

582 2012-01-24

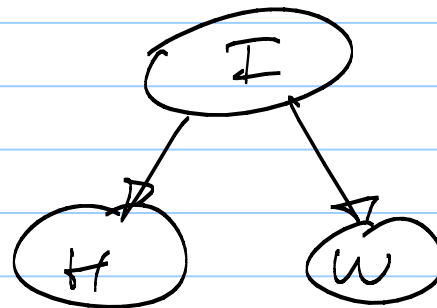
Note Title

2012-01-19

[first part from 1-19]

lay Roads

(In this variation, Holmes and Watson are equally bad drivers.)



$$P(I) = \begin{pmatrix} y & n \\ 0.7 & 0.3 \end{pmatrix}$$

$$P(H|I) = \begin{array}{c|cc} & I & y & n \\ \hline H & 0.8 & 0.1 \\ y & 0.2 & 0.9 \end{array}$$

$$P(W|I) = \begin{array}{c|cc} & I & y & n \\ \hline W & 0.8 & 0.1 \\ y & 0.2 & 0.9 \end{array}$$

Compute the initial probabilities of W and I.

$$P(H, I) = P(H|I) P(I) = \begin{array}{c|cc} & I & y & n \\ \hline H & 0.56 & 0.03 \\ y & 0.14 & 0.27 \end{array} = P(W, I)$$

Check: $\sum_H \sum_I P(H, I) = 1 \checkmark$

So, the initial probability of the crashes is.

$$P(H) = (0.59, 0.41) = \sum_I P(H, I) = P(W)$$

$$\text{Check: } \sum_H P(H) = 1 \checkmark$$

Now, we update $P(I)$ using the information (evidence) that Watson crashed ($W=y$)

$$\begin{aligned} P(I | W=y) &= (\text{use Bayes' rule}) = \frac{P(W=y | I) P(I)}{P(W=y)} = \\ &= \frac{1}{0.59} \times \overset{y}{(0.8, 0.1)} \overset{y}{\times} \overset{y}{(0.7, 0.3)} \overset{y}{=} \frac{1}{0.59} \times (0.56, 0.03) = \end{aligned}$$

pointwise table multiplication

$$= (0.95, 0.05) \quad (\text{approx.}) = \text{"updated } P(I)\text{"}$$

Now, we update the probability that Holmes crashed.

$P(H|I) \times P(I)$ to compute a joint probability. Then,

marginalize to H : $\sum_I P(H, I)$

$$P(H, I) = \begin{array}{c|cc} & \begin{array}{c} I \\ \hline y \quad n \end{array} & & \\ \begin{array}{c} H \\ \hline y \quad n \end{array} & \begin{array}{cc} 0.8 \times 0.95 & 0.1 \times 0.05 \end{array} \\ & \begin{array}{cc} 0.2 \times 0.95 & 0.9 \times 0.05 \end{array} \end{array}$$

$$\sum_I P(H, I) = (0.765, 0.235)$$

Smith's belief that Holmes crashed based on Watson's having crashed.

Then Smith is told that the roads are not dry

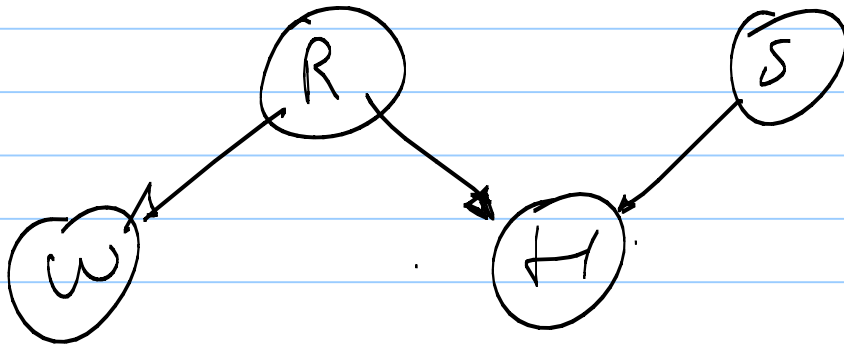
So, $P(H | I=n) = (0.1, 0.9)$

$$P(W|R) = \begin{array}{c|cc} & \begin{matrix} R & S \end{matrix} \\ \begin{matrix} W \\ H \end{matrix} & \begin{matrix} 1 & 0.2 \end{matrix} \\ & \begin{matrix} 0 & 0.8 \end{matrix} \end{array}$$

$$P(H|R,S) = \begin{array}{c|cc} & \begin{matrix} R & S \end{matrix} \\ \begin{matrix} Y \\ N \end{matrix} & \begin{matrix} (1, 0) & (0.9, 0.1) \end{matrix} \\ & \begin{matrix} (1, 0) & (0, 1) \end{matrix} \end{array}$$

$$P(R) = (0.2, 0.8)$$

$$P(S) = (0.1, 0.9)$$



Wet grass example - computations using tables

The initial (prior, i.e. in the absence of evidence) prob.
that Watson's lawn is wet is obtained from

$P(R, W) = P(W|R)P(R)$ and marginalization

$$P(W) = \sum_R P(R, W)$$

$$P(W, R) = \begin{array}{c|cc} & R & Y & n \\ \hline w & 1 & 0.2 \\ h & 0 & 0.8 \end{array} \times \begin{pmatrix} Y & n \\ 0.2 & 0.8 \end{pmatrix} = \begin{array}{c|cc} w & R & Y & n \\ \hline Y & 0.2 & 0.16 \\ h & 0 & 0.64 \end{array}$$

$$P(W) = \sum_R P(R, W) = (0.36, 0.64)$$

Do the same for $P(H)$ (the prior probability that Holmes's lawn is wet).

