PC with d-separation vs. CI testing

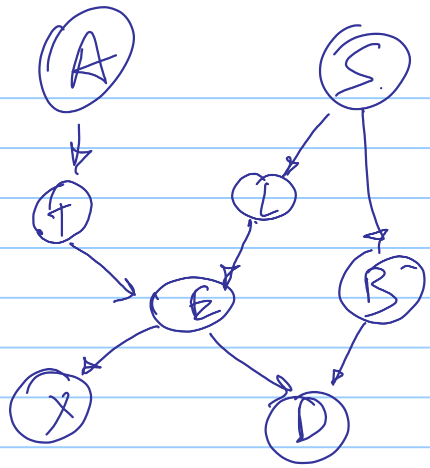
It has been observed that PC cannot learn the correct structure of the Asia network, b/c of the OR node E. Here is the explanation by Ahmad Alsuwat:

The main problem is that E depends deterministically on T and L through a logical OR. This means that:
(1) T and X are conditionally independent given E,
(2) L and X are conditionally independent given E, and
(3) E and X are conditionally independent given L and T.
Thus, the PC learning algorithm concludes that there should be no links between T and X, between L and X, and between E and X.

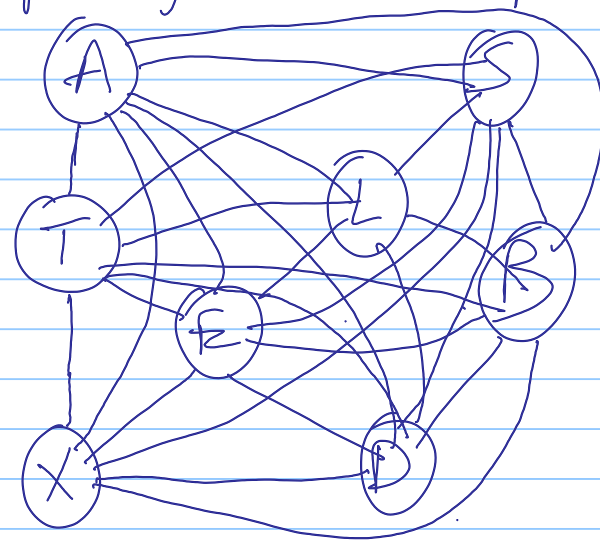
The exact same reasoning leads the PC algorithm to leave the node D unconnected to T, L, and E.

This is correct. However, the PC algorithm as described in Causation, Prediction, and Search uses d-separation tests, instead of independence tests. Such an algorithm, while clearly not practical, does not have the same problem, b/c E and X are not d-separated by L and T.

Here is how PC works w/ d-separation tests.



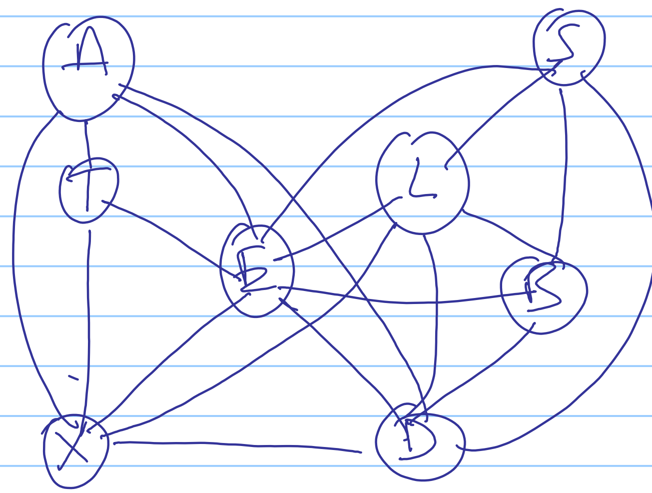
Step A (form the complete UG):



Step B (remove edges)

1. $n = \emptyset$ $A \perp\!\!\!\perp L$
$A \perp\!\!\!\perp S$ remove edges
$A \perp\!\!\!\perp B$
$T \perp\!\!\!\perp L$ (same for L)
$T \perp\!\!\!\perp S$
$T \perp\!\!\!\perp B$

sepsets $= \emptyset$



Note: $\perp\!\!\!\perp$ means "are d-separated"
$\perp$ means "are independent"

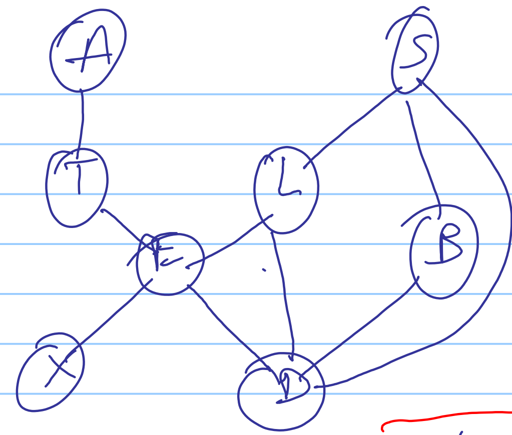
2. $n=1$ $A \perp\!\!\!\perp E \mid T$ $\quad L \perp\!\!\!\perp B \mid S$ remove edges

