Note Title 2/16/2009

The [illegible] method for constructing Bayesian networks

– based on this definition

Let V be a finite set of finite propositional variables, (Ω, F, P) be their joint probability distribution, and $G = (V,E)$ be a dag.
For each $v \in V$, let $c(v)$ be the set of all parents of v and $d(v)$ be the set of all descendents of v. Furthermore, for $v \in V$, let $a(v)$ be $V \setminus \{d(v) \cup \{v\}\}$, i.e., the set of propositional variables in V excluding v and v's descendents. Suppose for every subset $W \subseteq a(v)$, W and v are conditionally independent given c(v); that is, if $P(c(v)) > 0$, then

$$P(v \mid c(v)) = 0 \text{ or } P(W \mid c(v)) = 0 \text{ or } P(v \mid W \cup c(v)) = P(v \mid c(v)).$$

Then, $C = (V, E, P)$ is called a *Bayesian network* [Neapolitan, 1990].

The method [Russell & Norvig, Ch. 14]

Need a method such that a series of locally testable assertions of conditional independence guarantees the required global semantics

1. Choose an ordering of variables $X_1, \ldots, X_n$
2. For $i = 1$ to n
   add X_i to the network
   select parents from $X_1, \ldots, X_{i-1}$ such that
   $\mathbf{P}(X_i|Parents(X_i)) = \mathbf{P}(X_i|X_1, \ldots, X_{i-1})$

This choice of parents guarantees the global semantics:

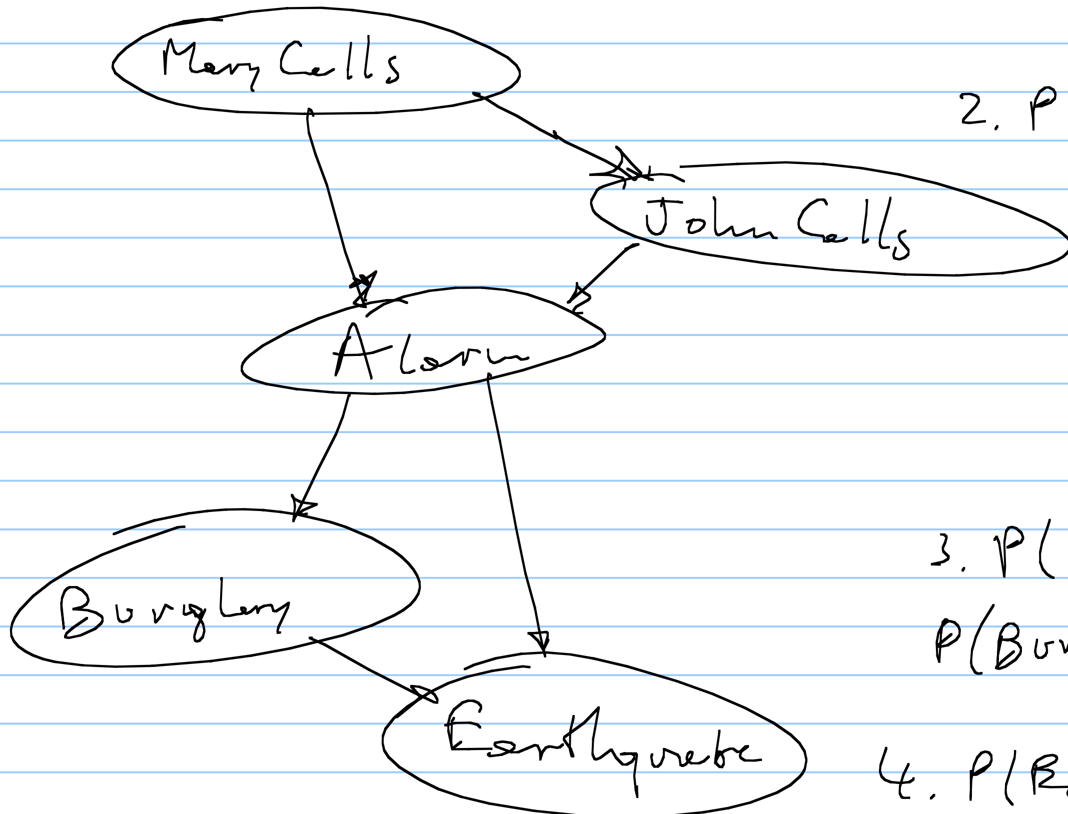
$$\begin{aligned}\mathbf{P}(X_1, \ldots, X_n) &= \Pi_{i=1}^{n} \mathbf{P}(X_i|X_1, \ldots, X_{i-1}) \quad \text{(chain rule)} \\ &= \Pi_{i=1}^{n} \mathbf{P}(X_i|Parents(X_i)) \quad \text{(by construction)}\end{aligned}$$

An example [Russell & Norvig, Ch. 14]

I'm at work, neighbor John calls to say my alarm is ringing, but neighbor Mary doesn't call. Sometimes it's set off by minor earthquakes. Is there a burglar?

