Icy Roads.

(In this variation, Holmes and Watson are equally bad drivers.)

Compute the initial probabilities of $W$ and $I$.

$$P(I) = \begin{array}{c} n \ 0.3 \end{array}$$

$$P(H|I) = \begin{array}{c} n \ 0.8 \ 0.2 \end{array}$$

$$P(W|I) = \begin{array}{c} n \ 0.8 \ 0.1 \ 0.2 \ 0.9 \end{array}$$

$$P(H|I) P(I) = \begin{array}{c} n \ 0.56 \ 0.03 \end{array} = P(W, I)$$

Check: $\sum_{H} P(H|I) P(I) = 1$
So, the initial probability of the crashes is

$$P(H) = (0.59, 0.41) = \sum_i P(H, I) = P(W)$$

Check: $$\sum_i P(H) = 1 \checkmark$$

Now, we update $$P(I)$$ using the information (evidence) that Watson crashed ($$W = y$$)

$$P(I | W = y) = \frac{P(W = y | I) P(I)}{P(W = y)} = \frac{\underbrace{P(W = y | I) P(I)}}{P(W = y)}$$

$$= \frac{1}{0.59} \times (0.8, 0.1) \times (0.7, 0.3) = \frac{1}{0.59} \times (0.56, 0.03)$$

pointwise table multiplication
Now, we update the probability that Holmes crashed.

\[ P(H|I) \times P(I) \]

To compute a joint probability. Then, marginalize to H:

\[ \sum_I P(H,I) \]
Then, Smith is told that the roads are not dry.

So, \( P(H | I=n) = (0.1, 0.9) \)