781 2013-04-16

Note Title 2013-04-16

Example of resolution proof: we will prove a property of groups, namely the existence of the right-inverse

Group axioms: $(P(x,y,z) \equiv x \circ y = z)$

operation. Examples: $\circ$ is $+$ or $\circ$ is $*$ and the universe is reals.

(1) closure under the operation

$\forall x \forall y \exists z\, P(x,y,z)$ (or: $\forall x \forall y \exists z\ x \circ y = z$).

(2) associativity

$\forall u \forall v \forall w \forall x \forall y \forall z$

$$\big((P(x,y,u) \wedge P(y,z,v)) \Rightarrow (P(x,v,w) \Leftrightarrow P(u,z,w))\big)$$

OR: $x \circ y = u \wedge y \circ z = v \Rightarrow (x \circ v = w \Leftrightarrow u \circ z = w)$

$y \circ z \qquad x \circ y$

(3) existence of left-neutral element and of left inverse;

$$\exists x \,(\forall y\, P(x,y,y) \wedge \forall y\, \exists z\, P(z,y,x))$$

in the example interpretation 0 or 1

OR $\exists x\,(x \circ y = y \wedge \exists z \; z \circ y = x)$ left-neutral element

left inverse

Show the existence of the right-inverse:

(4) $\exists x\,(\forall y\, P(x,y,y) \wedge \forall y\, \exists z\, P(y,z,x))$

$x \circ y = y$ ← left-neutral element

$y \circ z = x$ ← right-inverse

$(1) \wedge (2) \wedge (3) \models (4)$ or $(1) \wedge (2) \wedge (3) \wedge \sim(4)$ is inconsistent

We will show this by resolution.

We convert (1), (2), (3), $\sim$(4) into clausal form, and obtain (a), (b), (c), (d), (e), (f)

(a) comes from (1): $\{P(x, y, m(x,y))\}$
(by Skolemizing)

(b) from 2 ($\Rightarrow$): $\{\neg P(x,y,u), \neg P(y,z,v), \neg P(x,v,w), P(u,z,w)\}$

(c) from 2 ($\Leftarrow$): $\{\neg P(x,y,u), \neg P(y,z,v), \neg P(u,z,w), P(x,v,w)\}$

(d) from 3 $\{P(e,y,y)\}$ (e is the left neutral element)

(e) from 3 $\{P(i(y),y,e)\}$ ($i(y)$ is the left inverse of y)

(f) from $\neg$(4) $\{\neg P(x, j(x), j(x)), \neg P(k(x), z, x)\}$

OR: $\neg(x \circ j(x) = j(x) \wedge k(x) \circ z = x)$

(f) (d)

$$\{\neg P(k(e), z, e)\}.$$

Complete;

HW 6 showing the substitution for at least the first step. Due Thursday.