Designing Quantum Programming Languages with Types

Frank Fu

Computer Science and Engineering Department, UofSC

Why quantum programming languages?

- Researchers have shown quantum algorithms can offer substantial speed-up for certain computing tasks.
- Advances in quantum hardware from companies like IBM and Google.
- Quantum algorithms are usually expressed using quantum circuits.
- Quantum algorithms are commonly expressed at a high level.
- Debugging quantum algorithms can be expensive.

My research interest

Build tools to facilitate programming quantum computers.

- How to design a high-level programming language for quantum circuits?
- How to verify quantum programs?
- How to run a high-level programming language on actual quantum computer?
- What algorithms to run on current quantum computer?

Why types?

- Lightweight specifications of programs.
- Allow compiler to enforce invariants via type checking.
- A well-typed program satisfies certain properties.

Background on types: an idealized programming language

- Programs $M, N := \ x \mid \lambda x.M \mid MN$.
- Types $A, B := \ \mathcal{C} \mid A \to B$.
- Typing environment $\Gamma = x_1 : A_1, ..., x_n : A_n$.
- Typing judgment $\Gamma \vdash M : A$.
- Typing rules

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A} \qquad \frac{\Gamma, x : A \vdash M : B}{\Gamma \vdash \lambda x.M : A \to B} \qquad \frac{\Gamma \vdash M : A \to B \quad \Gamma \vdash N : A}{\Gamma \vdash MN : B}$$

Type safety

- A *type checker* checks $\Gamma \vdash M : A$.
- An *evaluator* performs evaluation $M \Downarrow V$.
- *Type safety*
  If $\Gamma \vdash M : A$ and $M \Downarrow V$, then $\Gamma \vdash V : A$.

Fancy types

- Linear types: $A \multimap B$.
- Dependent types: $(n : \mathsf{Nat}) \to \mathsf{Vec}\, A\, n \to \mathsf{Vec}\, A\, n$.
- Types with modalities: $A \to_\alpha B$.

Types for Quantum Computing

The basic types in Quantum Computing.

- **Bit**: $|0\rangle, |1\rangle$.
- **Qubit**: $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$, where $\alpha, \beta \in \mathbb{C}, |\alpha|^2 + |\beta|^2 = 1$.
- Multi-qubits are represented by a tensor product.
  $\text{Qubit} \otimes \text{Qubit}, \text{Qubit} \otimes \text{Qubit} \otimes \text{Qubit}, \text{Qubit} \otimes \text{Bit}$, etc.

Qubits are resource

- No cloning: *one can not duplicate a qubit.*

  ~~`dup x = (x, x)`~~

- Qubit does not exist in a vacuum.

  `Init0` : Unit ⊸ Qubit

  ```
  let x = Init0 () in ...
  ```

- Qubit does not disappear into the ether.

  `Discard` : Qubit ⊸ Unit

  ```
  let x = Init0 () in ...
  let _ = Discard x in ...
  ```

Updating Qubits: unitary operations

One way to update qubits is via *unitary operations*.

- Reversibility: $UU^\dagger = U^\dagger U = I$.
- Linearity: $U(\alpha|0\rangle + \beta|1\rangle) = \alpha U|0\rangle + \beta U|1\rangle$.

Common quantum gates

- Hadamard gate.

$$H|0\rangle = 1/\sqrt{2}(|0\rangle + |1\rangle)$$
$$H|1\rangle = 1/\sqrt{2}(|0\rangle - |1\rangle)$$

- Phase gate.

$$S|0\rangle = |0\rangle$$
$$S|1\rangle = i|1\rangle$$

- T gate.

$$T|0\rangle = |0\rangle$$
$$T|1\rangle = \omega|1\rangle, \text{where } \omega^2 = i$$

- CNOT gate.

$$\mathrm{CNOT}|00\rangle = |00\rangle \quad \mathrm{CNOT}|01\rangle = |01\rangle$$
$$\mathrm{CNOT}|10\rangle = |11\rangle \quad \mathrm{CNOT}|11\rangle = |10\rangle$$

Types for quantum gates

- Hadamard gate.

$$H : \mathrm{Qubit} \multimap \mathrm{Qubit}$$

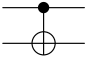
- Phase gate.

$$S : \mathrm{Qubit} \multimap \mathrm{Qubit}$$

- T gate.

