Integrity Constraints

- In the electrical domain, what if we predict that a light should be on, but observe that it isn’t? What can we conclude?
- We will expand the definite clause language to include integrity constraints which are rules that imply \textit{false}, where \textit{false} is an atom that is false in all interpretations.
- This will allow us to make conclusions from a contradiction.
- A definite clause knowledge base is always consistent. This won’t be true with the rules that imply \textit{false}.
Horn clauses

- An **integrity constraint** is a clause of the form
  
  $\textit{false} \leftarrow a_1 \land \ldots \land a_k$

  where the $a_i$ are atoms and $\textit{false}$ is a special atom that is false in all interpretations.

- A **Horn clause** is either a definite clause or an integrity constraint.
Negations can follow from a Horn clause KB.

The negation of $\alpha$, written $\neg\alpha$ is a formula that

- is true in interpretation $I$ if $\alpha$ is false in $I$, and
- is false in interpretation $I$ if $\alpha$ is true in $I$.

**Example:**

$$KB = \begin{cases} 
\text{false} \leftarrow a \land b. \\
a \leftarrow c. \\
b \leftarrow c. 
\end{cases} \quad KB \models \neg c.$$
Disjunctive Conclusions

Disjunctions can follow from a Horn clause KB.

The disjunction of \( \alpha \) and \( \beta \), written \( \alpha \lor \beta \), is
- true in interpretation \( I \) if \( \alpha \) is true in \( I \) or \( \beta \) is true in \( I \) (or both are true in \( I \)).
- false in interpretation \( I \) if \( \alpha \) and \( \beta \) are both false in \( I \).

Example:

\[
KB = \begin{cases} 
false \leftarrow a \land b. \\
   a \leftarrow c. \\
b \leftarrow d. 
\end{cases} \quad KB \models \neg c \lor \neg d.
\]
Questions and Answers in Horn KBs

- An **assumable** is an atom whose negation you are prepared to accept as part of a (disjunctive) answer.
- A **conflict** of $KB$ is a set of assumables that, given $KB$ imply $false$.
- A **minimal conflict** is a conflict such that no strict subset is also a conflict.
Example: If \( \{c, d, e, f, g, h\} \) are the assumables

\[
KB = \begin{cases} 
false \leftarrow a \land b. \\
a \leftarrow c. \\
b \leftarrow d. \\
b \leftarrow e.
\end{cases}
\]

- \( \{c, d\} \) is a conflict
- \( \{c, e\} \) is a conflict
- \( \{c, d, e, h\} \) is a conflict
Assume that the user is able to observe whether a light is lit or dark and whether a power outlet is dead or live.

A light can’t be both lit and dark. An outlet can’t be both live and dead:

\[
\text{false} \leftarrow \text{dark}_l \land \text{lit}_l. \\
\text{false} \leftarrow \text{dark}_l \land \text{lit}_l. \\
\text{false} \leftarrow \text{dead}_p \land \text{live}_p.
\]

Assume the individual components are working correctly:

\[
\text{assumable } \text{ok}_l. \\
\text{assumable } \text{ok}_s.
\]

\[\ldots\]

Suppose switches \(s_1, s_2,\) and \(s_3\) are all up:

\[
\text{up}_s_1. \text{up}_s_2. \text{up}_s_3.
\]
Representing the Electrical Environment

\[
\begin{align*}
\text{light}_1 & \leftarrow \text{live}_w_0 \land \text{ok}_l_1. \\
\text{live}_w_0 & \leftarrow \text{live}_w_1 \land \text{up}_s_2 \land \text{ok}_s_2. \\
\text{live}_w_1 & \leftarrow \text{live}_w_2 \land \text{down}_s_2 \land \text{ok}_s_2. \\
\text{live}_w_2 & \leftarrow \text{live}_w_3 \land \text{up}_s_1 \land \text{ok}_s_1. \\
\text{live}_w_3 & \leftarrow \text{live}_w_4 \land \text{up}_s_1 \land \text{ok}_s_3. \\
\text{live}_w_4 & \leftarrow \text{live}_w_5 \land \text{down}_s_1 \land \text{ok}_s_1. \\
\text{live}_w_5 & \leftarrow \text{live}_w_6 \land \text{down}_s_1 \land \text{ok}_s_1. \\
\text{live}_w_6 & \leftarrow \text{live}_w_5 \land \text{ok}_cb_1. \\
\text{live}_p_1 & \leftarrow \text{live}_w_3. \\
\text{live}_w_3 & \leftarrow \text{live}_w_5 \land \text{ok}_cb_1. \\
\text{live}_p_2 & \leftarrow \text{live}_w_6. \\
\text{live}_w_6 & \leftarrow \text{live}_w_5 \land \text{ok}_cb_2. \\
\text{live}_w_5 & \leftarrow \text{live}_\text{outside}. \\
\end{align*}
\]
If the user has observed $l_1$ and $l_2$ are both dark:

