2012-02-16 (from 2009-02-04)

Note Title 2009-02-04

HW2 — all exercises ~~from~~ [J07], ch. 2

2.1 (i) $\frac{2}{3}$, $\frac{1}{3}$, $\frac{1}{2}$

(ii) nearly ϕ, well below the minimum of the prob resulting from either individual action

2.4 Fig. 2.18. All variables are d-connected to A.

Note that for variables adjacent to A, e.g., C, there is no path (chain) with an intermediate variable V as required in defn. 2.1 on p. 30 [J07], so adjacent variables are not d-separated, so they are d-connected.

Fig. 2.19 All except C and F are d-connected to A. (C and F are d-separated from A because all paths between A and $\{C, F\}$ go through $E \to I \leftarrow F$, and there is no evidence on I or one of its descendants.

2.5

$$\begin{tikzcd}[column sep=small] & \text{Fuel?} \arrow[dl] \arrow[dr] & & \text{CleanSparkPlugs?} \arrow[dl] \\ \text{FuelMeterStanding} & & \text{Start?} & \end{tikzcd}$$

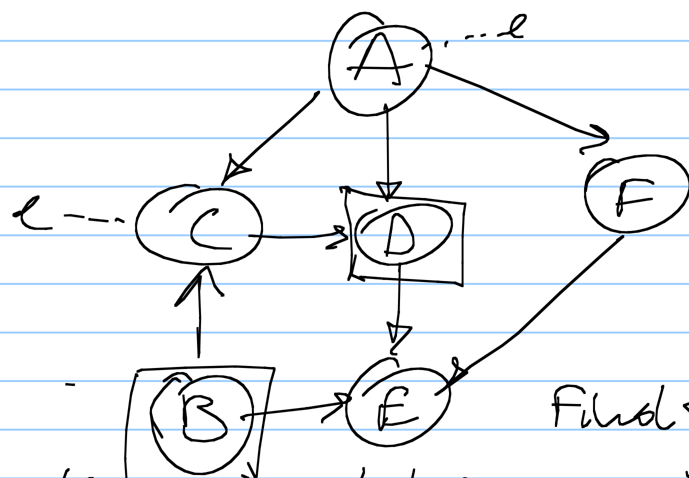
Each of the three pairs of adjacent vars cannot be d-seperated.

FuelMeterStanding can be d-seperated from Start? by {Fuel?}, {Fuel?, CleanSpark Plugs?}

" " " CleanSpark Plugs ? by {}, {Fuel?}, {Fuel?, Start?}

Fuel? can be d-separated from CleanSparkPlugs by {} and {FuelMaterStanding}.

Fig. 2.20



C ← A → F → E

C ← A → D → E

C → D → E

C ← B → E

min. hitting sets of the sets of nodes in the chains

Find the hitting set of

(no conv. connections)

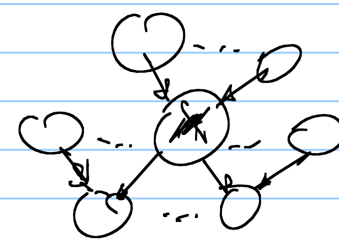
{ {A, F}, {A, D}, {D}, {B} }

2.6. C and E: $\{A, B, D\}$ and $\{B, D, F\}$

A and B: $\{\}$

C and E: $\{A, B, D, F\}$

A and B: $\{F\}$



2.7 A: $\underbrace{\{C, D, F\}}_{\text{children}} \cup \underbrace{\{\ \}}_{\text{parents}} \cup \underbrace{\{B\}}_{\text{parents of children, = spouses}} = \{B, C, D, F\}$

B: $\{C, E\} \cup \{\} \cup \{A, D, F\} = \{A, C, D, E, F\}$

C: $\{D\} \cup \{A, B\} \cup \{A\} = \{A, B, D\}$

D: $\{E\} \cup \{A, C\} \cup \{B, F\} = \{A, B, C, E, F\}$

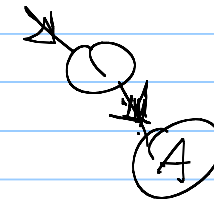
E: $\{\} \cup \{B, D, E\} \cup \{\} = \{B, D, E\}$

