2013-04-11

Note Title 2013-04-11

Exercise 86 [Schöning]

Every barber shaves all persons who do not shave themselves

$\forall x \forall y \left((\neg \text{Shaves}(x,x) \wedge \text{IsBarber}(y)) \Rightarrow \text{Shaves}(y,x) \right)$

$\forall x \forall y \left(\neg [\neg \text{Shaves}(x,x) \wedge \text{IsBarber}(y)] \vee \text{Shaves}(y,x) \right)$

$\forall x \forall y \left(\text{Shaves}(x,x) \vee \neg \text{IsBarber}(y) \right) \vee \text{Shaves}(y,x)$

$\forall x \forall y \left[\text{Shaves}(x,x) \vee \neg \text{IsBarber}(y) \vee \text{Shaves}(y,x) \right]$

(a) $\{Shaves(x,x), \neg IsBarber(y), Shaves(y,x)\}$

(one could have started with

$\forall x\,(B(x) \Rightarrow \forall y\,(\neg S(y,y) \Rightarrow S(x,y)))$

and obtained the same clause)

No barber shaves every person who shaves himself.

$\forall x\,(Shaves(x,x) \Rightarrow \neg \exists y\,(IsBarber(y) \wedge Shaves(y,x)))$

$\forall x\,(\neg Shaves(x,x) \vee \neg \exists y\,(IsBarber(y) \wedge Shaves(y,x)))$

$\forall x \left[\neg \text{Shaves}(x,x) \vee \forall y \neg \left(\text{IsBarber}(y) \wedge \text{Shaves}(y,x) \right) \right]$

$\forall x \forall y \left[\neg \text{Shaves}(x,x) \vee \neg \text{IsBarber}(y) \vee \neg \text{Shaves}(y,x) \right]$

(b) $\{ \neg \text{Shaves}(x,x), \neg \text{IsBarber}(y), \neg \text{Shaves}(y,x) \}$

You could have started with

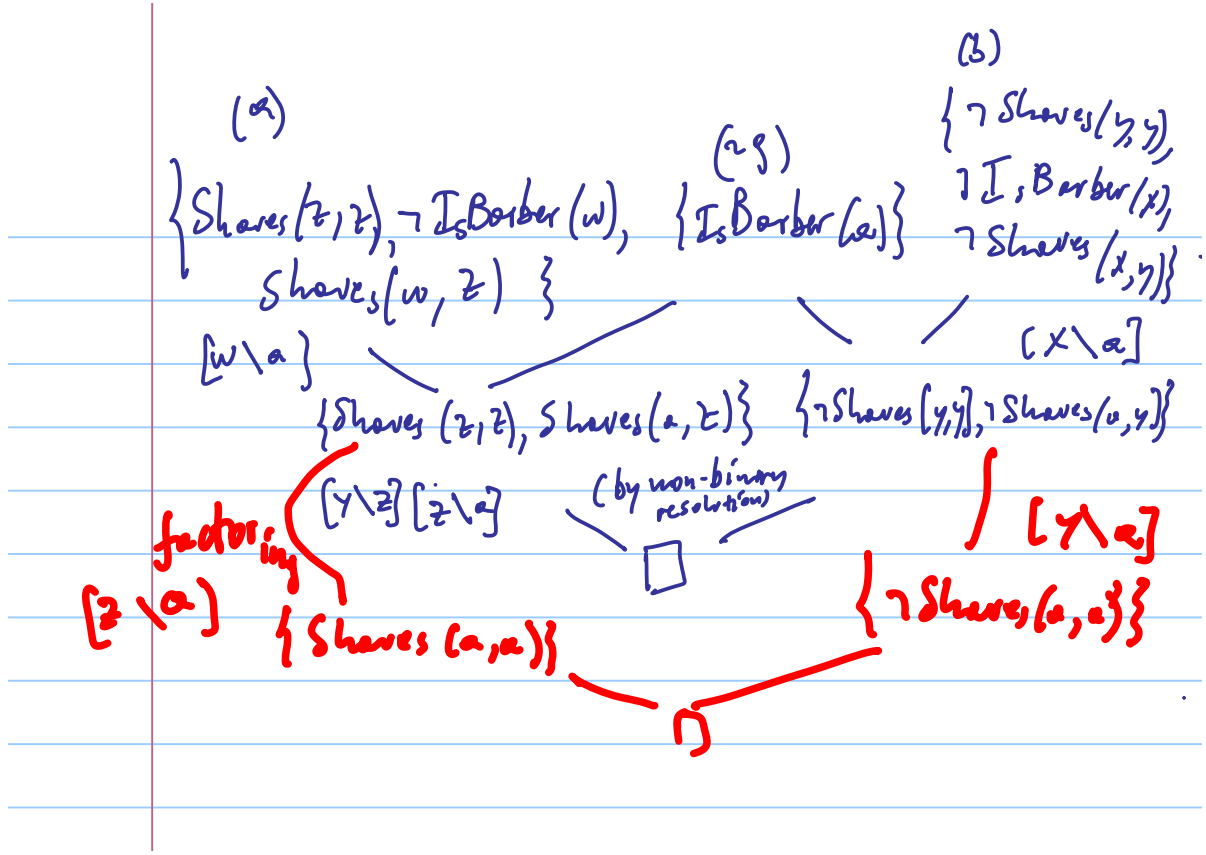
$\sim \exists x \exists y \left[B(x) \wedge S(y,y) \wedge S(x,y) \right]$ or with

$\forall x \forall y \left(B(x) \Rightarrow \left(S(y,y) \Rightarrow \neg S(x,y) \right) \right)$

and end up with the same clause

The goal: there are no barbers
$\forall x \neg IsBarber(x)$
Negated goal: $\neg\forall x \neg IsBarber(x)$
$\exists x \neg\neg IsBarber(x)$
$(\sim g)$ $\{ IsBarber(a) \}$

A resolution refutation proof:


(a)
{Shaves(z,z), ¬IsBarber(w), Shaves(w,z)}
(2g)
{IsBarber(a)}
(b)
{¬Shaves(y,y), ¬IsBarber(x), ¬Shaves(x,y)}
[w\a]
[x\a]
{Shaves(z,z), Shaves(a,z)}
{¬Shaves(y,y), ¬Shaves(a,y)}
[y\z][z\a]
(by non-binary resolution)
factoring
[z\a]
[y\a]
{Shaves(a,a)}
{¬Shaves(a,a)}
□