$$dark\_l_1. \ dark\_l_2.$$  

There are two minimal conflicts:

{\textit{ok}\_cb_1, \textit{ok}\_s_1, \textit{ok}\_s_2, \textit{ok}\_l_1} and
{\textit{ok}\_cb_1, \textit{ok}\_s_3, \textit{ok}\_l_2}.

You can derive:

$$\neg \textit{ok}\_cb_1 \lor \neg \textit{ok}\_s_1 \lor \neg \textit{ok}\_s_2 \lor \neg \textit{ok}\_l_1$$

$$\neg \textit{ok}\_cb_1 \lor \neg \textit{ok}\_s_3 \lor \neg \textit{ok}\_l_2.$$  

Either $cb_1$ is broken or there is one of six double faults.
A **consistency-based diagnosis** is a set of assumables that has at least one element in each conflict.

A **minimal diagnosis** is a diagnosis such that no subset is also a diagnosis.

Intuitively, one of the minimal diagnoses must hold. A diagnosis holds if all of its elements are false.

**Example:** For the proceeding example there are seven minimal diagnoses: \{ok\_cb\_1\}, \{ok\_s\_1, ok\_s\_3\}, \{ok\_s\_1, ok\_l\_2\}, \{ok\_s\_2, ok\_s\_3\}, \ldots
Recall: top-down consequence finding

To solve the query $\neg q_1 \land \ldots \land q_k$:

$$ac := \text{"yes } \leftarrow q_1 \land \ldots \land q_k\text{"}$$

repeat

select atom $a_i$ from the body of $ac$;
choose clause $C$ from $KB$ with $a_i$ as head;
replace $a_i$ in the body of $ac$ by the body of $C$

until $ac$ is an answer.
Implementing conflict finding: top down

- Query is false.
- Don’t select an atom that is assumable.
- Stop when all of the atoms in the body of the generalised query are assumable:
  - this is a conflict
false ← a.
a ← b & c.
b ← d.
b ← e.
c ← f.
c ← g.
e ← h & w.
e ← g.
w ← f.
assumable d, f, g, h.
Conclusions are pairs $\langle a, A \rangle$, where $a$ is an atom and $A$ is a set of assumables that imply $a$.

Initially, conclusion set $C = \{ \langle a, \{a\} \rangle : a$ is assumable $\}$.

If there is a rule $h \leftarrow b_1 \land \ldots \land b_m$ such that for each $b_i$ there is some $A_i$ such that $\langle b_i, A_i \rangle \in C$, then $\langle h, A_1 \cup \ldots \cup A_m \rangle$ can be added to $C$.

If $\langle a, A_1 \rangle$ and $\langle a, A_2 \rangle$ are in $C$, where $A_1 \subset A_2$, then $\langle a, A_2 \rangle$ can be removed from $C$.

If $\langle false, A_1 \rangle$ and $\langle a, A_2 \rangle$ are in $C$, where $A_1 \subseteq A_2$, then $\langle a, A_2 \rangle$ can be removed from $C$. 
Bottom-up Conflict Finding Code

\[ C := \{ \langle a, \{ a \} \rangle : a \text{ is assumable} \}; \]

repeat

select clause “\( h \leftarrow b_1 \land \ldots \land b_m \)” in \( T \) such that

\[ \langle b_i, A_i \rangle \in C \text{ for all } i \text{ and} \]

\[ \text{there is no } \langle h, A' \rangle \in C \text{ or } \langle \text{false}, A' \rangle \in C \]

\[ \text{such that } A' \subseteq A \text{ where } A = A_1 \cup \ldots \cup A_m; \]

\[ C := C \cup \{ \langle h, A \rangle \} \]

Remove any elements of \( C \) that can now be pruned;

until no more selections are possible