2011-02-08

The Compactness Theorem for the propositional calculus (Section 1.4 Sch"onig, handout).
The proof below is by Loveland [Theorem 1.2.3] in:

Loveland, Donald W.
Automated Theorem Proving: A Logical Basis.
Thm. A set $S$ of formulas is satisfiable if every finite subset $T$ of $S$ is satisfiable.

Proof.

It is immediate that, if $S$ is a satisfiable set of formulas, then any subset of $S$ is a satisfiable set.

To show the converse, we show that if $S$ is unsatisfiable, then there is a finite subset $T$ of $S$ that is also unsatisfiable.
Assume that $S$ is unsatisfiable.

$(S \models (\neg (A \supset A)))$ holds for $(\neg (A \supset A))$ is a

New theory and the propositional calculus is

complete (Theorem 9.5, page 19).

$S \vdash (\neg (A \supset A))$ converse of the deduction

Theorem

$T \vdash (\neg (A \supset A))$ for some finite $T \subseteq S$ by

Proofs/derivations are finite and

do, we can choose as hypotheses only

the formulas in $S$ that are actually

used in the proof.
\[ \vdash (T \vdash (\neg (A \to A))) \]  

\textit{deduction theorem}

\( T \) is unsatisfiable, since \( (\neg (A \to A)) \) is a contradiction and by the soundness of the proof calculus \( (T \vdash (\neg (A \to A))) \) is a flaw to logic.
Examples of resolvents

$c_1 = \{ A, \neg B \}$
$c_2 = \{ B \}$

$c_1 \setminus c_2 = \{ A, \neg B \} \setminus \{ B \}$

$c_3 = \{ A, \neg c \}$
$c_4 = \{ B, \neg c \}$

$c_3 \setminus c_4 = \{ A, \neg c \} \setminus \{ B, \neg c \}$

$c_5 = \{ c \}$

$c_6 = \{ A \}$
\( C_1, C_2, C_3 \) are just about, \( \{ A, \neg B \}, \{ B, \neg C \} \)

\[ C_4 = \{ \neg A \} \]

Exercises 30

\( F = \{ \{ A, \neg B, C \}, \{ B, C \}, \{ \neg A, C \}, \{ B, \neg C \}, \{ \neg C \} \} \)

\( \vdash \text{Res} \).
\[\{A, B, C\} \setminus \{B, C\}\]

\[\text{Res}_1 = \{\{A, C\}, \{\neg B, C\}, \{A, C \land \neg C\}, \{A, B, \neg B\}, \{A, \neg B\}\}\]

\[\text{Res}_2 = \{\{A, C\}, \{\neg C\}, \{A, B\}, \{A^3\}, \ldots\}\]

\[\text{Recall:}\]

\[K = G \iff \neg G \land K \text{ is unsatisfiable}\]

\[\neg G \lor K \text{ is unsatisfiable}\]
\[ K = 5 \iff K + 5 \iff 1 - K > 5 \]

Soundness

Completeness

Induction Theorem

and its

Counter

\[ K = 5 \iff K + 5 \iff K > 5 \iff K \leq 5 \iff K \leq 5 \iff \text{equivalent formula} \]
\( \neg (\neg \neg \psi) \) is a contradiction (i.e., it is unsatisfiable)

if \( K \vdash \psi \) is unsatisfiable

\[ \vdash \neg \psi \text{ is a proof (or derivation) of } \neg \psi \]

\[ \vdash \psi \text{ is a tautology} \]

\( K \vdash \psi \) then there is a proof of \( \psi \) from hypotheses \( K \)

\[ K \models \psi \text{ if } K \text{ is a model of } \psi \]

in every interpretation in which \( K \) holds, \( \psi \) also holds
\( \vdash \langle \kappa \rangle \) There is a proof of \( \kappa \) \( \vdash \)

If \( \kappa \vdash \phi \) then \( \kappa \models \phi \) is soundness

(Theme \#2 for the p.c.)

If \( \kappa \models \phi \) then \( \kappa + \phi \) is completeness

(Theme \#5)

Memo trick: \( \vdash \equiv \vdash \)

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