5.8.2 d-separation

We distinguish three kinds of transmission of influence in a causal network:

1. pipelined (serial) transmission

$$A \to B \to C$$

if B is not known, A and C may influence each other.

if B is known, A and C may not " " " .

2. diverging transmission

$$B \leftarrow A \to C$$

if A is not known, B and C may influence each other

if A is known, B and C may not influence each other

3. converging influence

$B \rightarrow A \leftarrow C$

if neither A nor any of its descendents is known, then B and C may not influence each other

if A or one of its descendents is known, then B and C may influence each other.

Defn. 2.1 [J07] (d-separation) (p. 30)

Two distinct variables A & B in a causal network are <u>d-separated</u> if for all paths between A and B there is and intermediate variable V [Note: therefore adjacent variables may not be d-separated] such that either:

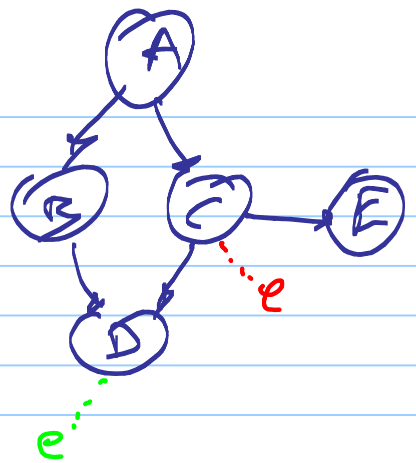
- the connection at V is serial or diverging and V is instantiated ("is known," "there is evidence on V," "there is a finding that V has some value")

or

- the connection is converging and neither V nor any of its descendants are not instantiated.

If A and B are not d-separated, we say that they are d-connected.

d



Are A, E d-separated in the e.g. on the left

(1) e is known

$A \to C \to E$ this path is blocked (at C)

e

$A \to B \to D \leftarrow C \to E$ " " " " (at C)

e

Yes, A and C are d-separated

(2) d is known

$A \to C \to E$ this path is not blocked

$A \to B \to D \leftarrow C \to E$ this path is also not blocked

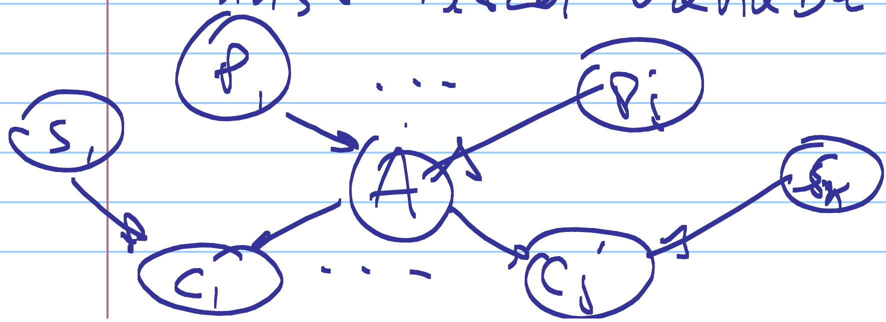
e

No, A are not d-separated; they are d-connected

HW3 (due 02-06): 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 2.10, 2.12, 2.14

Defn The Markov blanket of a variable A is the set consisting of the parents of A, the children of A, and the variables sharing a child with A

Claim (to be proved as part of HW3, ex. 2.8):
Let A be a variable in a causal network. Assume that all variables in A's Markov blanket are instantiated. Then, A is d-separated from all remaining uninstantiated variables.



Prove by cases. List each type of path starting at A.

The defn of d-separation leads to a simple but slow algorithm, because, in the worst case, it requires enumerating all paths between two nodes in a DAG.
Efficient algorithms exist. (One is called Bayesball).
One is based on a theorem by Lauritzen.