F: $\{E\} \cup \{A\} \cup \{B, D\} = \{A, B, D, E\}$

2.8 (Sketch) Consider all possible cases for chains ending at A:

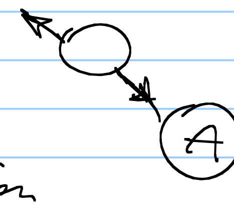
(1) chain into a parent of A (and into A)

blocked at the parent — serial connection



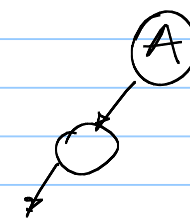
(2) chain out of a parent of A (and into A)

blocked at the parent — diverging connection



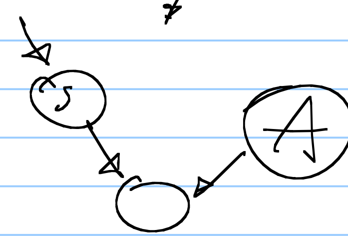
(3) chain out of a child of A (and out of A)

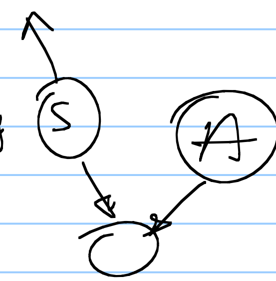
blocked at the child — serial connection



(4) chain into a child of A (and out of A)

blocked at the spouse S: serial or diverging





2.9 The ancestral graph of A, C, B for the graph of Figure 2.19 is

$$A \qquad B \qquad C.$$

There is no path from A to C in this graph, so there is all paths from A to C are intersected by $\{B\}$, so A and C are d-separated by B.

2.10. For the three networks ($\perp$ means: d-separated; $\not\perp$ means: d-connected)

$A \leftarrow B \rightarrow C$, $\quad A \rightarrow B \rightarrow C$, $\quad A \leftarrow B \leftarrow C$,

$A \perp C \mid B$

$A \not\perp C \mid \{\}$

$A \not\perp B \mid C$

$B \not\perp C \mid A$

$A \not\perp B \mid \{\}$

$B \not\perp C \mid \{\}$

so they are I-equivalent

For $A \rightarrow B \leftarrow C$,

$A \perp C \mid \{\}$

$A \not\perp C \mid \{B\}$

+, so this network is not I-equivalent to the other three.

2.12 $P(A)$, $P(B)$,

$P(C|A,B)$

$P(D|A,C)$

$P(E|B,D,F)$

$P(F|A)$

2.14 $P(A,B,C,D,E,F) = P(A)P(B)P(C|A,B)P(D|A,C)P(E|B,D,F) \times P(F|A)$

$$P(B,D,A,C) = \sum_E \sum_F P(A)P(B)P(C|A,B)P(D|A,C)P(E|B,D,F)P(F|A) =$$

$$= P(A)P(B)P(C|A,B)P(D|A,C)\sum_E \sum_F P(E|B,D,F)\,P(F|A) =$$

$$= P(A)P(B)P(C|A,B)P(D|A,C)\sum_F P(F|A)\sum_E P(E|B,D,F) =$$

$$= P(A)P(B)P(C|A,B)\,P(D|A,C)\sum_F P(F|A) =$$

$$= P(A)P(B)P(C|A,B)\,P(D|A,C)$$

$$P(B \mid D, A, C) = \frac{P(A)\,P(B)\,P(C \mid A, B)\,P(D \mid A, C)}{\sum_B P(A)\,P(B)\,P(C \mid A, B)\,P(D \mid A, C)} =$$

B is indep. of D given $\{A, C\}$

$$= \frac{\cancel{P(A)}\,P(B)\,P(C \mid A, B)\,\cancel{P(D \mid A, C)}}{\cancel{P(A)}\,\cancel{P(D \mid A, C)}\,\sum_B P(B)\,P(C \mid A, B)} = \frac{P(B, C \mid A)}{\sum_B P(B, C \mid A)} = \frac{P(B, C \mid A)}{P(C \mid A)} = P(B \mid C, A)$$

Note how much easier it is to show that $\{A, C\}$ d-separates B from D.

