2011-03-24

Note Title 2011-03-24

Exercise 57 [Schöning]

The following interpretation is a model of $\exists u \forall v\, P(v,u)$ but it is not a model of $\forall x \exists y\, P(x,y)$: $U_{\mathcal{A}} = \{a, b\}$, $P^{\mathcal{A}} = \{(a,b), (b,a)\}$

$\mathcal{A}(\exists u \forall v P(v,u)) = 1$ because for each assignment to v, there exists an assignment to u for which $P^{a}(v,u) \in P^{a}$. Namely, if v is assigned a, u is assigned b, and if v is assigned b, u is assigned a.

But, $\mathcal{a}(\forall x \exists y P(x,y)) = 0$ b/c no single assignment to y satisfies $P^{a}(x,y)$ for both possible assignments to x. In particular, if I choose $y = a$, then $(a,a) \notin P^{a}$ and if I choose ~~$y=b$~~, then $(b,b) \notin P^{a}$.

Exercise 59:

$$\underline{\forall x \exists y \forall z} \left[P(x, f(y)) \wedge (Q(w, z) \vee R(z))\right]$$

Exercise 61 Convert

$$F = (\forall x \exists y P(x, g(y, f(x))) \vee \neg Q(z)) \vee \neg \forall x R(x, y)$$

rename: $\vee \neg \forall w R(w, y)$

push negations in: $\vee \exists w \neg R(w, y)$

$$\forall x \exists y \exists w (P(x, g(y, f(x))) \vee \neg Q(z)) \vee \neg R(w, y)$$

Exercise 62 Find the Skolem form of the formula

$$\forall x \, \exists y \, \forall z \, \exists w \, (\neg P(a,w) \vee Q(f(x),y))$$

$$\forall x \, \forall z \, (\neg P(a, f_1(x,z)) \vee Q(f(x), f_2(x)))$$

Exercise 63

$$\forall z \, \exists y \, (P(x, g(y), z) \vee \neg \forall x \, Q(x)) \wedge \neg \forall z \, \exists x \, \neg R(f(x,z), z)$$

$$\vee \exists x \, \neg Q(x) \qquad \exists z \, \neg \exists x \, \neg R$$

$$\forall z \, \exists y \, (P(x, g(y), z) \vee \exists x \, \neg Q(x)) \wedge \exists z \, \forall x \, R(f(x,z), z)$$

$\forall z \exists y (P(x, g(y), z) \vee \exists w \neg Q(w)) \wedge \exists t \forall s\, R(f(s,t), t)$

$\forall z \exists y\, \exists w\, \exists t\, \forall s (P(x, g(y), z) \vee \neg Q(w)) \wedge R(f(s,t), t)$

$\forall z\, \forall s (P(x, g(f_1(z)), z) \vee \neg Q(f_2(z))) \wedge R(f(s, f_3(z)), f_3(z))$

Already in CNF ↗

Convert to CNF and write as a clause set; $\square$

$\{P(x, g(f_1(z)), z), \neg Q(f_2(z))\}$

$\{R(f(s, f_3(z)), f_3(z))\}$

two clauses