Note Title 1/23/2009

Example with Defn. 1.3 (Conditional independence for variables)

$P(A|C,B) = P(A|B) \triangleq$ A is independent of C given B

$\triangleq A \perp\!\!\!\perp C \mid B$ (Kjaerulff & Madsen notation)

Independence is symmetric ($A \perp\!\!\!\perp C|B$ iff $C \perp\!\!\!\perp A|B$)

$P(A|C,B) = P(A|B)$ iff $P(C|A,B) = P(C|B)$

defn of Cond Prop $\quad A \perp\!\!\!\perp C|B$

$$P(A|C,B) \overset{\downarrow}{=} \frac{P(A,C,B)}{P(C,B)} \overset{\downarrow}{=} \frac{P(A,B)}{P(B)}$$

$$P(C|A,B) = \frac{P(A,B,C)}{P(A,B)} = \frac{P(C,B)}{P(B)} \quad (2)$$

$$P(A,C,B) = \frac{P(A,B)}{P(B)} \times P(C,B) = \frac{P(\cancel{A,B})}{\cancel{P(B)}} \times \frac{P(A,B,C)}{\cancel{P(A,B)}} \times \cancel{P(B)}$$

I used (2) to rewrite $P(C,B)$ ✓

(May need to deal with possible division by zero.)

With variables, we define indep. of A ~~and B~~ given B as:

$P(a_i \mid c_k, b_j) = P(a_i \mid b_j)$ for each $a_i \in sp(A)$, $b_j \in sp(B)$, $c_k \in sp(C)$

The example on p. 11 uses Table 1.2 on p. 8

	b_1	b_2	b_3
a_1	.16	.12	.12
a_2	.24	.28	.08

$= P(A,B)$ $\quad P(B) = (0.4, 0.4, 0.2)$ (p. 9)

From $P(A,B)$, you can get $P(A \mid B)$: Table 1.1 $\frac{P(A,B)}{P(B)}$

	b_1	b_2	b_3
a_1	.4	.3	.6
a_2	.6	.7	.4

Always place the states (values) of the conditioning variable (here B) as column labels.

In this way, the numbers in each column sum to 1.
(Also, this is the way Hugin displays probability tables.)

$P(A|C,B)$ is not given. We could obtain it from $P(A,C|B)$ (table 1.6). Multiply that and obtain $P(A,B,C)$. Obtain $P(B,C) = \sum_A P(A,B,C)$ (i.e, marginalizing A out of $P(A,B,C)$),

Obtain $P(A|B,C) = \frac{P(A,B,C)}{P(B,C)}$.

Check that every entry in $P(A,B,C)$ where A and B have the same value is the same (does not depend on C).

Evidence (simple case) is a statement that a variable is in a particular state

Conditioning on evidence is most easily done by replacing the entries in a prob. table for variable states different from the evidence state with ϕ and then normalizing.

Ex.

	b_1	b_2	b_3
a_1	0.16	0.12	0.12
a_2	0.24	0.28	0.08

$= P(A,B)$

Enter the evidence $e = (B = b_2)$

$P(A|B=b_2) = P(A,B,e) =$

	b_1	b_2	b_2
a_1	0	0.12	0
a_2	0	0.28	0

$\Longrightarrow$

$$\frac{.12}{.4} = .12 \times 2.5 = .30$$

$$\frac{.28}{.4} = .28 \times 2.5 = .70$$

$$= P(A \mid B = b_2) = \frac{P(A,B)}{P(B=b_2)} = \frac{\boxed{\begin{matrix} 0.16 & .12 & .12 \\ .24 & .28 & .08 \end{matrix}}}{.4} = \begin{pmatrix} .3 \\ .7 \end{pmatrix}$$

in the b_2 place

Distributive law for potentials,

if $\mathrm{dom}(\varphi_1) \subseteq V$, then $(\varphi_1 \varphi_2)^{\downarrow V} = \varphi_1 (\varphi_2^{\downarrow V})$

Section 1.5 Random variables

A random variable is neither!

Please study Ch. 1.

Next time, Ch. 2!