$A \perp\!\!\!\perp X \mid T$ $\quad X \perp\!\!\!\perp D \mid E$

$A \perp\!\!\!\perp D \mid T$ $\quad L \perp\!\!\!\perp X \mid E$

$T \perp\!\!\!\perp X \mid E$ $\quad E \perp\!\!\!\perp B \mid S$

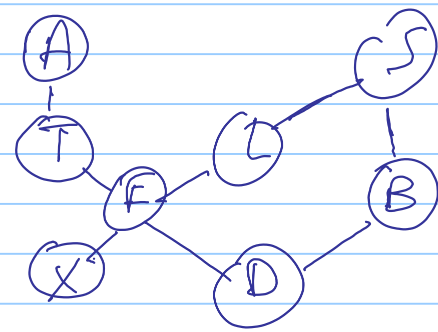
$S \perp\!\!\!\perp E \mid L$ $\quad$ (Same for L)



3. $n=2$ $S \perp\!\!\!\perp D \mid B, L$ remove edges

$L \perp\!\!\!\perp D \mid B, E$

$\boxed{\text{Also: } E \perp X \mid T, L \quad D \perp E \mid T, L}$



$\boxed{P(X \mid E, T, L) = P(X \mid T, L) \text{ since } E = f(T, L)}$

4. $n=3$ Nothing happens: no d-sep. involving E.

C. Orient triples.

$T - E - L \Rightarrow T \to E \leftarrow L$ b/c $E \notin \text{sepset}(T, L) = \{\}$

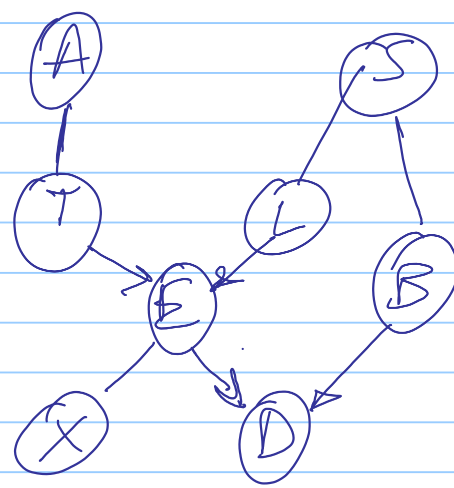
$L - S - B$ stays b/c $S \in \text{sepset}(L,B) = \{S\}$

$E - L - S$ stays b/c $L \in \text{sepset}(E,S) = \{L\}$

$S - B - D$ stays b/c $B \in \text{sepset}(S,D) = \{B, L\}$

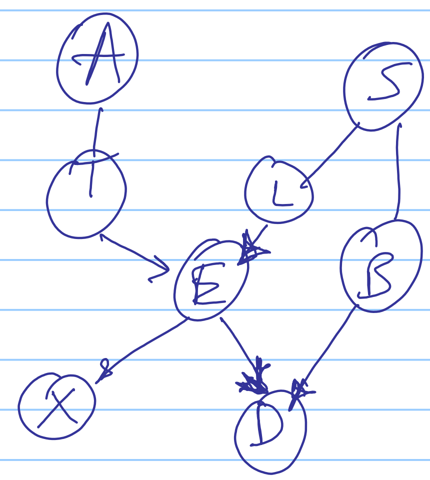
$E - D - B \Rightarrow E \rightarrow D \leftarrow B$, b/c $D \notin \text{sepset}(E,B) = \{S\}$ (or $\{L\}$)

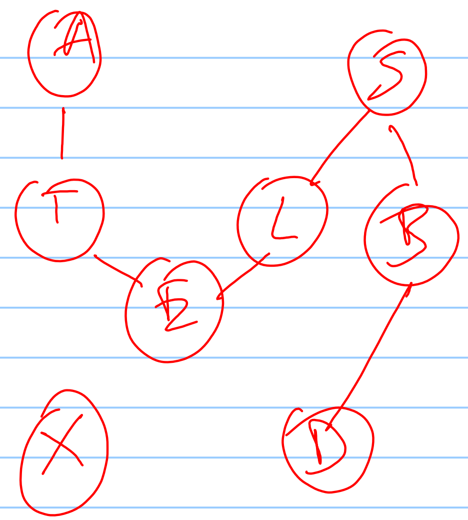
Therefore, after step 3:



Step 4 consists of the application of rules.

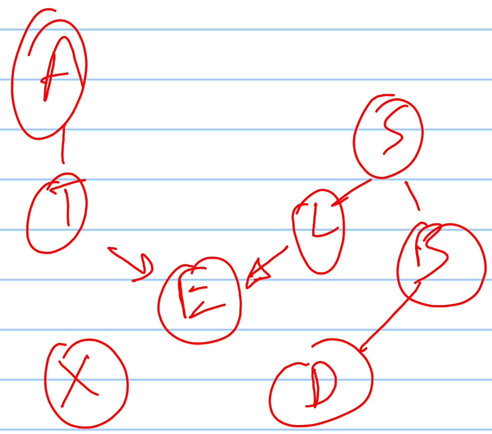
If I am not mistaken, the only edge that is oriented by application of the rules is E — X to E → X, so the result of applying PC to "Asia" is





Stop, b/c no adjacency > 2 (n = 3)

step 3



Step 4: nothing more