<u>Thm.</u> Let A, B, S be mutually exclusive sets of variables in a causal graph. Then, A and B are d-separated by S whenever
A and B are separated (in the undirected graph sense) in the graph

$$\left(\mathcal{G}_{An(A \cup B \cup S)}\right)^m,$$

the moral graph of the smallest ancestral set of $A \cup B \cup S$.

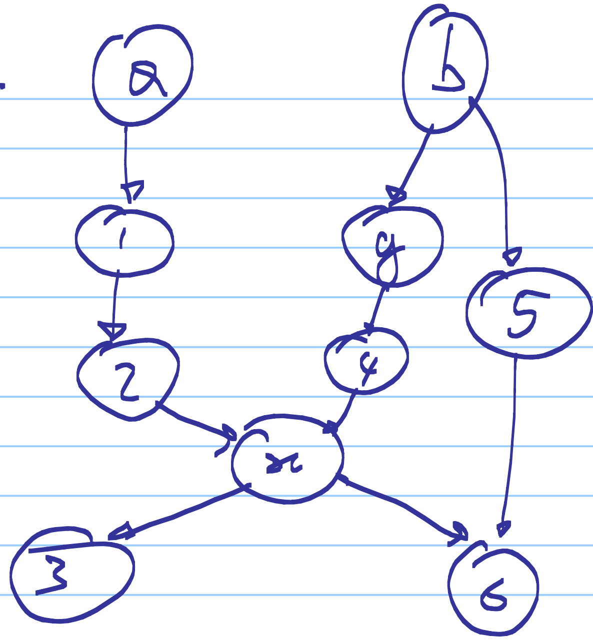
$An(U)$, where U is a set of variables in a causal graph, is the smallest ancestral set of U, i.e. the smallest set containing U and all its ancestors.

$\mathcal{G}_W$ is the induced subgraph of $\mathcal{G}_n$ the directed graph that includes all vertices of W

$\mathcal{G}^m$, where $\mathcal{G}$ is a directed graph, called the moral graph of $\mathcal{G}$, is an undirected graph with the same nodes as $\mathcal{G}$, an undirected edge for each directed edge of $\mathcal{G}$, and additional undirected edges that join ("marry") nodes in $\mathcal{G}$ that have a common child.

Example

$\mathcal{G}$:



$A = \{a\}$

$B = \{b\}$

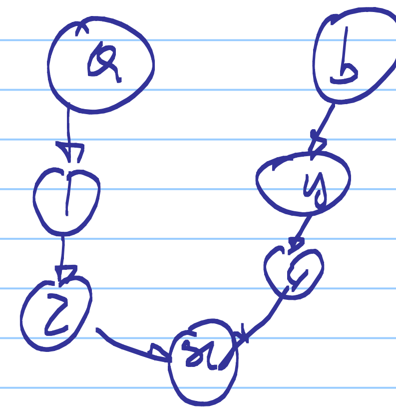
$S = \{x, y\}$

Are A and B d-separated by S?

$A \cup B \cup S = \{a, b, x, y\}$

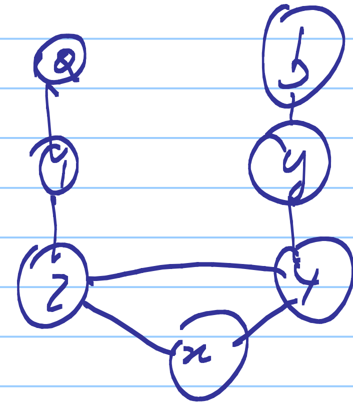
$An(\{a, b, x, y\}) = \{a, b, x, y, 1, 2, 4\}$

