2013-02-14

Note Title 2013-02-14

Simplified version of Theorem 9.7 [Yasuhara].

Let P be a propositional theory (based on the language $L(P_1)$) whose rules of inference include modus ponens. If the theorems of P include all tautologies and a contradiction, then for all formulas $A \in F(P_1)$, $\vdash_P A$.

Proof.

Let C be a contradiction that is a theorem of P.

(1) $\vdash_P C$ $\quad$ C is a theorem

(2) $\vdash_P \sim C$ $\quad$ $\sim C$ is a tautology

(3) $\vdash_P (\sim C \supset (C \supset A))$. $\quad$ a tautology (b/c it is lemma 9.1(3))

(comment: $(\sim (A \wedge C) \supset A))$)

(4) $\vdash_P (C \supset A)$ $\quad$ m.p. on (2) and (3)

(5) $\vdash_P A$ $\quad$ m.p. on (1) and (4)

Theorem 9.7 [Yasuhara] (full version):
replace "include a contradiction" with
"include more than those formulas that are
tautologies"

Proof: Let C be a non-tautology that is a theorem

C	A	$C \supset A$
t	t	t
t	f	f
f	t	t
f	f	t

$(C \supset A)$ holds
when either (1) A holds
or (2) $\sim C$ holds

Define $B_1, \ldots, B_k$ as in theorem 9.5 (Lemma 9.2) for $(C \supset A)$.

In the first case (A holds)

$B_1, \ldots, B_k \vdash_p (C \supset A)$. (using the construction of Lemma 9.2)

$B_1, \ldots B_k \vdash_p C$ is a theorem

$B_1, \ldots, B_k \vdash_p A$ by m.p

In the second case, ($\neg C$ holds)

$B_1, \ldots, B_k \vdash_p (C \supset A)$ using the construction of Lemma 9.2

$B_1, \ldots, B_k \vdash_p C$ C is a theorem

$B_1, \ldots, B_k \vdash A$ mp

Now, eliminate B_k using the construction of Thm. 9.5,

You can eliminate all B_i in the same way (again, as in Thm. 9.5). $\square$

$C = \sim p_1 \wedge p_1$, and $\vdash_p C$

Let A be an arbitrary formula, $\Updownarrow A = p_3$

(1) $\vdash_p \sim p_1 \wedge p_1$ (b/c it is a theorem)

(2) $\vdash_p \sim(\sim p_1 \wedge p_1)$ b/c it is a tautology

(3) $\vdash_p (((\sim p_1 \wedge p_1) \wedge \sim(\sim p_1 \wedge p_1)) \supset p_3)$ b/c it is a tautology

p_1	p_3	$(\ldots)$	$(\quad) \supset p_3$
t	t	f	t
t	f	f	t
		f	

f t t

f f f. t

(4) $\vdash_p ((\sim p_1 \wedge p_1) \supset (\sim(\sim p_1 \wedge p_1) \supset p_3))$ b/c it is a tautology

(5) $\vdash_p (\sim(\sim p_1 \wedge p_1) \supset p_3)$ mp on 1, 4

(6) $\vdash_p p_3$ m.p on 2, 5

End of example

Notes from 731, 2011-01-27

used now.