Note Title 1/30/2009

In a BN (V, E, P) (as defined by Neapolitan—see slides),

$$P(V) = \prod_{v \in V} P(v \mid c(v))$$

(notation, more commonly $c(v)$ is written $\pi(v)$, the parents of v).

Proof.

Order the variables in (V, E) in reverse topological order, $\langle v_1, v_2, \ldots, v_n \rangle$.

$$P(v_1, v_2, \ldots, v_n) \overset{\text{by the fundamental rule}}{=} P(v_1 \mid v_2, \ldots, v_n)\, P(v_2, \ldots, v_n) =$$

(v_1 has no descendants in $\{v_1, \ldots, v_n\}$) $= P(v_1 \mid \pi(v_1))\, P(v_2, \ldots, v_n) =$

By defn of BN: prob of a node given its non-descendants is the prob of that node given just the parents

$$= P(v_1 \mid \pi(v_1))\, P(v_2 \mid v_3, \dots, v_n)\, P(v_3, \dots v_n) =$$

$$= P(v_1 \mid \pi(v_1)) \times P(v_2 \mid \pi(v_2)) \times P(v_3, \dots v_n) = \dots$$

$$= P(v_1 \mid \pi(v_1)) \times P(v_2 \mid \pi(v_2)) \times \dots \times P(v_n).$$