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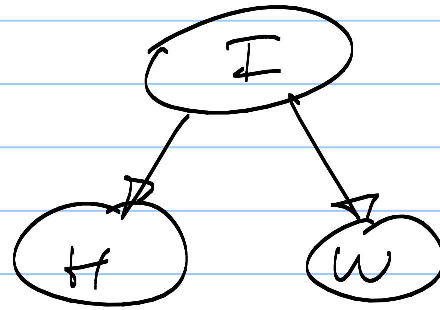
Note Title 2012-01-19

Icy Roads

(In this variation, Holmes and Watson are equally bad drivers.)



$P(I) = (\;\;, 0.3)$ (y, n)

$P(H|I) =$

H \ I	y	n
y	0.8	
n	0.2	

$P(W|I) =$

W \ I	y	n
y	0.8	0.1
n	0.2	0.9

Compute the initial probabilities of W and I.

$P(H, I) = P(H|I)\,P(I) =$

H \ I	y	n
y	0.56	0.03
n	0.14	0.27

$= P(W, I)$

Check: $\sum_H \sum_I P(H,I) = 1$ ✓

So, the initial probability of the crashes is.

$P(H) = (\boxed{0.59}, 0.41) = \sum_I P(H, I) = P(W)$

Check: $\sum_H P(H) = 1$ ✓

Now, we update $P(I)$ using the information (evidence) that Watson crashed $(W = y)$

$P(I \mid W=y) = $ (use Bayes' rule) $= \dfrac{P(W=y \mid I)\, P(I)}{P(W=y)} =$

$= \boxed{\dfrac{1}{0.59}} \times \begin{array}{cc} y & n \\ (0.8, & 0.1) \end{array} \times \begin{array}{cc} y & n \\ (0.7, & 0.3) \end{array} = \dfrac{1}{0.59} \times (0.56, 0.03) =$

pointwise table multiplication

$= (0.95,\ 0.05)$ (approx.) $=$ "updated $P(I)$"

Now, we update the probability that Holmes crashed.

$P(H|I) \times P(I)$ to compute a joint probability. Then, marginalize to H: $\sum_I P(H,I)$

$P(H,I) =$

$H \backslash I$	y	n
y	0.8×0.95	0.1×0.05
n	0.2×0.95	0.9×0.05

$\sum_I P(H,I) = (0.765, 0.235)$

Smith's belief that Holmes crashed based on Watson's having crashed.

Then Smith is told that the roads are not icy

So, $P(H \mid I = n) = (0.1, 0.9)$