$$P(H, R, S) = \underset{\substack{\uparrow \\ \text{chain rule}}}{P(H|R, S)} P(R, S) = \underset{\substack{\uparrow \\ \text{independence} \\ \text{of } R \text{ and } S \\ \text{in the absence of} \\ \text{evidence on } H \text{ or} \\ \text{any of its descendants}}}{P(H|R, S) P(R) P(S)}$$

$$P(H, R, S) = \begin{array}{c|cc} S & R & Y & n \\ \hline Y & (1, 0) & (0.9, 0.1) \\ n & (1, 0) & (0, 1) \end{array} \times \begin{array}{c|cc} R & Y & n \\ \hline Y & (0, 2) & (0.8) \\ n & (0, 1) & (0.9) \end{array} =$$

$$\begin{array}{c|cc} S & R & Y & n \\ \hline Y & (0.02, 0) & (0.072, 0.008) \\ n & (0.18, 0) & (0, 0.72) \end{array} \quad \left| \quad P(H) = \sum_{R, S} P(H, R, S) = \begin{array}{c|cc} Y & n \\ \hline (0.272, 0.728) \end{array} \right.$$

Compute $P(R, S | H=Y)$ and from this $P(R | H=Y)$ and

$$P(R, S | H=Y) = \frac{P(R, S, H)}{P(H=Y)} = \begin{array}{c|cc} S & R & Y & n \\ \hline Y & \frac{0.02}{0.272} & \frac{0.072}{0.272} \\ n & \frac{0.18}{0.272} & \frac{0}{0.272} \end{array} \sim \begin{array}{c|cc} S & R & Y & n \\ \hline Y & 0.074 & 0.265 \\ n & 0.662 & 0 \end{array}$$

$$P(R|H=y) = \sum_s P(R,s|H=y) = (0.736, 0.265) \text{ (approx)}$$

$$P(S|H=y) = \sum_R P(R,s|H=y) = (0.339, 0.662) \text{ (approx)}$$

Update $P(w)$, i.e., compute $P(w|H=y)$.

$$P(w, R | H=y) = P(w|R, H=y) P(R|H=y) \stackrel{\uparrow}{=} P(w|R) P(R|H=y) \stackrel{\uparrow}{=}$$

$\uparrow$
chain rule
(conditional
version with
context $H=y$)

$\uparrow$
 w is
independent
of H given R

R			R		
w \	y	n	w \	y	n
y	1	0.2	(0.736, 0.265)	.736	.053
n	0	0.8		0	.2120

$$P(W|H=y) = \sum_R P(W, R|H=y) = \begin{pmatrix} .789, .212 \end{pmatrix}$$

The evidence $W=y$ arrives. We compute

$P(R|W=y, H=y)$ using Bayes' rule

$$P(R|W=y, H=y) = \frac{P(W|H=y, R) P(R|H=y)}{P(W|H=y)}$$

$$= \frac{P(W|R) P(R|H=y)}{P(W|H=y)} = \begin{pmatrix} 0.93, 0.07 \end{pmatrix}$$

Holmes also wants $P(S|H=y, W=y)$.

Compute $P(R, S | H=y, W=y)$

$$P(S|R, H=y) = \frac{P(R, S | H=y)}{P(S | H=y)}$$

	R	
	y	n
S	y	0.074 0.736
	n	0.662 0.736
		0.265 0.265

	R	
	y	n
S	y	0.1 0.9
	n	1 0

Chain rule

$$P(R, S | H=y, W=y) = P(S | R, H=y, W=y) P(R | H=y, W=y) =$$

$$= P(S | R, H=y) P(R | H=y, W=y) =$$

	R	
	y	n
S	y	0.093 0.832
	n	0.07 0

$$P(S | H=y, W=y) = \sum_R P(R, S | H=y, W=y) = (0.163, 0.837)$$

larger than prior

Taxis in Athens.

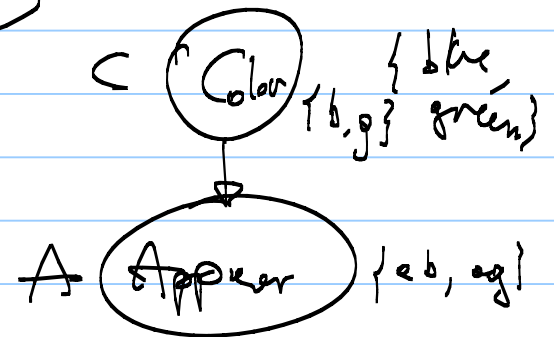
$$P(\text{Color}) = (0.9, 0.1)$$

$$P(A|C)$$

A \ C	b	g
ab	.7	.2
ag	.3	.8

$$P(A, C) = P(A|C)P(C) =$$

A \ C	b	g
ab	.63	.02
ag	.27	.08



$$P(C|A=ag) = \frac{P(A, C, e)}{P(e)} = \left(\frac{.27}{.35} \quad \frac{.08}{.35} \right) \approx (.77, .23)$$

Bayes' rule $\rightarrow$

$$= \frac{P(A=ag|C)P(C)}{P(A=ag)} = ab \begin{array}{c|cc} & b & g \\ \hline ag & \frac{.27}{.27+.08} & \frac{.08}{.27+.08} \end{array}$$

WA WA $\Rightarrow$

$$P(C|A) = \frac{P(A|C)P(C)}{P(A)} = ab \begin{array}{c|cc} & b & g \\ \hline b & \frac{.63}{.63+.02} & \frac{.02}{.63+.02} \\ ag & \frac{.27}{.27+.08} & \frac{.08}{.27+.08} \end{array}$$

The second row of this table is $P(C|A=ag)$.

