781- 2013 04 02, 20130404

Note Title 2013-04-02

Grading policy w/ project:

· Homework	40 →	30	→	25
Midterm	15	→	10	
Final	35 →	30 →	25 →	30
Presentation		10		
Project		15 →	25	

Exercise 61 (Schöning)

Convert $F = (\forall x \exists y\, P(x, g(y, f(x))) \vee \neg Q(z) \vee \neg \forall x\, R(x,y))$

$\forall x \exists u\, P(x, g(u, f(x))) \vee \neg Q(z) \vee \neg \forall w\, R(w,y)$ (rename)

$\forall x \exists u\, P(x, g(u, f(x))) \vee \neg Q(z) \vee \exists w \neg R(w,y)$ (push negation in)

$\forall x \exists u \exists w\, [P(x, g(u, f(x))) \vee \neg Q(z) \vee \neg R(x,y)]$ (prenex)

Exercise 62 (Schöning)

Find the Skolem form of the formula

$\forall x \exists y \forall z \exists w \, (\neg P(a, w) \vee Q(f(x), y))$

$\forall x \forall z \exists w \, (\neg P(a, w) \vee Q(f(x), g(x)))$ (replace y with $g(x)$)

$\forall x \forall z \, (\neg P(a, h(x, z)) \vee Q(f(x), g(x)))$ (replace w with $h(x, z)$)

Exercise 6.3. Transform to rectified prenex Skolem form.

$$\forall z\, \exists y \big(P(x, g(y), z) \vee \neg \forall x\, Q(x) \big) \wedge \neg \forall z\, \exists x\, \neg R(f(x,z), z)$$

$$\forall z\, \exists y \big(P(x, g(y), z) \vee \exists x\, \neg Q(x) \big) \wedge \exists z\, \forall x\, R(f(x,z), z)$$

$$\forall z\, \exists y \big(P(x, g(y), z) \vee \exists w\, \neg Q(w) \big) \wedge \exists t\, \forall s\, R(f(s,t), t)$$

$$\forall z\, \exists y\, \exists w\, \exists t\, \forall s\, [P(x, g(y), z) \vee \neg Q(w) \wedge R(f(s,t), t)]$$

$$\forall z\, \exists w\, \exists t\, \forall s\, [P(x, g(\underline{h(z)}), z) \vee \neg Q(w) \wedge R(f(s,t), t)]$$

$$\forall z\, \exists t\, \forall s\, [P(x, g(\underline{h(z)}), z) \vee \neg Q(\underline{h_1(z)}) \wedge R(f(s,t), t)]$$

$$\forall z \forall s \left[P(x, g(h(z)), z) \vee \neg Q(h_1(z)) \wedge R(f(s, h_2(z)), h_2(z)) \right]$$

Presentations

Tuesday, 4/16 ; Sakib & Arjun

Thursday, 4/18 ; Walker + Omar

Tuesday, 4/23 ; Selvi & McCoskh

Example of transformation of a formula to an S-equivalent formula in rectified prenex Skolem form.

Recall,

A formula F is S-equivalent to a formula G iff F is satisfiable iff G is satisfiable.

(Motivation: equivalence requires equality of models, models are a special kind of suitable structures, after Skolemizing new functions are introduced, so structures are no longer suitable. $F \Rightarrow F_{sk}$. The structures that are suitable for F are not suitable for F_{sk}, b/c F_{sk} has additional functions.)

(Example on pp. 60-61 of Schöning.)

$$F = (\neg \exists x\, P(y,z) \vee \forall y\, Q(x, f(y))) \vee \forall \underline{y}\, P(g(x,y), z))$$

1. Rename bound variables

$$(\neg \exists x(P(x,z) \vee \forall y\, Q(x, f(y)))) \vee \forall w\, P(g(x,w), z))$$

2. (Justified by exercise 48 :

$F(\dots x \dots)$ is satisfiable iff $\exists x\, F(\dots x \dots)$ is satisfiable, where x is free in F.)

$$\exists z\, ((\neg \exists x\, (P(x,z) \vee \forall y\, Q(x, f(y)))) \vee \forall w\, (P(g(x,w), z)))$$

3. Convert to prenex form

De Morgan's law as the negation is pushed in

$$\exists z\left(\forall x\left(\neg P(x,z) \wedge \exists y \neg Q(x,f(y))\right) \vee \forall w\, P(g(x,w),z)\right)$$

$$\exists z\, \forall x\, \exists y\, \forall w\, \left(\left(\neg P(x,z) \wedge \neg Q(x,f(y))\right) \vee P(g(x,w),z)\right)$$

4. Skolemize

$z \to a$ $\quad \forall x\, \exists \underline{\underline{y}}\, \forall w\, \left(\left(\neg P(x,a) \wedge \neg Q(x,f(y))\right) \vee P(g(x,w),a)\right)$

$y \to h(x)$ $\quad \forall x\, \forall w\, \left(\left(\neg P(x,a) \wedge \neg Q(x,f(h(x)))\right) \vee P(g(x,w),a)\right) = f_4$

5. Convert the matrix of the formula f_4 into CNF.

$F_5 = \forall x \forall w \{(\neg P(x, a) \vee P(g(x,w), a)) \wedge (\neg Q(x, f(h(x))) \vee P(g(x,w), a))\}$

We can write F_5 as a clause set

$$\{\neg P(x,a), P(g(x,w),a)\} \{\neg Q(x, f(h(x))), P(g(x,w),a)\}$$

This is the form needed for resolution refutation proofs in predicate calculus case.