general categorical model for dynamic lifting.
- A type system and operational semantics that are sound w.r.t. the categorical model.
- Demo.