781 2011_03_15

Note Title 2011-03-15

Predicate Logic (or: First-order Logic [FOL]
or First-order predicate logic
or Predicate Calculus)

P_i^k k is the arity of a predicate (symbol)

arity is the number of arguments

Terms

Ex. 43 matrix: $(Q(x) \vee (P(f(x), z) \wedge Q(a)) \vee R(x, z, g(x)))$

Shortcut:

$I_a(x) = x^a$

$I_a(f) = f^a$

$I_a(P) = P^a$

$I_a(b) = b^a$

$P(x, f(x))$ www.possson.org

student (coupon code)

$$I_a(P(x, f(x))) =$$

$$= P^a(x^a, f^a(x^a)) =$$

$$= <(2, succ(2)) = <(2, 3)$$

$$= 2 < 3 = 0 \quad (f)$$

If F has the form $F = \forall x G$, then

$$\mathcal{A}(F) = \begin{cases} 1, & \text{if for all } u \in U_{\mathcal{A}},\ \mathcal{A}_{[x/u]}(G) = 1 \\ 0, & \text{otherwise} \end{cases}$$

If F has the form $F = \exists x G$, then

$$\mathcal{A}(F) = \begin{cases} 1, & \text{if for some } u \in U_{\mathcal{A}},\ \mathcal{A}_{[x/u]}(G) = 1 \\ 0, & \text{otherwise} \end{cases}$$

$\mathcal{A} \models F$ iff $\mathcal{A}(F) = 1$

If $\mathcal{A}(F) = 1$ for every suitable structure $\mathcal{A}$, then F is valid, written $\models F$

If there is some suitable structure $\mathcal{A}$ for which $\mathcal{A}(F) = 1$, then F is satisfiable.

If there is no model for F (i.e., no suitable structure $\mathcal{A}$ s.t. $\mathcal{A}(F) = 1$), then F is unsatisfiable (or a contradiction).

Exercise 44

$F: \forall x \exists y \, P(x, y, f(z))$

A model for F is $\mathfrak{A} (U_{\mathfrak{a}}, I_{\mathfrak{a}})$, where

$U_{\mathfrak{a}} = \{c\}$ $\quad\quad z^{\mathfrak{a}} = c$

$f^{\mathfrak{a}} = c \to c$ $\quad\quad P^{\mathfrak{a}} = \{ (c, c, c) \}$

On interpretation $\mathcal{B}$ s.t. $\mathcal{B}(F) = 0$

$U_{\mathcal{B}} = \{c\}$ $\qquad z^{\mathcal{B}} = c$

$f^{\mathcal{B}} = c \to c$ $\qquad p^{\mathcal{B}} = \{\ \}$