Fig. 2.18

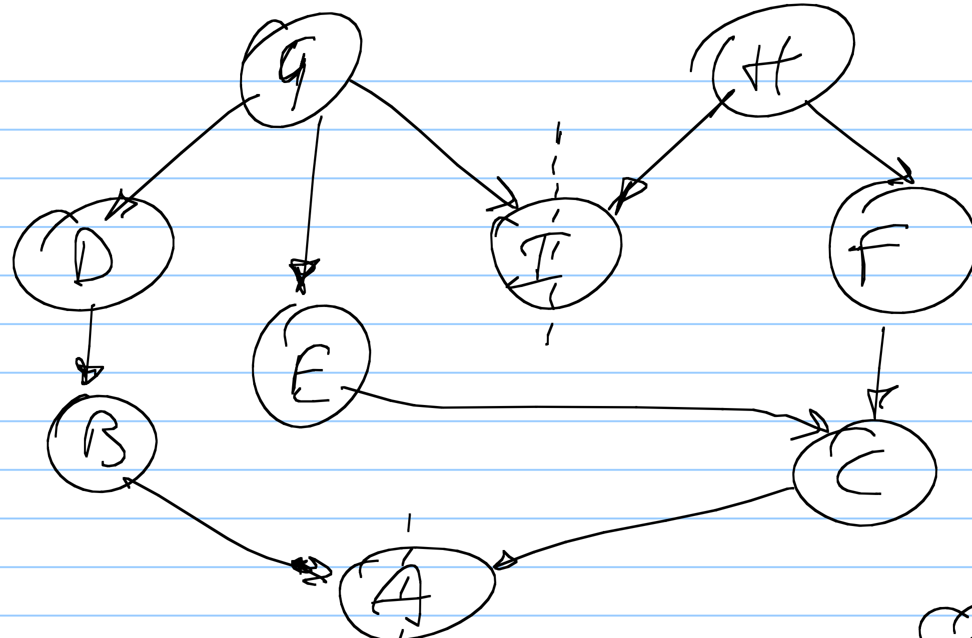
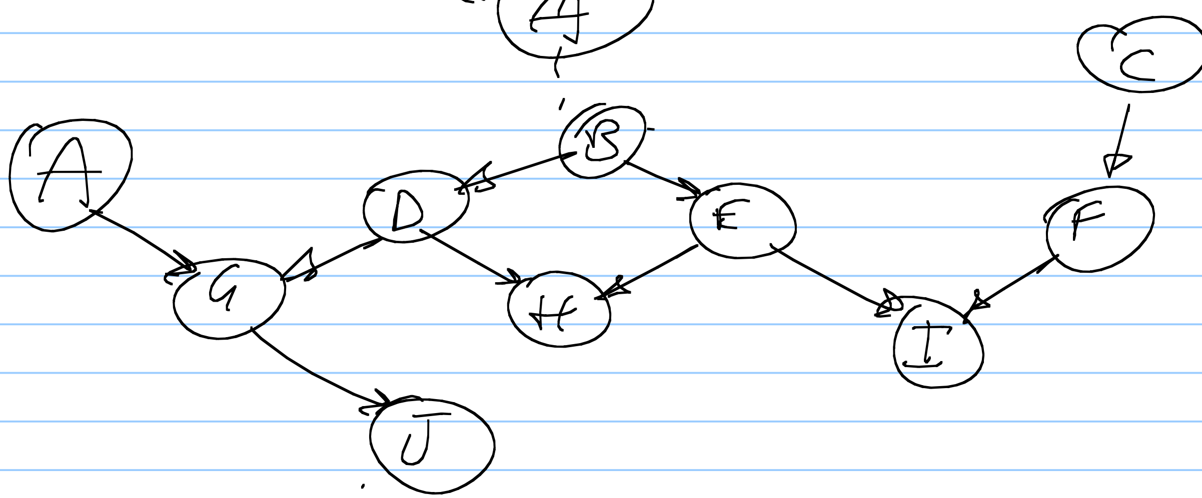


Fig. 2.19



Exercise 2.12

$P(A)$, $P(B)$, $P(C|A,B)$, $P(D|A,C)$, $P(E|B,D,F)$, $P(F|A)$.