$$T : \mathrm{Qubit} \multimap \mathrm{Qubit}$$

- CNOT gate.



$$\mathrm{CNOT} : \mathrm{Qubit} \otimes \mathrm{Qubit} \multimap \mathrm{Qubit} \otimes \mathrm{Qubit}$$

Measurement

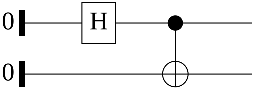
Measurement is needed to readout the bit information from qubit.

$$\mathrm{Meas} : \mathsf{Qubit} \multimap \mathsf{Bit}$$

- $M(\alpha|0\rangle + \beta|1\rangle) = |0\rangle$ with probability $|\alpha|^2$.
- $M(\alpha|0\rangle + \beta|1\rangle) = |1\rangle$ with probability $|\beta|^2$.

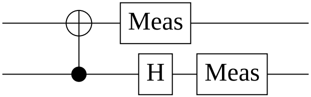
Programming quantum circuits in Proto-Quipper

```
bell00 : !(Unit -> Qubit * Qubit)
bell00 u =
  let a = Init0 ()
      b = Init0 ()
  in CNot b (H a)
```



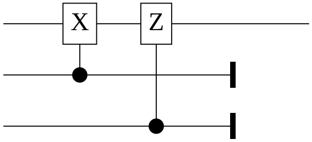
Programming quantum circuits in Proto-Quipper

```
alice : !(Qubit -> Qubit -> Bit * Bit)
alice a q =
  let (a, q) = CNot a q
      q = H q
  in (Meas a, Meas q)
```



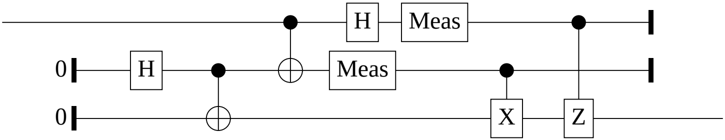
Programming quantum circuits in Proto-Quipper

```
bob : !(Qubit -> Bit -> Bit -> Qubit)
bob q x y =
  let (q, x) = C_X q x
      (q, y) = C_Z q y
      _ = Discard x
      _ = Discard y
  in q
```



Programming quantum circuits in Proto-Quipper

```
tele : !(Qubit -> Qubit)
tele q =
  let (b, a) = bell00 ()
      (x, y) = alice a q
      z = bob b x y
  in z
```



Interleaving circuit generation time and circuit execution via dynamic lifting

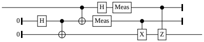
Code

```
alice2 : !(Qubit -> Qubit -> Bool * Bool)
alice2 a q =
  let (a, q) = CNot a q
      q = H q
  in (dynlift (Meas a), dynlift (Meas q))

bob2 : !(Qubit -> Bool -> Bool -> Qubit)
bob2 q x y =
  let q = if x then QNot q else q
      q = if y then ZGate q else q
  in q
```

Nat, <u>Bool</u>

Circuit



Qubit, <u>Bit</u>

Quantum computer

Types for dynamic lifting

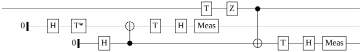
- $\Gamma \vdash_\alpha M : A$, where $\alpha = 0 \mid 1$.
- Dynamic lifting.

$$\frac{\Gamma \vdash_\alpha M : \mathsf{Bit}}{\Gamma \vdash_0 \mathsf{dynlift}\ M : \mathsf{Bool}}$$

- Type system distinguishes computation that uses dynamic lifting vs computation that corresponds to quantum circuits.

Programming with dynamic lifting

```
v3 : !(Qubit -> Qubit)
v3 q =
  let a1 = tgate_inv (H (Init0 ()))
      a2 = H (Init0 ())
      (a1, a2) = CNot a1 a2
      a1 = H (TGate a1)
  in if dynlift (Meas a1)
     then
       let _ = Discard (Meas a2)
       in v3 q
     else let q = ZGate (TGate q)
              (a2, q) = CNot a2 q
              a2 = H (TGate a2)
          in if dynlift (Meas a2)
             then v3 (ZGate q)
             else q
```



Future research

- How do we verify the correctness of a quantum program?
  - How to prove two quantum circuits are equal?
  - How to develop tests to ensure the programs perform correctly?
- How do we compile a high-level quantum programs to lower level languages (e.g., QIR, OpenQasm)?
- Suppose we have a 127 Qubits machine, what algorithms should we run on it?

Thank you!