Proto-Quipper with dynamic lifting

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Quipper and Proto-Quipper

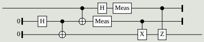
Code

```
alice2 : !(Qubit -> Qubit -> Bool * Bool)
alice2 a q =
  let (a, q) = CNot a q
      q = H q
  in (dynlift (Meas a), dynlift (Meas q))

bob2 : !(Qubit -> Bool -> Bool -> Qubit)
bob2 q x y =
  let q = if x then QNot q else q
      q = if y then ZGate q else q
  in q
```

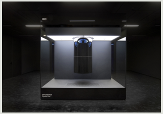
Circuit generation time

Circuit



Circuit execution time

Quantum computer



Dynamic Lifting

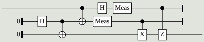
Code

```
alice2 : !(Qubit -> Qubit -> Bool * Bool)
alice2 a q =
  let (a, q) = CNot a q
      q = H q
  in (dynlift (Meas a), dynlift (Meas q))

bob2 : !(Qubit -> Bool -> Bool -> Qubit)
bob2 q x y =
  let q = if x then QNot q else q
      q = if y then ZGate q else q
  in q
```

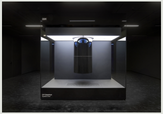
Nat, Bool

Circuit



Qubit, Bit

Quantum computer



The two runtimes assumption as categories

- A category of quantum circuits $\mathbf{M}$.
- A category of quantum operations $\mathbf{Q}$.
- An interpretation functor $J : \mathbf{M} \to \mathbf{Q}$.

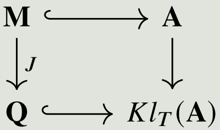
Categorical model for dynamic lifting

- A category $\mathbf{A}$ equipped with a monad T such that

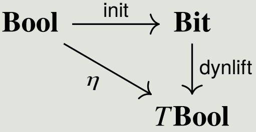
$$\begin{array}{ccc} \mathbf{M} & \hookrightarrow & \mathbf{A} \\ \downarrow{\scriptstyle J} & & \downarrow \\ \mathbf{Q} & \hookrightarrow & Kl_T(\mathbf{A}) \end{array}$$

Categorical model for dynamic lifting

- A category **A** equipped with a monad T such that



- Dynamic lifting is a morphism in **A** such that



Modalities for dynamic lifting

- $\Gamma \vdash_\alpha M : A$, where $\alpha = 0 \mid 1$.
- Dynamic lifting.

$$\frac{\Gamma \vdash_\alpha M : \mathbf{Bit}}{\Gamma \vdash_0 \mathsf{dynlift}\ M : \mathbf{Bool}}$$

- Modality indicates boxability.

$$!_1(S \multimap_1 U) \xrightarrow{\mathsf{box}} \mathrm{Circ}(S, U)$$

- Type system tracks modalities.

$$\frac{\Gamma, x : A \vdash_\alpha M : B}{\Gamma \vdash_1 \lambda x.M : A \multimap_\alpha B} \qquad \frac{\Gamma_1 \vdash_{\alpha_1} M : A \multimap_{\alpha_2} B \quad \Gamma_2 \vdash_{\alpha_3} N : A}{\Gamma_1, \Gamma_2 \vdash_{\alpha_1 \& \alpha_2 \& \alpha_3} MN : B}$$

Categorical semantics

- $\Gamma \vdash_1 M : A$ is a map in $\mathbf{A}$:

$$[\![\Gamma]\!] \stackrel{[\![M]\!]}{\rightarrow} [\![A]\!]$$

- $\Gamma \vdash_0 M : A$ is a map in $Kl_T(\mathbf{A})$:

$$[\![\Gamma]\!] \stackrel{[\![M]\!]}{\rightarrow} T[\![A]\!]$$

Operational Semantics

- Circuit generation time: $(C, M) \Downarrow (C', V)$
- Circuit execution time: $(Q, M) \Downarrow \sum_{i \in [n]} p_i(Q_i, V_i)$

$$\frac{(Q, M) \Downarrow (Q', \ell)}{(Q, \mathsf{dynlift}\, M) \Downarrow \mathrm{read}(Q', \ell)}$$

where $\mathrm{read}(Q', \ell) = p_1(Q_1, \mathrm{True}) + p_2(Q_2, \mathrm{False})$.

Dynamic lifting in practice: quick demo

Main results

- A general categorical model for dynamic lifting.
- A type system uses modality to track dynamic lifting.
- Operational semantics for the two runtimes.
- Type system and operational semantics are sound w.r.t. the categorical semantics.