Variables: $Burglar$, $Earthquake$, $Alarm$, $JohnCalls$, $MaryCalls$
Network topology reflects "causal" knowledge:
- A burglar can set the alarm off
- An earthquake can set the alarm off
- The alarm can cause Mary to call
- The alarm can cause John to call

Suppose we choose the ordering M, J, A, B, E



1. P(JohnCalls) = P(JohnCalls | MaryCalls) ? No

2. P(Alarm) = P(Alarm | MaryCalls, JohnCalls) ? no

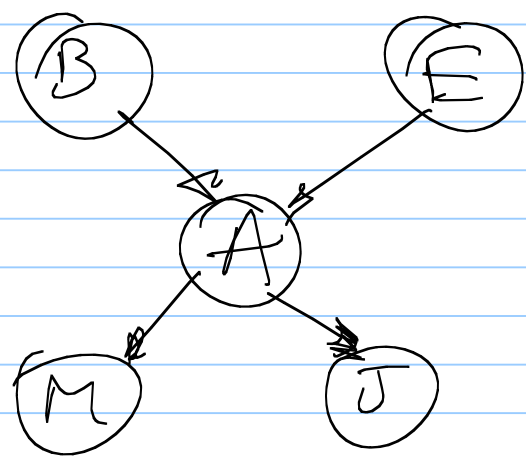
P(Alarm | MaryCalls) = P(Alarm | " , JohnCalls)? No

P(Alarm | JohnCalls) = P(Alarm | " , MaryCalls)? No

3. P(Burglary | Alarm) = P(Burglary | Alarm, MaryCalls, JohnCalls) ? Yes

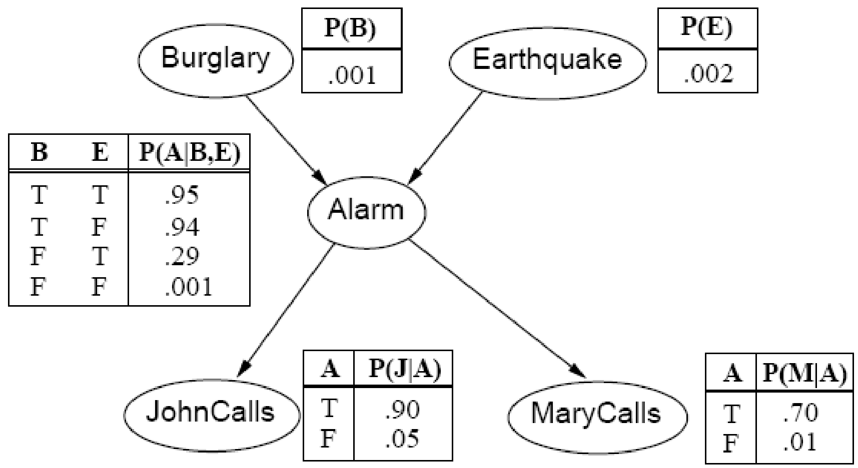
4. P(Earthquake | Alarm) $\neq$ P(Earthquake | M, J, A, B) = P(Earthquake | A, B).

Choose instead ⟨B, E, A, M, J⟩.



An order in which the edges are directed causally always results in a sparser network.

- Empirical observation



P(B)
.001

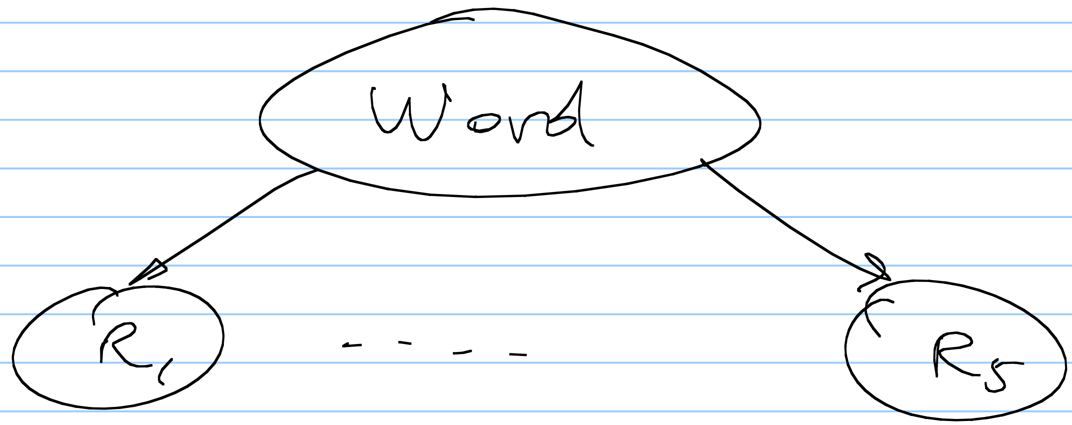
P(E)
.002

B	E	P(A|B,E)
T	T	.95
T	F	.94
F	T	.29
F	F	.001

A	P(J|A)
T	.90
F	.05

A	P(M|A)
T	.70
F	.01

Result with CPTs,



Bad, b/c the state space of Word is too large