$\mathcal{G}_{An(A \cup B \cup S)} = \mathcal{G}_{\{a,b,x,y,1,2,4\}} =$



We "moralize" $\mathcal{G}_{An(A\cup B\cup S)}$:

$$\left(\mathcal{G}_{An(A\cup B\cup S)}\right)^m =$$

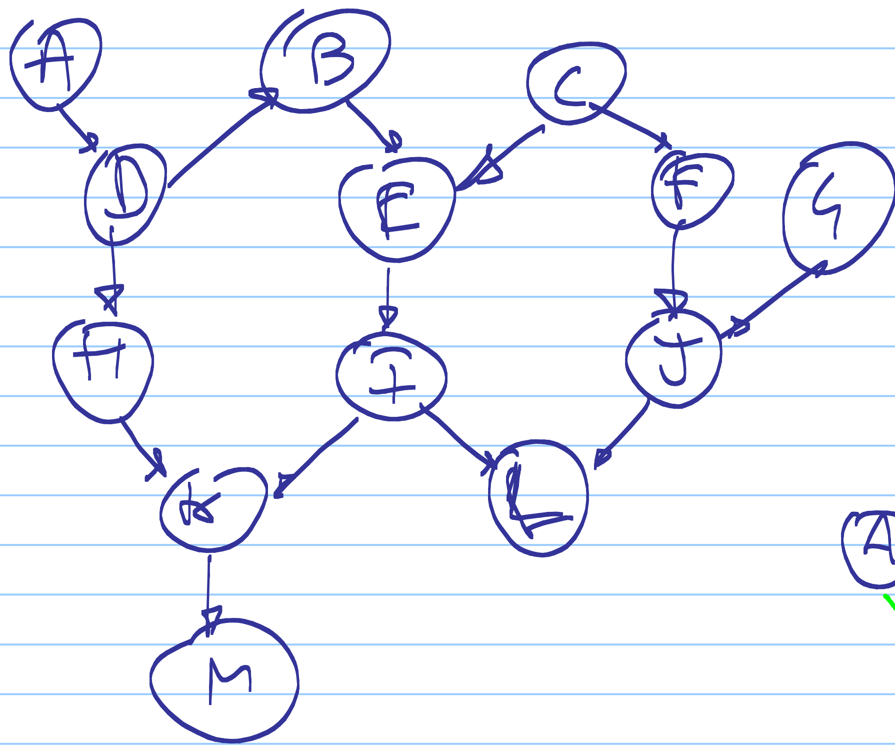


To answer the original question, we need to check whether a and b are graphically separated by $\{t, y\}$ in the graph above.

In this example, the answer is yes.

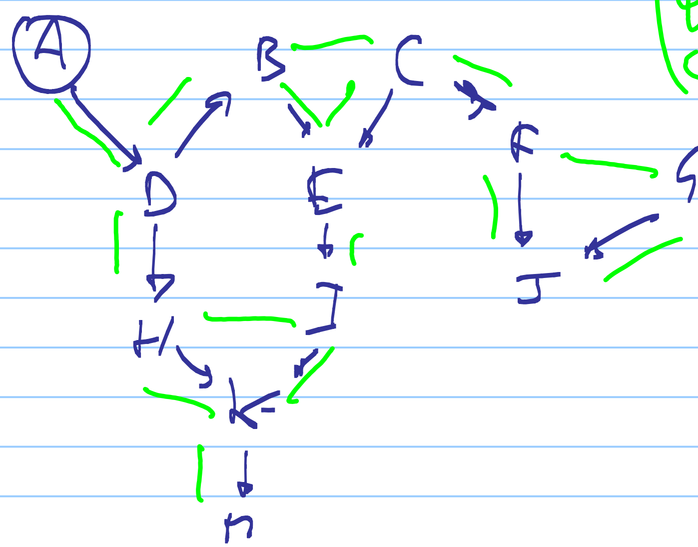
Another example (Fig 2.10 [J ϕ7])

$\mathcal{G}$



A and J d-separated by $\{B, M\}$

$\left(\mathcal{G}_{An\{A,J,B,M\}}\right)^m$



$\mathcal{G}_{An(\{A\} \cup \{J\} \cup \{B,M\})} =$

$= \mathcal{G}_{An(\{A,J,B,M\})} =$

$= \mathcal{G}_{\{A,J,B,M,F,G,C,K,I,H,D,E\}} =$