2013-01-31

Note Title 2013-01-31

PR1 new due date; Tuesday, Feb 5,

The deduction theorem.

If $B_1, \ldots B_t \vdash A$ [if there is a derivation (proof) of formula A from the axioms and the hypotheses $B_1, \ldots B_t$ }, then there is a proof of the formula $B_t \supset A$ from the hypotheses $B_1, \ldots, B_{t-1}$]

then

$$B_1, \ldots, B_{t-1} \vdash (B_t \supset A)$$

Exercise 8.6 [elsewhere]

The converse of the deduction theorem, i.e.

If $B_1, \ldots, B_{t-1} \vdash (B_t \supset A)$, then

$B_1, \ldots, B_{t-1}, B_t \vdash A$.

Proof

1. $B_1, \ldots B_{t-1} \vdash (B_t \supset A)$ given

2. $B_1, \ldots B_{t-1}, B_t \vdash (B_t \supset A)$ from defn of derivation and (1)

(Step 2 "formalizes" the notion that the propositional calculus is monotonic)

3. $B_1, \ldots B_{t-1}, B_t \vdash B_t$

4. $B_1, \ldots B_{t-1}, B_t \vdash A$ m.p on 2, 3

Lemma 9.1(1), alternate proof

(Need to show) $\vdash ((A \supset B) \supset ((B \supset C) \supset (A \supset C)))$

1. $(A \supset B), (B \supset C), A \vdash A$ hypothesis
2. $(A \supset B), (B \supset C), A \vdash (A \supset B)$ hypothesis
3. $(A \supset B), (B \supset C), A \vdash B$ mp on 1, 2
4. $(A \supset B), (B \supset C), A \vdash (B \supset C)$ hypothesis
5. $(A \supset B), (B \supset C), A \vdash C$ mp on 3, 4

6. $(A \supset B), (B \supset C) \vdash (A \supset C)$ deduction theorem

7. $(A \supset B) \vdash ((B \supset C) \supset (A \supset C))$ " "

8. $\vdash ((A \supset B) \supset ((B \supset C) \supset (A \supset C)))$ dom

Lemma 9.1(3) Show $(\sim B \supset (B \supset C))$

1. $\sim B \vdash \sim B$ hypothesis

2. $\sim B \vdash (\sim B \supset (\sim C \supset \sim B))$ axiom 1

3. $\sim B \vdash ((\sim C \supset \sim B) \supset (B \supset C))$ axiom 3

4 $\sim B \vdash (\sim C \supset \sim B)$ m. p. on 1, 2

5 $\sim B \vdash (B \supset C)$ m p on 4, 3

6. $\vdash (\sim B \supset (B \supset C))$ deduction theorem

Until now, we treated the propositional calculus (theory P_0) as a game on meaningless symbols.

Now, we introduce the semantics (meaning) of the propositional calculus.

Defn. (semantically) equivalent formulas.

Two formulas F and G are (semantically) equivalent if they have the same truth table. (I.e., they have the same truth value for each valuation.)

Theorem (substitution theorem)

Let F and G be equivalent formulas.
Let H be a formula with an occurrence of F as a subformula. Then H is equivalent to H' where H' is a formula obtained from H by substituting an occurrence of subformula F by G