HW3
Exercise 76, 85, 86 from Sch"{u}ning's Ch. 2 handout
Herbrand universe \( D (P) \) for a closed formula
in Skolem form
Ex. \( F = \forall x \forall y \forall z \; P(x, f(y), g(z, x)) \)

\[
D(F) = \{ a, f(a), g(a, a), f(g(a, a)), g(a, g(a, a)), g(g(a, a), g(a, a)), f(g(a, g(a, a))) \}
\]

Ex. \( G = \forall x \forall y \forall z \; Q(c, f(x), h(y, b)) \)

\[
D(G) = \{ b, c, f(b), f(c), h(b, b), h(c, b), \}
\]
\[ f(f(b)), f(f(c)), f(h(b, b)), \ldots \}

The herbrand structure \( \Theta \)

\[ D(f) = \{ a, f(a), g(\theta, a), \ldots \} \]

\[ f \to f \]

\[ g \to g \]

\[ D \quad \text{defined as} \quad (t_1, t_2, t_3) \in D \quad \text{iff} \quad g(t_1, t_2) = \quad \neg g(t_1, f(t_1)) \]
is not a model of $F = \forall x \forall y \exists z \phi(x, f(y), g(2, z))$

$b) \quad \alpha(F) = 0 \quad \text{for the assignment}
\quad x \to a, \ y \to a, \ z \to a \quad \text{and,}
\quad \alpha(F) = 1 \iff \alpha(\phi(x, f(y), g(2, z, x))) = 1
\quad \text{for every assignment to } x, y, z.$

Exercise 7.2: Call this structure $\mathcal{B}$

The universe and interpretation of $f$ and $g$
an set — they are the same for all Herbrand structures.

For \( P \), let \( P^B = \{ (\alpha, \beta, r) \mid \alpha, \beta, r \in D(F) \} \).

König's lemma.