

Theorem 3.27 . Let $G = (V, E)$ be a directed acyclic graph (dag).
For any probability distribution P on the variables of G , the following conditions are equivalent:

(1) P admits a recursive factorization according to G

$$P(V) = \prod_{v \in V} P(v | pa(v))$$

(2) P admits the directed global Markov property, relative to G .

If A and B are d-separated by C , then A and B are independent given C .

(3) P admits the directed local Markov property, relative to G

Each variable in G is independent of its non-descendants given its parents.